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移动环境下具有时滞的Lotka-Volterra合作模型的行波解

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摘 要

本文研究了移动环境下具有非局部扩散和时滞的Lotka-Volterra合作模型行波解的存在性。通过构造一对合适的上下解，再利用单调迭代，证明了当环境运动速度$c > \max\{c_1^*, c_2^*, 0\}$ 时，系统存在行波解。

关键词

Lotka-Volterra 合作模型，移动环境，时滞

Traveling Wave Solutions of a Lotka-Volterra Cooperative Model with Time Delay in a Shifting Environment

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Abstract

In this paper, we study the existence and uniqueness of forced traveling wave solution for Lotka-Volterra cooperative model with nonlocal diffusion and time delay in a shifting environment. By constructing a pair of appropriate upper and lower solutions and using the monotone iteration, we prove that there is a traveling wave solution if the speed of the environmental movement $c > \max\{c_1^*, c_2^*, 0\}$.

Keywords

Lotka-Volterra Cooperative Model, Shifting Environment, Time Delay

1. 引言

合作在自然界中是普遍存在的. 在生物数学中, Lotka-Volterra 模型通常被用来描述物种之间的合作关系. 气候变化引起的环境空间异质性会影响物种 [1] 之间的合作. 特别地, H. Berestycki [2] 考虑了如下的反应扩散方程来研究气候变化对物种扩散的影响：

$$\frac{\partial u}{\partial t} = d\frac{\partial^2 u}{\partial x^2} + f(u, x - ct), t > 0, x \in \mathbb{R}, \tag{1.1}$$

其中 $c > 0$为气候变化速率, $u(t,x)$ 为t时刻x 位置的种群密度, $d > 0$ 为扩散速率. 与系统(1.1) 相关的反应扩散方程行波解的存在性、稳定性和唯一性已经得到了广泛的研究 [3–5]. 系统(1.1)描述了单个物种在移动环境中的物种动态, 但不涉及物种之间的相互作用. 最近, Yang 和Wu [6] 研究了移动环境中的Lotka-Volterra 合作系统：

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = d_1 \frac{\partial^2 u(x,t)}{\partial x^2} + u(x,t)\left[r_1(x-ct) - u(x,t) + a_1 v(x,t)\right] \\ \frac{\partial v(x,t)}{\partial t} = d_2 \frac{\partial^2 v(x,t)}{\partial x^2} + v(x,t)\left[r_2(x-ct) - v(x,t) + a_2 u(x,t)\right] \end{cases}, t > 0, x \in \mathbb{R}, \tag{1.2}$$

其中所有参数都是正数, $a_i, i = 1, 2$表示物种间的合作强度, 内禀增长函数$r_i(.)$ 是一个连续的非减函数, 并且满足$-\infty < L_i = r_i(-\infty) < 0 < r_i(+\infty) = K_i < +\infty, i = 1, 2$. 通过构造合适的上下解和单调迭代方法, 证明了强迫波的存在性和渐近行为. 以上模型的更多结果参见 [7–9].

由于Laplace算子只能描述局部的人口相互作用. 然而, 在现实中种群的运动是随机的, 并不仅仅与相邻种群相互作用. 由于外力可能与远方种群发生相互作用, 参见 [10]. 因此, 许多研究者使用非局部扩散来描述物种的空间运动模式. 非局部扩散通常用卷积算子来表示:

$$[J*u-u](t,x)=\int_{\mathbb{R}}J(x-y)u(t,y)dy-u(t,x),$$

其中为概率密度函数. 最近, Wang和Li [11] 研究了具有非局部扩散的Lotka-Volterra合作系统:

$$\begin{cases}\frac{\partial u(x,t)}{\partial t}=d_1\left[\left(J_1*u\right)(x,t)-u(x,t)\right]+u(x,t)\left[r_1(x-ct)-u(x,t)+a_1v(x,t)\right]\\ \frac{\partial v(x,t)}{\partial t}=d_2\left[\left(J_2*v\right)(x,t)-v(x,t)\right]+v(x,t)\left[r_2(x-ct)-v(x,t)+a_2u(x,t)\right]\end{cases},t>0,x\in\mathbb{R}. \tag{1.3}$$

这里 $a_1a_2<1$表示两个物种之间的弱合作, 内禀增长函数 $r_i(.)$和核函数 $J_i(.)$满足以下假设:

(H1) $r_i(.)$是$\mathbb{R}$中的连续不减函数, $r_i(\pm\infty)$ 是有限的, 并满足$r_i(-\infty)<0<r_i(+\infty)$.

(J1) $J_i(.)\in C(\mathbb{R}),J(x)\geq 0,J(0)>0,\int_{\mathbb{R}}J_i(x)\mathrm{d}x=1$, 并对任意$\mu>0$ 有 $\int_{\mathbb{R}}J_i(x)e^{\mu|x|}\mathrm{d}x<\infty$.

作者通过经典的单调迭代和上下解方法得到了连接零平衡点和共存态的强迫波的存在性. 对上述模型的扩展, 如改变种间系数的符号和减弱假设条件等, 我们可以参考 [12–16].

众所周知, 在现实生活中由于怀孕、孵化和成熟等因素的影响, 时滞往往是不可避免的. 因此, 在反应扩散方程中加入时滞更具有实际意义. 许多具有时滞的反应扩散方程已被广泛研究, 参见 [17–26]. 例如, Yu和Yuan [27] 利用交互迭代技巧和Schauder 不动点定理研究了如下非局部时滞Lotka-Volterra竞争模型行波解的存在性:

$$\begin{cases}\frac{\partial u(x,t)}{\partial t}=d_1\left[\left(J_1*u\right)(x,t)-u(x,t)\right]+f_1\left(u_t(x),v_t(x)\right)\\ \frac{\partial v(x,t)}{\partial t}=d_2\left[\left(J_2*v\right)(x,t)-v(x,t)\right]+f_2\left(u_t(x),v_t(x)\right)\end{cases},t>0,x\in\mathbb{R}, \tag{1.4}$$

其中 $u_t(x)(s)=u(x,t+s),-\tau\leq s\leq 0,\tau$ 为最大时滞, $J_i:\mathbb{R}\to\mathbb{R}$ 和$\int_{\mathbb{R}}J_i(y)dy=1$为所有 $i=1,2$, $f_i:\mathbb{R}^2\to\mathbb{R},i=1,2$ 为连续函数且满足弱拟单调性(WQM) 或弱指数拟单调性(WQM^*). 在现有的文献中, 很少有同时考虑带有非局部扩散和时滞的反应扩散方程。对于单个物种, Cheng和Yuan [28]得到了强迫行波解的存在唯一性. 同样地, 对于二维情况, 同时考虑移动环境和时滞更具有实际意义.

受上述文献的启发, 我们考虑在移动环境中带有非局部扩散和时滞的Lotka-Volterra合作模型:

$$\begin{cases}\frac{\partial u(t,x)}{\partial t}=d_1\left[J_1*u-u\right](t,x)+u(t,x)\left[r_1(x+ct)-u\left(t,x\right)+a_1v\left(t-\tau_1,x\right)\right],\\ \frac{\partial v(t,x)}{\partial t}=d_2\left[J_2*v-v\right](t,x)+v(t,x)\left[r_2(x+ct)-v\left(t,x\right)+a_2u\left(t-\tau_2,x\right)\right].\end{cases} \tag{1.5}$$

其中$c>0,x\in\mathbb{R},d_i>0,u(t,x)$和 $v(t,x)$ 表示两个合作物种, 时滞 $\tau_i,i=1,2$ 为非负常数, $a_i,i=1,2$ 表示种群间的合作强度. 在接下来的讨论中, 我们做了如下假设.

(A1) $0<a_1a_2<1$.

(A2) $r_i(.)$是$\mathbb{R}$中的连续不增函数, $r_i(\pm\infty)$ 有限, 且 $r_i(+\infty)<0<r_i(-\infty)$.

(A3) $J_i\in C\left(\mathbb{R},\mathbb{R}\right)$, 并满足$\int_{\mathbb{R}}J_i(x)dx=1$, 对任意 $\mu>0$有 $\int_{\mathbb{R}}J_i(x)e^{\mu|x|}\mathrm{d}x<\infty$.

此外，我们给予下列假设.

(A4) $r_1(+\infty) < -a_1k_2$, $r_2(+\infty) < -a_2k_1$.

这意味着在正无穷远处，两个物种的环境恶化的非常严重. 我们主要研究系统(1.5)行波解的存在性. 系统(1.5)的行波解是具有如下特殊形式的平移不变解：

$$(u,v)(t,x) = (U,V)(\xi), \xi := x + ct. \tag{1.6}$$

将(1.6)代入(1.5), 我们可以得到如下的波剖系统:

$$\begin{cases} cU'(\xi) = d_1\left[J_1 * U(\xi) - U(\xi)\right] + U(\xi)\left[r_1(\xi) - U\left(\xi\right) + a_1V\left(\xi - c\tau_1\right)\right], \\ cV'(\xi) = d_2\left[J_2 * V(\xi) - V(\xi)\right] + V(\xi)\left[r_2(\xi) - V\left(\xi\right) + a_2U\left(\xi - c\tau_2\right)\right], \end{cases} \xi \in \mathbb{R}. \tag{1.7}$$

通过计算，我们发现式(1.7) 的极限系统在负无穷远处存在唯一的正平衡点$E^*\left(k_1,k_2\right)$, 其中

$$k_1 = \frac{r_1(-\infty) + a_1r_2(-\infty)}{1 - a_1a_2}, \quad k_2 = \frac{r_2(-\infty) + a_2r_1(-\infty)}{1 - a_1a_2}.$$

我们将证明连接平衡态$E^*\left(k_1,k_2\right)$ 和$E_0(0,0)$ 的行波解, 并且满足边界条件

$$\begin{cases} \lim_{\xi\to-\infty}(U(\xi),V(\xi)) = \left(k_1,k_2\right), \\ \lim_{\xi\to\infty}(U(\xi),V(\xi)) = (0,0). \end{cases} \tag{1.8}$$

论文的其余部分组织如下. 在第二节中, 我们介绍了一些准备引理. 在第三节中, 通过构造合适的上下解, 然后再利用单调迭代, 我们证明了系统行波解的存在性.

2. 预备知识

我们首先介绍一些记号. 对于任意的$u = (u_1,u_2)$ 和$v = (v_1,v_2)$, 若$u_i \leqslant v_i, i = 1,2$, 则记$u \leqslant v$, 若$u \leqslant v$ 但$u \neq v$, 则记$u < v$.

定义2.1. *若存在一列有限点列$\{\xi_j\}_{j=1}^N$ 使得连续函数$(U(\xi),V(\xi))$在$\mathbb{R}\backslash\{\xi_j\}$ 上满足*

$$\begin{cases} cU'(\xi) \geq (\leq)d_1\left[J_1 * U(\xi) - U(\xi)\right] + U(\xi)\left[r_1(\xi) - U\left(\xi\right) + a_1V\left(\xi - c\tau_1\right)\right], \\ cV'(\xi) \geq (\leq)d_2\left[J_2 * V(\xi) - V(\xi)\right] + V(\xi)\left[r_2(\xi) - V\left(\xi\right) + a_2U\left(\xi - c\tau_2\right)\right]. \end{cases}$$

其中$\xi \in \mathbb{R}$, 且$(U(\xi),V(\xi))$ 在ξ_j处连续, 则称$(U(\xi),V(\xi))$ 为系统(1.7)的上(下)解.

我们首先构造一对上下解. 定义函数

$$\Delta_i(\lambda,c) := d_i\left[\int_{\mathbb{R}} J_i(y)e^{-\lambda y}dy - 1\right] - c\lambda + r_i(-\infty), i = 1,2. \tag{2.1}$$

以及参数

$$c_i^*(\infty) := \inf_{\lambda>0} \frac{d_i\left(\int_{\mathbb{R}} J_i(y)e^{-\lambda y}\,\mathrm{d}y - 1\right) + r_i(-\infty)}{\lambda}$$

由函数的凸性可知, 有以下结论成立.

引理2.2 对任意$c > \max\{c_1^*, c_2^*\}$，$\Delta_1(\lambda, c)$ 和$\Delta_2(\lambda, c)$ 分别有两个不同的正根λ_1, λ_2 和λ_3, λ_4 ，并满足

$$\Delta_1(\lambda, c) = \begin{cases} > 0, \lambda < \lambda_1 \\ < 0, \lambda \in (\lambda_1, \lambda_2) \\ > 0, \lambda > \lambda_2 \end{cases}, \quad \Delta_2(\lambda, c) = \begin{cases} > 0, \lambda < \lambda_3 \\ < 0, \lambda \in (\lambda_3, \lambda_4) \\ > 0, \lambda > \lambda_4 \end{cases}.$$

令$l_1(\xi) = e^{\lambda_1 \xi} - q e^{\eta \lambda_1 \xi}, l_2(\xi) = e^{\lambda_3 \xi} - q e^{\eta \lambda_3 \xi}$, 其中常数 $\eta \in \left(1, \min\left\{2, \frac{\lambda_2}{\lambda_1}, \frac{\lambda_4}{\lambda_3}\right\}\right)$, $q > 1$. 易知$l_i(\xi_i) = 0$, 此时$\xi_i = \frac{1}{\lambda_i(\eta-1)} \ln \frac{1}{q} < 0$. 选择 η 使得 $\Delta_1(\lambda_1 \eta, c) < 0$ 和 $\Delta_2(\lambda_3 \eta, c) < 0$. 由此定义连续函数

$$\underline{U}(\xi) = \begin{cases} k_1 \left(e^{\lambda_1 \xi} - q e^{\eta \lambda_1 \xi}\right), \xi \le \xi_1 \\ 0, \xi > \xi_1 \end{cases}, \underline{V}(\xi)) = \begin{cases} k_2 \left(e^{\lambda_3 \xi} - q e^{\eta \lambda_3 \xi}\right), \xi \le \xi_2 \\ 0, \xi > \xi_2 \end{cases}.$$

引理2.3 对任意$c > \max\{c_1^*, c_2^*\}$, 当 $q > 1$ 足够大时，$(\underline{U}(\xi), \underline{V}(\xi))$ 是系统(1.7)的一个下解.

证明. 方便起见, 令

$$P_1(\xi) := d_1 [J_1 * \underline{U}(\xi) - \underline{U}(\xi)] - c\underline{U}'(\xi) + \underline{U}(\xi) [r_1(\xi) - \underline{U}(\xi) + a_1 \underline{V}(\xi - c\tau_1)],$$

$$P_2(\xi) := d_2 [J_2 * \underline{V}(\xi) - \underline{V}(\xi)] - c\underline{V}'(\xi) + \underline{V}(\xi) [r_2(\xi) - \underline{V}(\xi) + a_1 \underline{U}(\xi - c\tau_2)].$$

为了证明 $(\underline{U}(\xi), \underline{V}(\xi))$是一个下解, 只需证明 $P_1(\xi) \ge 0$ 和 $P_2(\xi) \ge 0$. 我们先证 $P_1(\xi) \ge 0$.

(i)当 $\xi > \xi_1$，$\underline{U}(\xi) = 0$, 此时 $P_1(\xi) = 0$.

(ii)当 $\xi \le \xi_1$，$\underline{U}(\xi) = k_1 \left(e^{\lambda_1 \xi} - q e^{\eta \lambda_1 \xi}\right)$，$\underline{V}(\xi - c\tau_1) = k_2 \left(e^{\lambda_3(\xi - c\tau_1)} - q e^{\eta \lambda_3 (\xi - c\tau_1)}\right) > 0$ 则

$$\begin{aligned}
P_1(\xi) &= k_1 e^{\lambda_1 \xi} \left[d_1 \left(\int_{\mathbb{R}} J(y) e^{-\lambda_1 y} dy - 1\right) - c\lambda_1 + r_1(\xi)\right] \\
&\quad - k_1 q e^{\eta \lambda_1 \xi} \left[d_1 \left(\int_{\mathbb{R}} J(y) e^{-\eta \lambda_1 y} dy - 1\right) - c\eta\lambda_1 + r_1(\xi)\right] + U(\xi) [-U(\xi) + a_1 V(\xi - c\tau_1)] \\
&\ge k_1 e^{\lambda_1 \xi} \left[d_1 \left(\int_{\mathbb{R}} J(y) e^{-\lambda_1 y} dy - 1\right) - c\lambda_1 + r_1(-\infty) - r_1(-\infty) + r_1(\xi)\right] \\
&\quad - k_1 q e^{\eta \lambda_1 \xi} \left[d_1 \left(\int_{\mathbb{R}} J(y) e^{-\eta \lambda_1 y} dy - 1\right) - c\eta\lambda_1 + r_1(-\infty) - r_1(-\infty) + r_1(\xi)\right] \\
&\quad - \left(k_1 e^{\lambda_1 \xi} - k_1 q e^{\eta \lambda_1 \xi}\right)^2 \\
&\ge -k_1 e^{\lambda_1 \xi} [r_1(-\infty) - r_1(\xi)] + k_1 q e^{\eta \lambda_1 \xi} [r_1(-\infty) - r_1(\xi)] - k_1 q e^{\eta \lambda_1 \xi} [\Delta_1(\lambda_1 \eta, c)] - k_1^2 e^{2\lambda_1 \xi} \\
&\ge -k_1 e^{\eta \lambda_1 \xi} \left[q \Delta_1(\lambda_1 \eta, c) + k_1 e^{\lambda_1 \xi (2-\eta)}\right] \\
&= -k_1 e^{\eta \lambda_1 \xi} \left[q \Delta_1(\lambda_1 \eta, c) + \frac{1}{q} k_1 e^{\frac{2-\eta}{\eta-1}}\right] \\
&\ge 0.
\end{aligned}$$

当 $q > 1$ 足够大时, 最后一个不等式成立. 用类似的方法，我们可以证明对任意的$\xi \in \mathbb{R}$,

$P_2(\xi) \geq 0$. 所以, $(\underline{U}(\xi), \underline{U}(\xi))$ 是系统(1.7)的一个下解, 证毕. □

由假设 (A3), 我们可以选择 $\xi_i^0 > 0$ 足够大, 其中 $i = 12$使得$r_1(\xi_i^0) + a_1 k_2 < 0$, $r_2(\xi_i^0) + a_2 k_1 < 0$. 定义

$$h_i(\mu) = d_i \left[\int_{\mathbb{R}} J_i(y) e^{\mu y} dy - 1 \right] + c\mu + r_i\left(\xi_i^0\right) + a_i k_j, i \neq j \in \{1, 2\}$$

易知 $h_i(0) < 0$, 当 $\mu \to \infty$时 $h_i(\mu) \to +\infty$. 又对任意 $\mu \in \mathbb{R}$ 有 $h_i(\mu) = d_i \int_{\mathbb{R}} J_i(y) y^2 e^{\mu y} dy \geq 0$, 这意味着存在 $\mu_i > 0$ 使得 $h_i(\mu_i) = 0$. 由此定义连续函数

$$\overline{U}(\xi) = \begin{cases} k_1 e^{-\mu_1\left(\xi - \xi_1^0\right)}, \xi \geq \xi_1^0 \\ k_1, \xi < \xi_1^0 \end{cases}, \overline{V}(\xi) = \begin{cases} k_2 e^{-\mu_2\left(\xi - \xi_2^0\right)}, \xi \geq \xi_2^0. \\ k_2, \xi < \xi_2^0 \end{cases}.$$

引理2.4 对任意$c > 0$, $(\overline{U}(\xi), \overline{U}(\xi)) \geq (\underline{U}(\xi), \underline{U}(\xi))$是系统(1.7)的一个上解.

证明. 因为 $\xi_1^0 \geq 0 \geq \xi_i$, 所以$(\overline{U}(\xi), \overline{U}(\xi)) \geq (\underline{U}(\xi), \underline{U}(\xi))$. 下面证明$(\overline{U}(\xi), \overline{V}(\xi))$是系统(1.7)的一个上解, 只需证明

$$Q_1(\xi) := d_1\left[J_1 * \overline{U}(\xi) - \overline{U}(\xi)\right] - c\overline{U}'(\xi) + \overline{U}(\xi)\left[r_1(\xi) - \overline{U}(\xi) + a_1 \overline{V}(\xi - c\tau_1)\right] \leq 0,$$

$$Q_2(\xi) := d_2\left[J_2 * \overline{V}(\xi) - \overline{V}(\xi)\right] - c\overline{V}'(\xi) + \overline{V}(\xi)\left[r_2(\xi) - \overline{V}(\xi) + a_1 \overline{U}(\xi - c\tau_2)\right] \leq 0.$$

我们先证$Q_1(\xi) \leq 0$. (i) 当 $\xi < \xi_1^0$ 时, $\overline{U}(\xi) = k_1, \overline{V}(\xi - c\tau_1) \leq k_2$, 有

$$Q_1(\xi) = k_1\left[r_1(\xi) - k_1 + a_1 \overline{V}(\xi - c\tau_1)\right] \leq k_1\left[r_1(-\infty) - k_1 + a_1 k_2\right] = 0$$

(ii)当 $\xi \geq \xi_1^0$ 时, $\overline{U}(\xi) = k_1 e^{-\mu_1\left(\xi - \xi_1^0\right)}, \overline{V}(\xi - c\tau_1) \leq k_2$, 有

$$\begin{aligned} Q_1(\xi) &= k_1 e^{-\mu_1\left(\xi - \xi_1^0\right)}\left[d_1\left(\int_{\mathbb{R}} J(y) e^{\mu_1 y} dy - 1\right) + c\mu_1 + r_1(\xi) - k_1 e^{-\mu_1\left(\xi - \xi_1^0\right)} + a_1 \overline{V}(\xi - c\tau_1)\right] \\ &\leq k_1 e^{-\mu_1\left(\xi - \xi_1^0\right)}\left[d_1\left(\int_{\mathbb{R}} J(y) e^{\mu_1 y} dy - 1\right) + c\mu_1 + r_1\left(\xi_1^0\right) + a_1 k_2\right] \\ &= 0. \end{aligned}$$

用类似的方法，我们可以证明对任意的$\xi \in \mathbb{R}$, $Q_2(\xi) \leq 0$. 所以, $(\overline{U}(\xi), \overline{U}(\xi))$是系统(1.7)的一个上解, 证毕. □

现在我们定义波形剖面集 Γ:

$$\Gamma : \left\{(U(\xi), V(\xi)) \in C(\mathbb{R}, \mathbb{R}) : (\overline{U}(\xi), \overline{V}(\xi)) \geq (U(\xi), V(\xi)) \geq (\underline{U}(\xi), \underline{V}(\xi))\right\} \tag{2.2}$$

选定常数 $\beta_1 = d_1 + 2k_1 - r_1(+\infty), \beta_2 = d_2 + 2k_2 - r_2(+\infty)$, 定义算子:

$$\begin{aligned} H_1(U, V) &:= d_1 J_1 * U(\xi) + (\beta_1 - d_1) U(\xi) + U(\xi)\left[r_1(\xi) - U(\xi) + a_1 V(\xi - c\tau_1)\right], \\ H_2(U, V) &:= d_2 J_2 * V(\xi) + (\beta_2 - d_2) V(\xi) + V(\xi)\left[r_2(\xi) - V(\xi) + a_2 U(\xi - c\tau_2)\right]. \end{aligned}$$

得到如下积分方程,

$$\begin{cases} U(\xi)=\frac{1}{c}\int_{-\infty}^{\xi} H_1(U,V)(z)e^{\frac{\beta_1}{c}(z-\xi)}dz := F_1(U,V)(\xi), \\ V(\xi)=\frac{1}{c}\int_{-\infty}^{\xi} H_2(U,V)(z)e^{\frac{\beta_2}{c}(z-\xi)}dz := F_2(U,V)(\xi). \end{cases} \tag{2.3}$$

由此定义算子 $F(U(\xi),V(\xi))=(F_1(U(\xi),V(\xi)),F_2(U(\xi),V(\xi))),i=1,2$. 此时, 系统(1.7)解的存在性问题转化为算子F 的不动点的存在性问题. 因此, 我们有以下结果.

引理2.5 当 $c>\max\{c_1^*,c_2^*,0\}$，则 F 是一个非减算子且 $F(\Gamma)\subseteq\Gamma$. 进一步地, 如果 $(U,V)\in\Gamma$是非增的，那么 $F(U,V)(\xi)$关于 $\xi\in\mathbb{R}$ 也是非增的.

证明. 首先证明F是一个非减算子. 对任意 (U_1,V_1), $(U_2,V_2)\in\Gamma$ 且 $(U_1,V_1)\geq(U_2,V_2)$, 则有

$$\begin{aligned}
&H_1\left(U_1,V_1\right)(\xi)-H_1\left(U_2,V_2\right)(\xi)\\
=&d_1\left[\int_{\mathbb{R}}J_1(y)\left(U_1(\xi-y)-U_2(\xi-y)\right)dy\right]+\left(U_1(\xi)-U_2(\xi)\right)\left(\beta_1-d_1+r_1(\xi)\right)\\
&+a_1\left[U_1(\xi)V_1\left(\xi-c\tau_1\right)-U_2(\xi)V_2\left(\xi-c\tau_1\right)\right]-\left[U_1(\xi)^2-U_2(\xi)^2\right]\\
\geq&d_1\left[\int_{\mathbb{R}}J_1(y)\left(U_1(\xi-y)-U_2(\xi-y)\right)dy\right]+\left(U_1(\xi)-U_2(\xi)\right)\left[\beta_1-d_1+r_1(\xi)-U_1(\xi)-U_2(\xi)\right]\\
\geq&0.
\end{aligned}$$

用类似的方法可得, $H_2\left(U_1,V_1\right)(\xi)-H_2\left(U_2,V_2\right)(\xi)\geq 0$. 从$F_1$和$F_2$的定义可知：

$$F_i\left(U_1,V_1\right)(\xi)-F_i\left(U_2,V_2\right)(\xi)=\frac{1}{c}\int_{-\infty}^{\xi}\left[H_i\left(U_1,V_1\right)(z)-H_i\left(U_2,V_2\right)(z)\right]e^{\frac{\beta_i}{c}(z-\xi)}dz\geq 0, i=1,2.$$

因此，F 是一个非减算子.

如果$(U,V)\in\Gamma$ 是非增函数，则对任意$s\geq 0,\xi\in\mathbb{R}$，我们有

$$\begin{aligned}
&H_1(U,V)(\xi+s)-H_1(U,V)(\xi)\\
=&d_1\left[\int_{\mathbb{R}}J_1(y)(U(\xi+s-y)-U(\xi-y))dy\right]+\left(U(\xi+s)-U(\xi)\right)\left(\beta_1-d_1\right)\\
&+U(\xi+s)r_1(\xi+s)-U(\xi)r_1(\xi)+a_1\left[U(\xi+s)V\left(\xi+s-c\tau_1\right)-U(\xi)V\left(\xi-c\tau_1\right)\right]\\
&-\left[U(\xi+s)^2-U(\xi)^2\right]\\
\leq&d_1\left[\int_{\mathbb{R}}J_1(y)(U(\xi+s-y)-U(\xi-y))dy\right]+\left(U(\xi+s)-U(\xi)\right)\left(\beta_1-d_1-U(\xi+s)-U(\xi)\right)\\
&+U(\xi+s)r_1(\xi+s)-U(\xi)r_1(\xi+s)\\
=&d_1\left[\int_{\mathbb{R}}J_1(y)(U(\xi+s-y)-U(\xi-y))dy\right]\\
&+\left(U(\xi+s)-U(\xi)\right)\left(\beta_1-d_1+r_1(\xi+s)-U(\xi+s)-U(\xi)\right)\\
\leq&0.
\end{aligned}$$

类似的有 $H_2(U,V)(\xi+s)-H_2(U,V)(\xi)\le 0$. 因此,

$$\begin{aligned}F_i(U,V)(\xi+s) &= \frac{1}{c}\int_{-\infty}^{\xi+s} H_i(U,V)(z)e^{\frac{\beta_i}{c}(z-(\xi+s))}dz\\ &= \frac{1}{c}\int_{-\infty}^{\xi} H_i(U,V)(z+s)e^{\frac{\beta_i}{c}(z-\xi)}dz\\ &\le \frac{1}{c}\int_{-\infty}^{\xi} H_i(U,V)(z)e^{\frac{\beta_i(z-\xi)}{c}}dz\\ &= F_i(U,V)(\xi).\end{aligned}$$

下面证明 $F(\Gamma)\subseteq\Gamma$. 根据上解和F的定义, 可以得到

$$\begin{aligned}&F_1(\overline{U},\overline{V})(\xi)\\ &= \frac{1}{c}\int_{-\infty}^{\xi} H_1(\overline{U},\overline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\\ &= \frac{1}{c}\left[\int_{-\infty}^{\xi_N} H_1(\overline{U},\overline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz+\cdots+\int_{\xi_2}^{\xi_1} H_1(\overline{U},\overline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz+\int_{\xi_1}^{\xi} H_1(\overline{U},\overline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\right]\\ &\le \frac{1}{c}\left[\int_{-\infty}^{\xi_N}\left(c\overline{U}'(z)+\beta_1\overline{U}(z)\right)e^{\frac{\beta_1}{c}(z-\xi)}dz+\cdots+\int_{\xi_1}^{\xi}\left(c\bar{U}'(z)+\beta_1\overline{U}(z)\right)e^{\frac{\beta_1}{c}(z-\xi)}dz\right]\\ &= \overline{U}(\xi),\end{aligned}$$

$$\begin{aligned}&F_2(\overline{U},\overline{V})(\xi)\\ &= \frac{1}{c}\int_{-\infty}^{\xi_N} H_2(\overline{U},\overline{V})(z)e^{\frac{\beta_1(z-\xi)}{c}}dz+\cdots+\int_{\xi_2}^{\xi_1} H_2(\overline{U},\overline{V})(z)e^{\frac{\beta_1(z-\xi)}{c}}dz+\int_{\xi_1}^{\xi} H_2(\overline{U},\overline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\Bigg]\\ &\le \overline{V}(\xi).\end{aligned}$$

这表明 $F(\overline{U}(\xi),\overline{V}(\xi))\le(\overline{U}(\xi),\overline{V}(\xi))$. 同样, 根据下解和$F$的定义可知

$$\begin{aligned}&F_1(\underline{U},\underline{V})(\xi)\\ &= \frac{1}{c}\int_{-\infty}^{\xi} H_1(\underline{U},\underline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\\ &= \frac{1}{c}\left[\int_{-\infty}^{\xi_N} H_1(\underline{U},\underline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz+\cdots+\int_{\xi_2}^{\xi_1} H_1(\underline{U},\underline{V})(z)e^{\frac{\hat{\beta}_1(z-\xi)}{c}})dz+\int_{\xi_1}^{\xi} H_1(\underline{U},\underline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\right]\\ &\ge \frac{1}{c}\left[\int_{-\infty}^{\xi_N}\left(c\underline{U}'(z)+\beta_1\underline{U}(z)\right)e^{\frac{\beta_1}{c}(z-\xi)}dz+\cdots+\int_{\xi_1}^{\xi}\left(c\underline{U}'(z)+\beta_1\underline{U}(z)\right)e^{\frac{\beta_1}{c}(z-\xi)}dz\right]\\ &= \underline{U}(\xi)\end{aligned}$$

$$
\begin{aligned}
&F_2(\underline{U},\underline{V})(\xi)\\
&=\frac{1}{c}\left[\int_{-\infty}^{\xi_N} H_2(\underline{U},\underline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz+\cdots+\int_{\xi_2}^{\xi_1} H_2(\underline{U},\underline{V})(z)e^{-\frac{\beta_1}{c}(z-\xi)}dz+\int_{\xi_1}^{\xi} H_2(\underline{U},\underline{V})(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\right]\\
&\geq \underline{V}(\xi)
\end{aligned}
$$

这表明 $F(\underline{U},\underline{V})\geq(\underline{U},\underline{V})$. 由于 F 是非减算子, 那么对所有的 $(U,V)\in\Gamma$，我们有

$$
(\underline{U},\underline{V})\leq F(\underline{U},\underline{V})\leq F(U,V)\leq F(\overline{U}(\xi),\overline{V}(\xi))\leq(\overline{U}(\xi),\overline{V}(\xi)), \tag{2.4}
$$

因此, $F(\Gamma)\subseteq\Gamma$. 证毕. □

3. 行波解的存在性

定理3.1. *假设(A1)-(A4)成立, 则对于任意给定的 $c>\max\{c_1^*,c_2^*,0\}$, 系统(1.7)存在连接 $E^*(k_1,k_2)$和 $E_0(0,0)$ 的行波解$(U(\xi),V(\xi))$, 且满足边界条件(1.8).*

证明. 考虑下面的迭代

$$
U_{n+1}=F_1\left(U_n,V_n\right), V_{n+1}=F_2\left(U_n,V_n\right), n\geq 1,
$$

选取初始迭代为

$$
U_1=F_1(\overline{U},\overline{V}), V_1=F_2(\overline{U},\overline{V}).
$$

由于$\overline{U}(\xi),\overline{V}(\xi)$是非增函数, 由引理2.5可知, 对每个给定的 n, $(U_n(\xi),V_n(\xi))$关于$\xi\in\mathbb{R}$是非增的. 令

$$
\lim_{n\to+\infty}(U_n(\xi),V_n(\xi))=(U(\xi),V(\xi)).
$$

显然, $(U(\xi),V(\xi))$ 是一个非增函数且

$$
(\underline{U}(\xi),\underline{V}(\xi))\leq(U(\xi),V(\xi))\leq(\overline{U}(\xi),\overline{V}(\xi)).
$$

此外, $H_i(U_n(\xi),V_n(\xi))$ 逐点收敛到 $H_i(U(\xi),V(\xi))$, 其中$i=1,2$. 由于

$$
H_1(U_n(\xi),V_n(\xi))\leq k_1\left[r_1(+\infty)+\beta_1+2d_1+k_1+a_1k_2\right],
$$

$$
H_2(U_n(\xi),V_n(\xi))\leq k_2\left[r_2(+\infty)+\beta_2+2d_2+k_2+a_2k_1\right].
$$

根据Lebesgue控制收敛定理, 我们得到

$$
\begin{aligned}
U(\xi)&=\lim_{n\to\infty}U_n(\xi)=\lim_{n\to\infty}F_1\left(U_{n-1},V_{n-1}\right)(\xi)\\
&=\lim_{n\to\infty}\frac{1}{c}\int_{-\infty}^{\xi}H_1\left(U_{n-1},V_{n-1}\right)(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\qquad.\\
&=\frac{1}{c}\int_{-\infty}^{\xi}H_1(U,V)(z)e^{\frac{\beta_1}{c}(z-\xi)}dz=F_1(U,V)(\xi)
\end{aligned}
$$

类似可得$V(\xi)=F_2(U,V)(\xi)$. 容易看出, $(U(\xi),V(\xi))\in C^1(\mathbb{R},\mathbb{R})$ 满足系统(1.7).

接下来我们将证明 $(U(\xi),V(\xi))$ 满足边界条件(1.8). 我们注意到

$$\lim_{\xi\to+\infty}(\overline{U}(\xi),\overline{V}(\xi))=\lim_{\xi\to+\infty}(\underline{U}(\xi),\underline{V}(\xi))=(0,0)\,.$$

这表明 $\lim_{\xi\to+\infty}(U(\xi),V(\xi))=(0,0)$. 由于 $(U(\xi),V(\xi))$ 是非增有界的, 因此存在常数 $A_i\in(0,k_i], i=1,2$ 使得

$$\lim_{\xi\to-\infty}(U(\xi),V(\xi))=(A_1,A_2).$$

则有

$$\lim_{\xi\to-\infty}H_1(U,V)(\xi)=\beta_1A_1+A_1\left[r_1(-\infty)-A_1+a_1A_2\right],$$
$$\lim_{\xi\to-\infty}H_2(U,V)(\xi)=\beta_2A_2+A_2\left[r_2(-\infty)-A_2+a_2A_1\right].$$

根据L'Hopital法则可得,

$$\begin{aligned}A_1&=\lim_{\xi\to-\infty}U(\xi)\\&=\lim_{\xi\to-\infty}\frac{1}{c}\int_{-\infty}^{\xi}H_1(U,V)(z)e^{\frac{\beta_1}{c}(z-\xi)}dz\\&=\lim_{\xi\to-\infty}\frac{H_1(U,V)(\xi)}{\beta_1}\\&=A_1+\frac{A_1\left[r_1(-\infty)-A_1+a_1A_2\right]}{\beta_1}\end{aligned}$$

$$\begin{aligned}A_2&=\lim_{\xi\to-\infty}V(\xi)\\&=\lim_{\xi\to-\infty}\frac{1}{c}\int_{-\infty}^{\xi}H_2(U,V)(z)e^{\frac{\beta_2}{c}(z-\xi)}dz\\&=\lim_{\xi\to-\infty}\frac{H_2(U,V)(\xi)}{\beta_2}\\&=A_2+\frac{A_2\left[r_2(-\infty)-A_2+a_2A_1\right]}{\beta_2}.\end{aligned}$$

所以有

$$\begin{cases}A_1[r_1(-\infty)-A_1+a_1A_2]=0,\\A_2[r_2(-\infty)-A_2+a_2A_1]=0.\end{cases}$$

对上述公式直接进行计算得

$$\begin{cases}A_1=\frac{r_1(-\infty)+a_1r_2(-\infty)}{1-a_1a_2},\\A_2=\frac{r_2(-\infty)+a_2r_1(-\infty)}{1-a_1a_2}.\end{cases}\tag{3.1}$$

即 $\lim_{\xi\to-\infty}(U(\xi),V(\xi))=(k_1,k_2)$. 所以 $(U(\xi),V(\xi))$ 满足边界条件(1.8). 证毕. □

4. 总结与展望

本文研究了在轻度恶化的环境下, 一类带有时滞的Lotka-Volterra合作系统行波解的存在性. 通过上下解结合单调迭代方法, 证明了该系统存在行波解. 此外, 本文的模型仍有一些待进一步研究的问题, 以下是对本文工作的研究展望:

1.假设条件(A2)中可取 $r_i(.)$不变号, 即 $0 \leq r_i(+\infty) < r_i(-\infty)$, 然后再考虑系统(1.7)是否存在行波解.

2.本文仅研究了系统(1.7)行波解的存在性, 可进一步研究该系统的唯一性、稳定性和不存在性.

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