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Stability Analysis for Neutral System with Time-Varying Delays and Nonlinear Perturbations

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Abstract

This paper studies the stability of neutral system with time-varying delays and nonlinear perturbations. The stability of the system with time-varying delays and nonlinear perturbations is analysed by choosing a proper Lyapunov-Krasovskii functional, applying the Linear Matrix Inequality (LMI), and using the Lyapunov-Krasovskii stabilization theorem.

Keywords

Nonlinear Perturbation, Lyapunov-Krasovskii Functional, Stability Analysis, Linear Matrix Inequality (LMI)

具有非线性扰动的时变时滞中立型系统的稳定性分析

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摘　要

本文研究具有非线性扰动的时变时滞中立性系统的稳定性。通过选取合适的Lyapunov-Krasovskii泛函，应用LMI不等式和Lyapunov-Krasovskii稳定性定理对时滞相关的非线性扰动系统进行稳定性分析。

关键词

非线性扰动，Lyapunov-Krasovskii泛函，稳定性分析，LMI不等式

1. 引言

时变时滞中立系统是指中立时滞和状态时变时滞均为时变的中立型系统.时变时滞中立系统已有许多深刻的研究结果，例如文献[1]-[2]用莱布尼茨公式和自由权矩阵方法研究时变时滞中立系统的稳定性。具有非线性扰动的时变时滞中立型系统引起了许多研究者的兴趣，例如文献[3]研究具有非线性扰动的指数稳定性，文献[4]研究具有非线性扰动的中立型系统的鲁棒稳定性。此外，文献[5]-[7]所研究的系统不仅具有非线性扰动，而且有不同的中立时滞和时变时滞。本文研究一类更一般的具有非线性扰动的时变时滞中立型系统，对此系统进行稳定性分析，并给出渐近稳定的充分条件。

本文考虑如下具有时变时滞中立型系统：

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\left[x(t)-g\left(t,x\left(t-h(t)\right)\right)\right]=Ax(t)+Bx\left(t-\tau(t)\right)+f\left(t,x\left(t-\tau(t)\right)\right), \\ x(t)=\varphi(t),\ \ t\in\left[-\max\left(\bar{h},\tau_2\right),0\right]. \end{cases} \tag{1.1}$$

其中 $x(t)\in\mathbf{R}^n$ 表示系统的状态向量，$f,g:[0,+\infty)\times\mathbf{R}^n\to\mathbf{R}^n$，$A,B\in\mathbf{R}^{n\times n}$ 是常数矩阵，$h(t),\tau(t)$ 分别是中立时滞和时变时滞，满足：

$$0\le h(t)\le\bar{h},\quad 0\le\dot{h}(t)\le h_d<1;\ \ \tau_1\le\tau(t)\le\tau_2,\quad 0\le\dot{\tau}(t)\le\tau_d<1. \tag{1.2}$$

这里 $\bar{h},\tau_1,\tau_2,h_d,\tau_d$ 都是正常数，$\varphi(t)$ 是在 $\left[-\max\left(\bar{h},\tau_2\right)\right]$ 上连续的初始函数。并且

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[g\left(t,x\left(t-h(t)\right)\right)\right]=g_t\left(t,x\left(t-h(t)\right)\right)+\nabla_x g\left(t,x\left(t-h(t)\right)\right)\dot{x}\left(t-h(t)\right)\left(1-\dot{h}(t)\right)$$

记 $G_1:=G_1\left(t,x\left(t-h(t)\right)\right)=g_t\left(t,x\left(t-h(t)\right)\right)$，$G_2:=G_2\left(t,x\left(t-h(t)\right)\right)=\nabla_x g\left(t,x\left(t-h(t)\right)\right)$，其中非线性函数

$$f\left(t,x\left(t-\tau(t)\right)\right),\ \ G_1\left(t,x\left(t-h(t)\right)\right),G_2\left(t,x\left(t-h(t)\right)\right)$$

满足：$f(t,0)=0,\ G_1(t,0)=0,\ G_2(t,0)=0,$ 及

$$f^{\mathrm{T}}f\le\alpha^2x^{\mathrm{T}}(t)x(t),\quad G_1^{\mathrm{T}}G_1\le\beta^2x^{\mathrm{T}}\left(t-h(t)\right)x\left(t-h(t)\right),\quad G_2^{\mathrm{T}}G_2\le aI, \tag{1.3}$$

其中 $\alpha>0,\ \beta>0,0<a<1$ 是给定的常数。

2. 具有非线性扰动的时变时滞中立系统的稳定性分析

系统(1.1)改写为：

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau(t)) + f(t, x(t-\tau(t))) + G_1(t, x(t-h(t))) \\ \qquad + G_2(t, x(t-h(t)))\dot{x}(t-h(t))(1-\dot{h}(t)), \\ x(t) = \varphi(t),\ t \in [-\max(\bar{h}, \tau_2), 0]. \end{cases} \tag{2.1}$$

为了得到系统(2.1)的稳定性条件，我们需要下面的引理。

引理 2.1 [5]对于任意常矩阵 $Z^{\mathrm{T}} = Z > 0$，常数 $\bar{h} > 0, \tau_1 > 0, \tau_2 > 0$ 有

$$-\int_{t-\bar{h}}^{t} \rho^{\mathrm{T}}(s) Z \rho(s) \mathrm{d}s \le -\frac{1}{\bar{h}}\left(\int_{t-\bar{h}}^{t} \rho(s)\mathrm{d}s\right)^{\mathrm{T}} Z \left(\int_{t-\bar{h}}^{t} \rho(s)\mathrm{d}s\right)$$

$$-\int_{t-\tau_2}^{t-\tau_1} \rho^{\mathrm{T}}(s) Z \rho(s) \mathrm{d}s \le -\frac{1}{(\tau_2-\tau_1)}\left(\int_{t-\tau_2}^{t-\tau_1} \rho(s)\mathrm{d}s\right)^{\mathrm{T}} Z \left(\int_{t-\tau_2}^{t-\tau_1} \rho(s)\mathrm{d}s\right).$$

引理 2.2(Schur 补)[8] 给定适当维数的常矩阵 $\Omega_1, \Omega_2, \Omega_3$，$\Omega_1^{\mathrm{T}} = \Omega_1$，$\Omega_2^{\mathrm{T}} = \Omega_2 > 0$，那么 $\Omega_1 + \Omega_3^{\mathrm{T}} \Omega_2^{-1} \Omega_3 < 0$ 成立，当且仅当

$$\begin{bmatrix} \Omega_1 & \Omega_3^{\mathrm{T}} \\ * & -\Omega_2 \end{bmatrix} < 0 \text{ 或者 } \begin{bmatrix} -\Omega_2 & \Omega_3^{\mathrm{T}} \\ * & \Omega_1 \end{bmatrix} < 0 \text{。}$$

引理 2.3[9] 给定适当维数的矩阵 $Q = Q^{\mathrm{T}}, E, D$，那么对于所有满足 $F^{\mathrm{T}}(t) F(t) \le I$ 的 $F(t)$，不等式 $Q + DF(t)E + E^{\mathrm{T}} F^{\mathrm{T}}(t) D^{\mathrm{T}} < 0$ 成立的充分必要条件是存在一个常数 $\delta > 0$ 使得 $Q + \delta^{-1} DD^{\mathrm{T}} + \delta E^{\mathrm{T}} E < 0$。

根据 Lyapunov 理论，对于系统(2.1)的 Lyapunov-Krasovkii 泛函 $V(t)$，如果 $\frac{\mathrm{d}V(t)}{\mathrm{d}t} < 0$，那么系统(2.1)是渐近稳定的。从而有如下结果：

定理 2.1. 对于给定的常数 $\bar{h} > 0, \tau_1 > 0, \tau_2 > 0, \alpha > 0, \beta > 0,\ 0 < a < 1$ 满足条件(1.2)、(1.3)，且存在线性矩阵不等式：

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} \\ * & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ * & * & P_{33} & P_{34} & P_{35} & P_{36} \\ * & * & * & P_{44} & P_{45} & P_{46} \\ * & * & * & * & P_{55} & P_{56} \\ * & * & * & * & * & P_{66} \end{bmatrix} > 0,$$

$$R = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0,\quad S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0,\quad T = \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix} > 0,$$

$$W = \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} > 0,\quad X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0,\quad Y = \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} > 0,$$

和存在常数：$\varepsilon_i \ge 0, i = 1,2,3$，$\lambda > 0$ 使得下列线性矩阵不等式成立：

$$\begin{bmatrix} \Omega + \lambda D^{\mathrm{T}} D + \lambda^{-1} E^{\mathrm{T}} E & A_c^{\mathrm{T}} Z \\ * & -Z \end{bmatrix} < 0 \tag{2.2}$$

其中

$\Omega_{1,1} = P_{11}A + A^{\mathrm{T}} P_{11} + P_{15} + P_{15}^{\mathrm{T}} + S_{11} + W_{11} + (\tau_2 - \tau_1) X_{11} + \bar{h} Y_{11} + A^{\mathrm{T}} H^{\mathrm{T}} + HA,$

$\Omega_{1,2} = P_{11}B + A^{\mathrm{T}} P_{12} + P_{14} + P_{25}^{\mathrm{T}} - (1-\tau_d)(P_{15} + P_{16}) + HB,$

$\Omega_{1,3}=0$， $\Omega_{1,4}=-P_{14}$， $\Omega_{1,5}=P_{16}$， $\Omega_{1,6}=P_{12}$， $\Omega_{1,7}=0$， $\Omega_{1,8}=P_{15}$， $\Omega_{1,9}=0$，

$\Omega_{1,10}=P_{13}$， $\Omega_{1,11}=0$， $\Omega_{1,12}=A^{\mathrm{T}}P_{14}+P_{45}^{\mathrm{T}}$， $\Omega_{1,13}=A^{\mathrm{T}}P_{15}+P_{55}^{\mathrm{T}}$，

$\Omega_{1,14}=A^{\mathrm{T}}P_{16}+P_{56}$， $\Omega_{1,15}=P_{11}+H$， $\Omega_{1,16}=P_{11}+H$，

$\Omega_{2,2}=P_{12}^{\mathrm{T}}B+BP_{12}+P_{24}+P_{24}^{\mathrm{T}}-(1-\tau_d)P_{25}-(1-\tau_d)P_{25}^{\mathrm{T}}-(1-\tau_d)P_{26}-(1-\tau_d)P_{26}^{\mathrm{T}}-(1-\tau_d)R_{11}+\varepsilon_1\alpha^2 I$，

$\Omega_{2,3}=0$， $\Omega_{2,4}=-P_{24}$， $\Omega_{2,5}=P_{26}$， $\Omega_{2,6}=P_{22}-(1-\tau_d)R_{12}$， $\Omega_{2,7}=0$

$\Omega_{2,8}=B^{\mathrm{T}}P_{13}+P_{34}^{\mathrm{T}}-(1-\tau_d)P_{35}^{\mathrm{T}}-(1-\tau_d)P_{36}^{\mathrm{T}}$， $\Omega_{2,9}=0$， $\Omega_{2,10}=P_{23}$， $\Omega_{2,11}=0$，

$\Omega_{2,12}=B^{\mathrm{T}}P_{14}+P_{44}^{\mathrm{T}}-(1-\tau_d)P_{45}^{\mathrm{T}}-(1-\tau_d)P_{46}^{\mathrm{T}}$，

$\Omega_{2,13}=B^{\mathrm{T}}P_{15}+P_{45}-(1-\tau_d)P_{55}^{\mathrm{T}}-(1-\tau_d)P_{56}^{\mathrm{T}}$，

$\Omega_{2,14}=B^{\mathrm{T}}P_{16}+P_{46}^{\mathrm{T}}-(1-\tau_d)P_{56}^{\mathrm{T}}-(1-\tau_d)P_{66}^{\mathrm{T}}$， $\Omega_{2,15}=P_{12}^{\mathrm{T}}$， $\Omega_{2,16}=P_{12}^{\mathrm{T}}$，

$\Omega_{3,3}=R_{11}-(1-\tau_d)S_{11}$， $\Omega_{3,7}=R_{12}-\left(1-\frac{1}{2}\tau_d\right)S_{12}$，

$\Omega_{3,4}=\Omega_{3,5}=\Omega_{3,6}=\Omega_{3,8}=\Omega_{3,9}=\Omega_{3,10}=\Omega_{3,11}=\Omega_{3,12}=\Omega_{3,13}=\Omega_{3,14}=\Omega_{3,15}=\Omega_{3,16}=0$

$\Omega_{4,4}=\Omega_{4,5}=\Omega_{4,6}=\Omega_{4,7}=0$， $\Omega_{4,8}=-P_{34}^{\mathrm{T}}$， $\Omega_{4,9}=\Omega_{4,10}=\Omega_{4,11}=0$，

$\Omega_{4,12}=-P_{44}^{\mathrm{T}}$， $\Omega_{4,13}=-P_{45}$， $\Omega_{4,14}=-P_{46}$， $\Omega_{4,15}=\Omega_{4,16}=0$，

$\Omega_{5,5}=\Omega_{5,6}=\Omega_{5,7}=0$， $\Omega_{5,8}=P_{36}^{\mathrm{T}}$， $\Omega_{5,9}=\Omega_{5,10}=\Omega_{5,11}=0$，

$\Omega_{5,12}=P_{46}^{\mathrm{T}}$， $\Omega_{5,13}=P_{56}^{\mathrm{T}}$， $\Omega_{5,14}=P_{66}^{\mathrm{T}}$， $\Omega_{5,15}=\Omega_{5,16}=0$，

$\Omega_{6,6}=-(1-\tau_d)R_{22}$， $\Omega_{6,7}=0$， $\Omega_{6,8}=P_{23}$， $\Omega_{6,9}=\Omega_{6,10}=\Omega_{6,11}=0$，

$\Omega_{6,12}=P_{24}$， $\Omega_{6,13}=P_{25}$， $\Omega_{6,14}=P_{26}$， $\Omega_{6,15}=0$， $\Omega_{6,16}=0$，

$\Omega_{7,7}=R_{22}-\left(1-\frac{1}{2}\tau_d\right)S_{22}$， $\Omega_{7,8}=\Omega_{7,9}=\Omega_{7,10}=\Omega_{7,11}=\Omega_{7,12}=\Omega_{7,13}=\Omega_{7,14}=\Omega_{7,15}=\Omega_{7,16}=0$，

$\Omega_{8,8}=-(1-h_d)T_{11}+\varepsilon_2\beta^2 I$， $\Omega_{8,9}=0$， $\Omega_{8,10}=P_{33}-(1-h_d)T_{12}$， $\Omega_{8,11}=0$，

$\Omega_{8,12}=\Omega_{8,13}=\Omega_{8,14}=0$， $\Omega_{8,15}=\Omega_{8,16}=P_{13}^{\mathrm{T}}$， $\Omega_{9,9}=T_{11}-\left(1-\frac{1}{2}h_d\right)W_{11}$，

$\Omega_{9,10}=0$， $\Omega_{9,11}=T_{12}-\left(1-\frac{1}{2}h_d\right)W_{12}$， $\Omega_{9,12}=\Omega_{9,13}=\Omega_{9,14}=\Omega_{9,15}=\Omega_{9,16}=0$，

$\Omega_{10,10}=-(1-h_d)T_{22}+G_2^{\mathrm{T}}ZG_2$， $\Omega_{10,11}=0$， $\Omega_{10,12}=P_{34}$， $\Omega_{10,13}=P_{35}$， $\Omega_{10,14}=P_{36}$， $\Omega_{10,15}=\Omega_{10,16}=0$，

$\Omega_{11,11}=T_{22}-\left(1-\frac{1}{2}h_d\right)W_{22}$， $\Omega_{11,12}=\Omega_{11,13}=\Omega_{11,14}=\Omega_{11,15}=\Omega_{11,16}=0$，$\Omega_{12,12}=\Omega_{12,13}=\Omega_{12,14}=0$，

$\Omega_{12,15}=\Omega_{12,16}=P_{14}^{\mathrm{T}}$， $\Omega_{13,13}=\Omega_{13,14}=0$， $\Omega_{13,15}=\Omega_{13,16}=P_{15}^{\mathrm{T}}$， $\Omega_{14,14}=0$， $\Omega_{14,15}=\Omega_{14,16}=P_{16}^{\mathrm{T}}$， $\Omega_{15,15}=-\varepsilon_1 I$，

$\Omega_{15,16}=0$， $\Omega_{16,16}=-\varepsilon_2 I$，

$$D=\begin{bmatrix}\Gamma_{1,10} & \Gamma_{2,10} & 0 & 0 & 0 & 0 & 0 & P_{13} & 0 & 0 & 0 & P_{14} & P_{15} & P_{16} & Z & Z\end{bmatrix}^{\mathrm{T}},$$

$$E=\begin{bmatrix}I & I & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & I & I & I & I & I\end{bmatrix},$$

$\Gamma_{1,10}=P_{11}+H+A^{\mathrm{T}}Z^{\mathrm{T}}$， $\Gamma_{2,10}=P_{12}+B^{\mathrm{T}}Z^{\mathrm{T}}$， $H=S_{12}+W_{12}+(\tau_2-\tau_1)X_{12}+\bar{h}Y_{12}$，

$Z = S_{22} + W_{22} + (\tau_2 - \tau_1) X_{22} + \bar{h} Y_{22}$，$A_c = \begin{bmatrix} A & B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & I \end{bmatrix}$.

则系统(2.1)是渐近稳定的。

证明：选取恰当的 Lyapunov-Krasovkii 泛函：$V = \sum_{i=1}^{4} V_i$

$$
\begin{aligned}
V_1 &= \zeta^{\mathrm{T}}(t) P \zeta(t) \\
V_2 &= \int_{t-\tau(t)}^{t-\frac{1}{2}\tau(t)} \rho^{\mathrm{T}}(s) R \rho(s) \mathrm{d}s + \int_{t-\frac{1}{2}\tau(t)}^{t} \rho^{\mathrm{T}}(s) S \rho(s) \mathrm{d}s \\
V_3 &= \int_{t-h(t)}^{t-\frac{1}{2}h(t)} \rho^{\mathrm{T}}(s) T \rho(s) \mathrm{d}s + \int_{t-\frac{1}{2}h(t)}^{t} \rho^{\mathrm{T}}(s) W \rho(s) \mathrm{d}s \\
V_4 &= \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} \rho^{\mathrm{T}}(s) X \rho(s) \mathrm{d}s + \int_{-\bar{h}}^{0} \int_{t+\theta}^{t} \rho^{\mathrm{T}}(s) Y \rho(s) \mathrm{d}s
\end{aligned}
\tag{2.3}
$$

其中

$$
\zeta^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-\tau(t)) \quad x^{\mathrm{T}}(t-h(t)) \quad \int_{t-\tau_2}^{t-\tau(t)} x(s)^{\mathrm{T}} \mathrm{d}s \quad \int_{t-\tau(t)}^{t} x(s)^{\mathrm{T}} \mathrm{d}s \quad \int_{t-\tau(t)}^{t-\tau_1} x(s)^{\mathrm{T}} \mathrm{d}s \right],
$$

$$
\rho^{\mathrm{T}}(s) = \left[x^{\mathrm{T}}(s) \quad \dot{x}^{\mathrm{T}}(s) \right]
$$

对 $V(t)$ 关于 t 沿着系统(2.1)求导：

$$
\dot{V}(t) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4
$$

$$
\begin{aligned}
\dot{V}_1 &= 2\zeta^{\mathrm{T}}(t) P \dot{\zeta}(t) \\
&= 2\zeta^{\mathrm{T}}(t) P \begin{bmatrix} Ax(t) + Bx(t-\tau(t)) + f + G_1 + G_2 \dot{x}(t-h(t))(1-\dot{h}(t)) \\ \dot{x}(t-\tau(t))(1-\dot{\tau}(t)) \\ \dot{x}(t-h(t))(1-\dot{h}(t)) \\ x(t-\tau(t))(1-\dot{\tau}(t)) - x(t-\tau_2) \\ x(t) - x(t-\tau(t))(1-\dot{\tau}(t)) \\ x(t-\tau_1) - x(t-\tau(t))(1-\dot{\tau}(t)) \end{bmatrix} \\
&\le 2\zeta^{\mathrm{T}}(t) P \begin{bmatrix} Ax(t) + Bx(t-\tau(t)) + f + G_1 + G_2 \dot{x}(t-h(t)) \\ \dot{x}(t-\tau(t)) \\ \dot{x}(t-h(t)) \\ x(t-\tau(t)) - x(t-\tau_2) \\ x(t) - (1-\tau_d) x(t-\tau(t)) \\ x(t-\tau_1) - (1-\tau_d) x(t-\tau(t)) \end{bmatrix}
\end{aligned}
\tag{2.4}
$$

其中，$\zeta^{\mathrm{T}}(t)$ 中各项都是正定的，且 $P > 0$。

$$
\begin{aligned}
\dot{V}_2 &= \left(1 - \frac{1}{2}\dot{\tau}(t)\right) \rho^{\mathrm{T}}\left(t - \frac{1}{2}\tau(t)\right) R \rho\left(t - \frac{1}{2}\tau(t)\right) - (1-\dot{\tau}(t)) \rho^{\mathrm{T}}(t-\tau(t)) R \rho(t-\tau(t)) \\
&\quad + \rho^{\mathrm{T}}(t) S \rho(t) - \left(1 - \frac{1}{2}\dot{\tau}(t)\right) \rho^{\mathrm{T}}\left(t - \frac{1}{2}\tau(t)\right) S \rho\left(t - \frac{1}{2}\tau(t)\right) \\
&\le \rho^{\mathrm{T}}\left(t - \frac{1}{2}\tau(t)\right) R \rho\left(t - \frac{1}{2}\tau(t)\right) - (1-\tau_d) \rho^{\mathrm{T}}(t-\tau(t)) R \rho(t-\tau(t)) \\
&\quad + \rho^{\mathrm{T}}(t) S \rho(t) - \left(1 - \frac{1}{2}\tau_d\right) \rho^{\mathrm{T}}\left(t - \frac{1}{2}\tau(t)\right) S \rho\left(t - \frac{1}{2}\tau(t)\right)
\end{aligned}
\tag{2.5}
$$

$$\begin{aligned}\dot{V}_3 &= \left(1-\frac{1}{2}\dot{h}(t)\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}h(t)\right)T\rho\left(t-\frac{1}{2}h(t)\right)-\left(1-\dot{h}(t)\right)\rho^{\mathrm{T}}\left(t-h(t)\right)T\rho\left(t-h(t)\right)\\ &\quad+\rho^{\mathrm{T}}(t)S\rho(t)-\left(1-\frac{1}{2}\dot{\tau}(t)\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}\tau(t)\right)S\rho\left(t-\frac{1}{2}\tau(t)\right)\\ &\le \rho^{\mathrm{T}}\left(t-\frac{1}{2}h(t)\right)T\rho\left(t-\frac{1}{2}h(t)\right)-(1-h_d)\rho^{\mathrm{T}}\left(t-h(t)\right)T\rho\left(t-h(t)\right)\\ &\quad+\rho^{\mathrm{T}}(t)W\rho(t)-\left(1-\frac{1}{2}\dot{\tau}_d\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}\tau(t)\right)W\rho\left(t-\frac{1}{2}\tau(t)\right)\end{aligned} \tag{2.6}$$

$$\begin{aligned}\dot{V}_4 &= (\tau_2-\tau_1)\rho^{\mathrm{T}}(t)X\rho(t)-\int_{t-\tau_2}^{t-\tau_1}\rho^{\mathrm{T}}(s)X\rho(s)\mathrm{d}s+\bar{h}\rho^{\mathrm{T}}(t)Y\rho(t)-\int_{t-\bar{h}}^{t}\rho^{\mathrm{T}}(s)Y\rho(s)\mathrm{d}s\\ &\le (\tau_2-\tau_1)\rho^{\mathrm{T}}(t)X\rho(t)-\frac{1}{(\tau_2-\tau_1)}\left(\int_{t-\tau_2}^{t-\tau_1}\rho(s)\mathrm{d}s\right)^{\mathrm{T}}X\left(\int_{t-\tau_2}^{t-\tau_1}\rho(s)\mathrm{d}s\right)\\ &\quad+\bar{h}\rho^{\mathrm{T}}(t)Y\rho(t)-\frac{1}{\bar{h}}\left(\int_{t-\bar{h}}^{t}\rho(s)\mathrm{d}s\right)^{\mathrm{T}}Y\left(\int_{t-\bar{h}}^{t}\rho(s)\mathrm{d}s\right)\\ &\le (\tau_2-\tau_1)\rho^{\mathrm{T}}(t)X\rho(t)+\bar{h}\rho^{\mathrm{T}}(t)Y\rho(t)\end{aligned} \tag{2.7}$$

$$\begin{aligned}\dot{V}(t) &= 2\zeta^{\mathrm{T}}(t)P\dot{\zeta}(t)+\left(1-\frac{1}{2}\dot{\tau}(t)\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}\tau(t)\right)R\rho\left(t-\frac{1}{2}\tau(t)\right)-\left(1-\dot{\tau}(t)\right)\rho^{\mathrm{T}}\left(t-\tau(t)\right)R\rho\left(t-\tau(t)\right)\\ &\quad+\rho^{\mathrm{T}}(t)S\rho(t)-\left(1-\frac{1}{2}\dot{\tau}(t)\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}\tau(t)\right)S\rho\left(t-\frac{1}{2}\tau(t)\right)\\ &\quad+\left(1-\frac{1}{2}\dot{h}(t)\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}h(t)\right)T\rho\left(t-\frac{1}{2}h(t)\right)-\left(1-\dot{h}(t)\right)\rho^{\mathrm{T}}\left(t-h(t)\right)T\rho\left(t-h(t)\right)\\ &\quad+\rho^{\mathrm{T}}(t)W\rho(t)-\left(1-\frac{1}{2}\dot{h}(t)\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}h(t)\right)W\rho\left(t-\frac{1}{2}h(t)\right)\\ &\quad+(\tau_2-\tau_1)\rho^{\mathrm{T}}(t)X\rho(t)-\int_{t-\tau_2}^{t-\tau_1}\rho^{\mathrm{T}}(s)X\rho(s)\mathrm{d}s+\bar{h}\rho^{\mathrm{T}}(t)Y\rho(t)-\int_{t-\bar{h}}^{t}\rho^{\mathrm{T}}(s)Y\rho(s)\mathrm{d}s\\ &\le 2\zeta^{\mathrm{T}}(t)P\begin{bmatrix}Ax(t)+Bx\left(t-\tau(t)\right)+f+G_1+G_2\dot{x}\left(t-h(t)\right)\\ \dot{x}\left(t-\tau(t)\right)\\ \dot{x}\left(t-h(t)\right)\\ x\left(t-\tau(t)\right)-x(t-\tau_2)\\ x(t)-(1-\tau_d)x\left(t-\tau(t)\right)\\ x(t-\tau_1)-(1-\tau_d)x\left(t-\tau(t)\right)\end{bmatrix}\\ &\quad+\rho^{\mathrm{T}}\left(t-\frac{1}{2}\tau(t)\right)R\rho\left(t-\frac{1}{2}\tau(t)\right)-(1-\tau_d)\rho^{\mathrm{T}}\left(t-\tau(t)\right)R\rho\left(t-\tau(t)\right)+\rho^{\mathrm{T}}(t)S\rho(t)\\ &\quad-\left(1-\frac{1}{2}\tau_d\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}\tau(t)\right)S\rho\left(t-\frac{1}{2}\tau(t)\right)+\rho^{\mathrm{T}}\left(t-\frac{1}{2}h(t)\right)T\rho\left(t-\frac{1}{2}h(t)\right)\\ &\quad-(1-h_d)\rho^{\mathrm{T}}\left(t-h(t)\right)T\rho\left(t-h(t)\right)+\rho^{\mathrm{T}}(t)S\rho(t)\\ &\quad-\left(1-\frac{1}{2}\dot{\tau}_d\right)\rho^{\mathrm{T}}\left(t-\frac{1}{2}\tau(t)\right)S\rho\left(t-\frac{1}{2}\tau(t)\right)+(\tau_2-\tau_1)\rho^{\mathrm{T}}(t)X\rho(t)+\bar{h}\rho^{\mathrm{T}}(t)Y\rho(t)\end{aligned} \tag{2.8}$$

由(1.3)可得，对于任意常数 $\varepsilon_1>0,\varepsilon_2>0,$ 有

$$\varepsilon_1\left(\alpha^2x^{\mathrm{T}}(t)x(t)-f^{\mathrm{T}}f\right)\ge 0,\quad \varepsilon_2\left(\beta^2x^{\mathrm{T}}\left(t-h(t)\right)x\left(t-h(t)\right)-G_1^{\mathrm{T}}G_1\right)\ge 0. \tag{2.9}$$

然后结合方程(2.8)~(2.9)和引理 2.1 可知：

$$\dot{V}(t)\le\xi^{\mathrm{T}}(t)\left(\Omega+A_c^{\mathrm{T}}ZA_c+\Gamma\right)\xi(t)\le\xi^{\mathrm{T}}(t)\left(\Omega+A_c^{\mathrm{T}}ZA_c+DG_2E+E^{\mathrm{T}}G_2^{\mathrm{T}}D^{\mathrm{T}}\right)\xi(t) \tag{2.10}$$

其中 $\Omega=\left(\Omega_{ij}\right)_{16\times16}$， $\Gamma^{\mathrm{T}}=\Gamma$，

$$\Gamma=\begin{bmatrix}
0&0&0&0&0&0&0&0&0&\Gamma_{1,10}G_2&0&0&0&0&0&0\\
*&0&0&0&0&0&0&0&0&\Gamma_{2,10}G_2&0&0&0&0&0&0\\
&&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
&&*&0&0&0&0&0&0&0&0&0&0&0&0&0\\
&&*&*&0&0&0&0&0&0&0&0&0&0&0&0\\
&&*&*&*&0&0&0&0&0&0&0&0&0&0&0\\
&&*&*&*&*&0&0&0&0&0&0&0&0&0&0\\
&&*&*&*&*&*&0&0&0&0&0&0&0&0&0\\
&&*&*&*&*&*&*&0&0&0&0&0&0&0&0\\
&&*&*&*&*&*&*&*&0&0&P_{14}^{\mathrm{T}}G_2&P_{15}^{\mathrm{T}}G_2&P_{16}^{\mathrm{T}}G_2&ZG_2&ZG_2\\
&&*&*&*&*&*&*&*&*&0&0&0&0&0&0\\
&&*&*&*&*&*&*&*&*&*&0&0&0&0&0\\
&&*&*&*&*&*&*&*&*&*&*&0&0&0&0\\
&&*&*&*&*&*&*&*&*&*&*&*&0&0&0\\
&&*&*&*&*&*&*&*&*&*&*&*&*&0&0\\
&&*&*&*&*&*&*&*&*&*&*&*&*&*&0
\end{bmatrix}$$

$$\Gamma_{1,10}=P_{11}+H+A^{\mathrm{T}}Z^{\mathrm{T}},\quad \Gamma_{2,10}=P_{12}+B^{\mathrm{T}}Z^{\mathrm{T}},$$

$$D=\begin{bmatrix}\Gamma_{1,10} & \Gamma_{2,10} & 0 & 0 & 0 & 0 & 0 & P_{13} & 0 & 0 & 0 & P_{14} & P_{15} & P_{16} & Z & Z\end{bmatrix}^{\mathrm{T}},$$

$$E=\begin{bmatrix}I & I & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & I & I & I & I & I\end{bmatrix},$$

$$\begin{aligned}\xi^{\mathrm{T}}(t)=\Big[&x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau(t))\ \ x^{\mathrm{T}}\left(t-\tfrac{1}{2}\tau(t)\right)\ \ x^{\mathrm{T}}(t-\tau_2)\ \ x^{\mathrm{T}}(t-\tau_1)\ \ \dot{x}^{\mathrm{T}}(t-\tau(t))\\
&\dot{x}^{\mathrm{T}}\left(t-\tfrac{1}{2}\tau(t)\right)\ \ x^{\mathrm{T}}(t-h(t))\ \ x^{\mathrm{T}}\left(t-\tfrac{1}{2}h(t)\right)\ \ \dot{x}^{\mathrm{T}}(t-h(t))\ \ \dot{x}^{\mathrm{T}}\left(t-\tfrac{1}{2}h(t)\right)\\
&\left(\int_{t-\tau_2}^{t-\tau(t)}x(s)\mathrm{d}s\right)^{\mathrm{T}}\ \ \left(\int_{t-\tau(t)}^{t}x(s)\mathrm{d}s\right)^{\mathrm{T}}\ \ \left(\int_{t-\tau(t)}^{t-\tau_1}x(s)\mathrm{d}s\right)^{\mathrm{T}}\ \ f^{\mathrm{T}}\ \ G_1^{\mathrm{T}}\Big],\end{aligned}$$

结合引理 2.3，存在常数 $\lambda>0$，使得

$$\Omega+A_c^{\mathrm{T}}ZA_c+\lambda^{-1}D^{\mathrm{T}}D+\lambda E^{\mathrm{T}}E<0 \tag{2.11}$$

由(1.3)，引理 2.2 和引理 2.3 得(2.11)等价于(2.2)，即 $\dot{V}(t)\le 0$。

因此，带有非线性扰动的时变时滞中立型系统(2.1)是渐近稳定的，证毕。

3. 注记

本文对于构造的 Lyapunov-Krasovskii 泛函做出了改进，即，把中立时滞和时变时滞分成两段 $\int_{t-\tau(t)}^{t-\frac{1}{2}\tau(t)}\rho^{\mathrm{T}}(s)R\rho(s)\mathrm{d}s$，$\int_{t-\frac{1}{2}\tau(t)}^{t}\rho^{\mathrm{T}}(s)S\rho(s)\mathrm{d}s$，$\int_{t-h(t)}^{t-\frac{1}{2}h(t)}\rho^{\mathrm{T}}(s)T\rho(s)\mathrm{d}s$，$\int_{t-\frac{1}{2}h(t)}^{t}\rho^{\mathrm{T}}(s)W\rho(s)\mathrm{d}s$，因此 Lyapunov 泛函的时域被划分得更精细。同时，本文用 $t-h(t)$ 来代替积分下限 $t-\bar{h}$，这样处理有利于降低系统的保守性。

对于 Lyapunov 泛函的导数，产生了非对称项如 $x(t)P_{11}G_2x(t-h(t))(1-\dot{h}(t))$，并不能直接使用 LMI 处理问题。因此，对非对称项进行有效的放缩以保证系统的可控制性，并且尽可能降低系统的保守性。应用引理 2.3，非对称项可被对称项有效控制，从而使得 Lyapunov 泛函的导数被约束在一个可控制的线性矩阵不等式中，最后得到判定系统稳定性的充分条件。

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