Advances in Condensed Matter Physics 凝聚态物理学进展, 2017, 6(4), 81-85
Published Online November 2017 in Hans. http://www.hanspub.org/journal/cmp
https://doi.org/10.12677/cmp.2017.64011

Hans汉斯

Geometric Phase of Rabi Oscillations in a Large Josephson-Junction Qubit

Yuanxin Qiao, Zhaoxian Yu

Department of Physics, Beijing Information Science and Technology University, Beijing
Email: zxyu1965@163.com

Received: Sep. 1st, 2017; accepted: Sep. 13th, 2017; published: Sep. 19th, 2017

Abstract

By using the Lewis–Riesenfeld invariant theory, we have studied the geometric phase of Rabi oscillations in a large Josephson-junction qubit. The geometric phase has nothing to do with the dc and microwave pulses of bias current, and is independent of the transitions frequencies of microwaves.

Keywords

Lewis-Riesenfeld Invariant Theory, Geometric Phase, Rabi Oscillations, Large Josephson-Junction Qubit

大约瑟夫逊结量子比特中拉比振荡的几何相位

乔元新，于肇贤

北京信息科技大学理学院，北京
Email: zxyu1965@163.com

收稿日期：2017年9月1日；录用日期：2017年9月13日；发布日期：2017年9月19日

摘　要

通过使用Lewis-Riesenfeld不变量理论，我们研究了大约瑟夫逊结量子比特中拉比振荡的几何相位。发现几何相位与偏置电流的直流和微波脉冲无关，也与微波的转换频率无关。

关键词

Lewis-Riesenfeld不变量理论，几何相位，拉比振荡，大约瑟夫逊结量子比特

文章引用：乔元新，于肇贤．大约瑟夫逊结量子比特中拉比振荡的几何相位[J]．凝聚态物理学进展，2017，6(4)：81-85．
DOI: 10.12677/cmp.2017.64011

1. 引言

1984年Berry研究发现，在绝热过程中量子力学波函数存在一个不可积的具有几何性质的相位因子，它不同于通常的动力学相位因子。这一发现在许多物理领域的几何相位因子已被广泛研究和证实，并逐渐通过相关实验证实。在最近20多年里，几何相位因子的研究已经成为量子力学中最重要的基本问题之一，其基本概念几乎渗透到物理学的各个领域[1]-[31]。本文将通过使用 Lewis-Riesenfeld 不变理论，研究大约瑟夫逊结量子位中拉比振荡的几何相位。

2. 物理模型

大约瑟夫逊结量子比特中拉比振荡的哈密顿算子可以写为[32]

$$\hat{H}=\hat{\sigma}_x I_{\mu\omega c}(t)\sqrt{\hbar/2\omega_{10}C}/2+\hat{\sigma}_y I_{\mu\omega s}(t)\sqrt{\hbar/2\omega_{10}C}/2+\hat{\sigma}_z\frac{\delta I_{dc}(t)}{2}\frac{\partial E_{10}}{\partial I_{dc}}, \tag{1}$$

其中 $\hat{\sigma}_{x,y,z}$ 是泡利算符。用 dc 和微波脉冲的偏置电流可以完全操作量子位状态。ω_{10} 是量子位状态之间的跃迁频率，C 是结电容。此时

$$I(t)=I_{dc}+\delta I_{dc}(t)+I_{\mu\omega c}(t)\cos\omega_{10}t+I_{\mu\omega s}(t)\sin\omega_{10}t, \tag{2}$$

为方便期间，我们令

$$A(t)=I_{\mu\omega c}(t)\sqrt{\hbar/2\omega_{10}C}/2,\ B(t)=I_{\mu\omega s}(t)\sqrt{\hbar/2\omega_{10}C}/2,\ C(t)=\frac{\delta I_{dc}(t)}{2}\frac{\partial E_{10}}{\partial I_{dc}}. \tag{3}$$

方程(1)式变为

$$\hat{H}=A(t)\hat{\sigma}_x+B(t)\sigma_y+C(t)\hat{\sigma}_z. \tag{4}$$

引入 $\sigma_{\pm}=\frac{1}{2}\left[\sigma_x\pm i\sigma_y\right]$，有

$$\hat{H}=D(t)\hat{\sigma}_{+}+D(t)\hat{\sigma}_{-}+C(t)\hat{\sigma}_z. \tag{5}$$

其中 $D(t)=A(t)-iB(t)$，并且满足以下关系

$$\left[\hat{\sigma}_{+},\hat{\sigma}_{-}\right]=\hat{\sigma}_z,\quad \left[\hat{\sigma}_z,\hat{\sigma}_{+}\right]=2\hat{\sigma}_{+},\quad \left[\hat{\sigma}_z,\hat{\sigma}_{-}\right]=-2\hat{\sigma}_{-}. \tag{6}$$

3. 动力学相位与几何相位

为了自洽，我们首先简要介绍 Lewis-Riesenfeld(L-R)不变理[27]。对与时间有关的一维系统的哈密顿量 $\hat{H}(t)$，如果存在不变量运算符 $\hat{I}(t)$

$$i\frac{\partial\hat{I}(t)}{\partial t}+\left[\hat{I}(t),\hat{H}(t)\right]=0. \tag{7}$$

则可以给出与时间有关的不变量 $|\lambda_n,t\rangle$ 的特征值方程

$$\hat{I}(t)|\lambda_n,t\rangle=\lambda_n|\lambda_n,t\rangle, \tag{8}$$

当$\frac{\partial\lambda_n}{\partial t}=0$，此系统有时间有关的薛定谔方程是

$$i\frac{\partial\left|\psi(t)\right\rangle_s}{\partial t}=\hat{H}(t)\left|\psi(t)\right\rangle_s. \tag{9}$$

根据L-R不变理论，方程(9)的特定解$\left|\lambda_n,t\right\rangle_s$只有相位因子$\exp\left[i\delta_n(t)\right]$与$\hat{I}(t)$的本征函数$\left|\lambda_n,t\right\rangle$不同，即

$$\left|\lambda_n,t\right\rangle_s=\exp\left[i\delta_n(t)\right]\left|\lambda_n,t\right\rangle, \tag{10}$$

这表明$\left|\lambda_n,t\right\rangle_s(n=1,2,\cdots)$构成方程(9)解的完整集合。那么薛定谔方程(9)的一般解可以写成

$$\left|\psi(t)\right\rangle_s=\sum_n C_n\exp\left[i\delta_n(t)\right]\left|\lambda_n,t\right\rangle, \tag{11}$$

其中

$$\delta_n(t)=\int_0^t \mathrm{d}t'\left\langle\lambda_n,t'\right|i\frac{\partial}{\partial t'}-\hat{H}(t')\left|\lambda_n,t\right\rangle, \tag{12}$$

且$C_n=\left\langle\lambda_n,0\middle|\psi(0)\right\rangle_s$。

对于由方程(5)哈密顿量描述的系统，我们可以定义以下不变量

$$\hat{I}(t)=\alpha(t)\hat{\sigma}_++\hat{\alpha}^*(t)\hat{\sigma}_-+\beta(t)\hat{\sigma}_z. \tag{13}$$

将方程(5)和(13)代入到方程(7)，可以得到一个辅助方程

$$i\dot{\alpha}(t)+2(D\beta-\alpha\beta)=0,\quad i\dot{\beta}+D\left(\alpha-\alpha^*\right)=0. \tag{14}$$

为了获得一个与时间无关的不变量，我们可以引入么正变换算子$\hat{V}(t)=\exp\left[\mu(t)\hat{\sigma}_+-\mu^*(t)\hat{\sigma}_-\right]$，当满足以下关系式时

$$\beta\cos\left(2|\mu|\right)+\alpha\mu\sqrt{2|\mu|}\left[\sin\sqrt{2|\mu|}+\frac{1}{\sqrt{2|\mu|}}-1\right]=1, \tag{15}$$

$$\alpha+\frac{\mu|\beta|}{|\mu|}\left[\sin\left(2|\mu|\right)+2|\mu|-1\right]=1, \tag{16}$$

有

$$\hat{I}_V\equiv\hat{V}^\dagger(t)\hat{I}(t)\hat{V}(t)=\hat{\sigma}_z. \tag{17}$$

通过使用Baker-Campbell-Hausdoff公式[33]

$$\hat{V}^\dagger(t)\frac{\partial\hat{V}(t)}{\partial t}=\frac{\partial\hat{L}}{\partial t}+\frac{1}{2!}\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right]+\frac{1}{3!}\left[\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right],\hat{L}\right]+\frac{1}{4}\left[\left[\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right],\hat{L}\right],\hat{L}\right]+\cdots, \tag{18}$$

且$\hat{V}(t)=\exp\left[\hat{L}(t)\right]$。满足下面方程

$$D+C+\frac{\beta\mu}{|\mu|}\left[\sin2|\mu|+2|\mu|-1\right]+\frac{2\mu\left(\mu\dot{\mu}^*-\dot{\mu}\mu^*\right)}{\left(2|\mu|\right)^3}\left[\sin\left(2|\mu|-2|\mu|^3\right)\right]=0, \tag{19}$$

有

$$\hat{H}_V(t)=\hat{V}^{\dagger}(t)\hat{H}(t)\hat{V}(t)-i\hat{V}^{\dagger}(t)\frac{\partial\hat{V}(t)}{\partial t}=\lambda\hat{\sigma}_z, \tag{20}$$

其中：

$$\lambda=i\left[1-\cos\left(2|\mu|\right)\right]\left(\mu\dot{\mu}^*-\dot{\mu}\mu^*\right)+D\left[\sin\sqrt{2|\mu|}+\frac{1}{2\sqrt{2|\mu|}}-1\right]\sqrt{2|\mu|}\left[\alpha\mu+\alpha^*\mu^*\right], \tag{21}$$

该量子系统具有的几何相位为：

$$\dot{\delta}^g(t)=i\left[1-\cos\left(2|\mu|\right)\right]\left(\mu\dot{\mu}^*-\dot{\mu}\mu^*\right). \tag{22}$$

4. 结论

在本文中，我们研究了大约瑟夫逊结量子比特中拉比振荡的几何相位。发现几何相位与偏置电流的直流和微波脉冲无关，也与微波的转换频率无关。很明显，几何相具有纯粹的几何和拓扑特征。这种几何因子是一种新的量子效应，且体现在介观电路中。我们期待该结论在将来的实验中得以证实。

基金项目

本文得到北京信息科技大学研究生院基金支持。

参考文献 (References)

[1] Berry, M.V. (1984) Quantal Phase Factors Accompanying Adibatic Changes. *Proceedings of the Royal Society of London*, **392**, 45-57. https://doi.org/10.1098/rspa.1984.0023

[2] Aharonov, Y. and Bohm, D. (1959) Significance of Electromagnetic Potentials in the Quantum Theory. *Physics Review*, **115**, 485-491. https://doi.org/10.1103/PhysRev.115.485

[3] Aharonov, Y. and Anandan, J. (1987) Phase Change during a Cyclic Quantum Evolution. *Physical Review Letters*, **58**, 1593. https://doi.org/10.1103/PhysRevLett.58.1593

[4] Samuel, J. and Bhandari, R. (1988) General Setting for Berry's Phase. *Physical Review Letters*, **60**, 2339. https://doi.org/10.1103/PhysRevLett.60.2339

[5] Yu, Z.X. and Zhang, D.X. (1995) Phase Factor for Quantum Systems with Time-Bound Conditions. *Journal of Qingdao University*, 849.

[6] Pati, A.K. (1995) Geometric Aspects of Noncyclic Quantum Evolution. *Physical Review A*, **52**, 2576-2584. https://doi.org/10.1103/PhysRevA.52.2576

[7] Uhlmann, A. (1986) Parallel Transport and Quantum Holonomy along Density Operators. *Reports on Mathematical Physics*, **24**, 229-240. https://doi.org/10.1016/0034-4877(86)90055-8

[8] Sj'oqvist, E., *et al*. (2000) Geometric Phases for Mixed States in Interferometry. *Physical Review Letters*, **85**, 2845-2849. https://doi.org/10.1103/PhysRevLett.85.2845

[9] Gao, X.C., Xu, J.B. and Qian, T.Z. (1991) Geometric Phase and the Generalized Invariant Formulation. *Physical Review A*, **44**, 7016. https://doi.org/10.1103/PhysRevA.44.7016

[10] Yu, G., Yu, Z.X. and Zhang, D.X. (1996) Berry Phase of the Finite Deep Column Potential Well with Motive Boundary. *Yantai Teachers University Journal*, **12**, 117.

[11] Gao, X.C., *et al*. (1996) Quantum-Invariant Theory and the Evolution of a Quantum Scalar Field in Robertson-Walker Flat Spacetimes. *Physical Review D*, **53**, 4374. https://doi.org/10.1103/PhysRevD.53.4374

[12] Hou, Y.Z., Yu, Z.X. and Liu, Y.H. (1994) Using Quantum-Positive Transformations to Cause the Scalability of Topological Items to Obtain the Perturbation of Perturbation on the Contribution of B-e: Ry Phase Factors. *Journal of Xinyan Teachers College*, **7**, 37.

[13] Liang, J.Q. and Ding, X.X. (1991) Broken Gauge Equivalence of Hamiltonians Due to Time-Evolution and the Berry Phase. *Physics Letters A*, **153**, 273.

[14] Sun, C.P. and Ge, M.L. (1990) Generalizing Born-Oppenheimer Approximations and Observable Effects of an Induced Gauge Field. *Physical Review D*, **41**, 1349. https://doi.org/10.1103/PhysRevD.41.1349

[15] Sun, C.P. (1993) Quantum Dynamical Model for Wave-Function Reduction in Classical and Macroscopic Limits. *Physical Review A*, **48**, 898.

[16] Sun, C.P. (1988) Analytic Treatment of High-Order Adiabatic Approximations of 2-Neutrino Oscillations in Matter. *Physical Review D*, **38**, 2908. https://doi.org/10.1103/PhysRevD.38.2908

[17] Sun, C.P., *et al*. (1988) High-Order Quantum Adiabatic Approximation and Berrys Phase Factor. *Journal of Physics A*, **21**, 1595. https://doi.org/10.1088/0305-4470/21/7/023

[18] Chen, G., Li, J. and Liang, J.-Q. (2006) Critical Property of the Geometric Phase in the Dicke Model. *Physical Review A*, **74**, 150. https://doi.org/10.1103/PhysRevA.74.054101

[19] Fan, H.Y. and Ruan, T.N. (1984) Some New Applications of Coherent States. *China Science*, **42**, 27.

[20] Tsui, *et al*. (1982) Two-Dimensional Magnetotransport in the Extreme Quantum Limit. *Physical Review Letters*, **48**, 1559. https://doi.org/10.1103/PhysRevLett.48.1559

[21] Semenoff, *et al*. (1986) Non-Abelian Adiabatic Phases and the Fractional Quantum Hall Effect. *Physical Review Letters*, **57**, 1195. https://doi.org/10.1103/PhysRevLett.57.1195

[22] Chen, C.M., *et al*. (1991) Quantum Hall Effect and Bery Phase Factor Physics. *Acta Physica Sinica*, **40**, 345.

[23] Fan, H.Y. and Li, L.S. (1996) Supersymmetric Unitary Operator for Some Generalized Jaynes-Cummings Models. *Communications in Theoretical Physics*, **25**, 105.

[24] Wu, Y. and Yang, X. (1997) Jaynes-Cummings Model for a Trapped Ion in Any Position of a Standing Wave. *Physical Review Letters*, **78**, 3086. https://doi.org/10.1103/PhysRevLett.78.3086

[25] Wu, Y. (1996) Simple Algebraic Method to Solve a Coupled-Channel Cavity QED Model. *Physical Review A*, **54**, 4534. https://doi.org/10.1103/PhysRevA.54.4534

[26] Yang, X., Wu, Y. and Li, Y. (1997) Unified and Standardized Procedure to Solve Various Nonlinear Jaynes-Cummings Models. *Physical Review A*, **55**, 4545. https://doi.org/10.1103/PhysRevA.55.4545

[27] Lewis, H.R. and Riesenfeld, W.B. (1969) An Exact Quantum Theory of the Time-Dependent Harmonic Oscillator and of a Charged Particle in a Time-Dependent Electromagnetic Field. *Journal of Mathematical Physics*, **10**, 1458. https://doi.org/10.1063/1.1664991

[28] Yu, Z.X. and Liu, Y.H. (1991) Aharonov-Bohm Effect of Inducing Gauge Field and Gometric Phase Factors of Suter Experiment. *Journal of Yantai Teachers University*, **10**, 42.

[29] Wilczek, F., *et al*. (1984) Appearance of Gauge Structure in Simple Dynamical Systems. *Physical Review Letters*, **52**, 2111. https://doi.org/10.1103/PhysRevLett.52.2111

[30] Moody, J., *et al*. (1986) Realizations of Magnetic-Monopole Gauge Fields: Diatoms and Spin Precession. *Physical Review Letters*, **56**, 893. https://doi.org/10.1103/PhysRevLett.56.893

[31] Shapere, A. and Wilczek, F. (1989) Geometric Phases in Physics. World Scientific, Singapore.

[32] Martinis, J.M., Nam, S. and Aumentado, J. (2002) Rabi Oscillations in a Large Josephson-Junction Qubit. *Physical Review Letters*, **89**, Article ID: 117901. https://doi.org/10.1103/PhysRevLett.89.117901

[33] Wei, J. and Norman, E. (1963) Lie Algebraic Solution of Linear Differential Equations. *Journal of Mathematical Physics*, **4**, 575. https://doi.org/10.1063/1.1703993

知网检索的两种方式：

1. 打开知网页面 http://kns.cnki.net/kns/brief/result.aspx?dbPrefix=WWJD
   下拉列表框选择：[ISSN]，输入期刊 ISSN：2326-3512，即可查询
2. 打开知网首页 http://cnki.net/
   左侧“国际文献总库”进入，输入文章标题，即可查询

投稿请点击：http://www.hanspub.org/Submission.aspx
期刊邮箱：cmp@hanspub.org