Advances in Applied Mathematics 应用数学进展, **2019, 8(6), 1058-1063**
Published Online June 2019 in Hans. http://www.hanspub.org/journal/aam
https://doi.org/10.12677/aam.2019.86121

Hans汉斯

Lump Solutions to the (3+1)-Dimensional Mel'nikov Equation

Xiaoyu Li, Xuelin Yong*, Yehui Huang

School of Mathematical Sciences and Physics, North China Electric Power University, Beijing
Email: ncepumath@sohu.com, *yongxuelin@ncepu.edu.cn, yhhuang@ncepu.edu.cn

Received: May 6th, 2019; accepted: May 28th, 2019; published: Jun. 4th, 2019

Abstract

In this article, lump solutions of the (3+1)-dimensional Mel'nikov are obtained via the Hirota bilinear method and symbolic computation with Maple. A class of lump solutions rationally localized in all directions in the space is obtained. And we propose the conditions for the analyticity and rational localization of the lump solutions. By selecting special value of parameters involved, the dynamic characteristics of the solutions are illustrated.

Keywords

Lump Solution, (3+1)-Dimensional Mel'nikov, Hirota Bilinear Method

(3+1)维Mel'nikov方程的Lump解研究

李晓玉，雍雪林*，黄晔辉

华北电力大学数理学院，北京
Email: ncepumath@sohu.com, *yongxuelin@ncepu.edu.cn, yhhuang@ncepu.edu.cn

收稿日期：2019年5月6日；录用日期：2019年5月28日；发布日期：2019年6月4日

摘 要

本文运用Hirota双线性方法和符号计算研究了(3+1)维Mel'nikov方程的Lump解。我们给出了一阶Lump解的表达式，并讨论了相应的解析性和局部性条件。最后，我们作出了解的图像并分析了解的动态性质。

*通讯作者。

文章引用: 李晓玉, 雍雪林, 黄晔辉. (3+1)维 Mel'nikov 方程的 Lump 解研究[J]. 应用数学进展, 2019, 8(6): 1058-1063.
DOI: 10.12677/aam.2019.86121

关键词

Lump解，(3+1)维Mel'nikov方程，Hirota双线性方法

1. 引言

作为孤子方程的可积推广，带自相容源的孤子方程是一类重要的数学物理模型，它涉及到流体力学、等离子体物理和固体物理中的很多问题[1]。带自相容源的孤子方程可以通过 Sato 理论[2]，反散射方法[3]，广义达布变换方法[4]和源生成法得到[5]。研究发现，在带自相容源的非线性可积系统中孤波是以变速度运动的[6]-[11]。

近年来在非线性科学中对于 Lump 波的研究引起了越来越多学者的广泛关注[12]-[18]。Lump 解是一类局域的有理函数解，并且大量孤子方程的 Lump 解已经被找到[19] [20] [21] [22]。特别地，通过符号计算方法也给出了一些高维的非线性系统的有理解[23] [24] [25] [26] [27]。然而，对于(3+1)维的情况，得到的解往往不是在所有方向上都是局域有理化的，因此只能称为类 Lump 型解[28] [29] [30]。因此求解(3+1)维偏微分方程的 Lump 解是非常重要并且有意义的。

本文我们将研究如下的(3+1)维 Mel'nikov 方程：

$$\begin{cases}\left(u_t+6uu_x+u_{xxx}+8\left|\phi\right|_x^2\right)_x-u_{yy}+u_{zz}=0,\\ i\phi_y=2\phi_{xx}+2u\phi,\\ i\phi_z=\phi_{xx}+u\phi,\end{cases}\tag{1}$$

文献[31]中通过 Hirota 方法和 Wronskian 技巧讨论了它的行列式解和双线性贝克隆变换。本文结构如下：在第二部分，通过变量变换并且利用 Hirota 双线性法得到了该方程的 Lump 解，同时作出了两组特解的图像并分析了它们的动力学性质。最后给出本文的结论。

2. Mel'nikov 的 Lump 解

引入变量代换

$$\begin{cases}u=2\left(\ln f\right)_{xx},\\ \phi=\dfrac{g}{f},\end{cases}\tag{2}$$

方程(1)可以转化为如下双线性形式：

$$\begin{cases}\left(D_xD_t+D_x^4-D_y^2+D_z^2\right)f\cdot f+8\left(g\bar{g}-f^2\right)=0,\\ \left(iD_y-2D_x^2\right)g\cdot f=0,\\ \left(iD_z-D_x^2\right)g\cdot f=0.\end{cases}\tag{3}$$

其中，f 是实函数，g 是关于 x,y,z 和 t 的复函数，$\bar{g}$ 是 g 的复共轭函数。这里 D 是双线。

性算子定义如下[32]：

$$D_x^m D_t^n(a,b)=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^m\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^n a(x,t)b(x',t')\bigg|_{x'=x,t'=t} \tag{4}$$

为了求解双线性方程，我们作如下假设：

$$\begin{cases} f=1+s^2+h^2, \\ g_R=b_0+b_1s+b_2h+b_3s^2+b_4h^2, \\ g_I=c_0+c_1s+c_2h+c_3s^2+c_4h^2, \end{cases} \tag{5}$$

其中

$$\begin{cases} s=a_1x+a_2y+a_3z+a_4t+a_5, \\ h=a_6x+a_7y+a_8z+a_9t+a_{10}, \end{cases} \tag{6}$$

这里的待定系数 $a_i\,(1\le i\le 10), b_j, c_j\,(0\le j\le 4)$ 全都是实数。

将(5)和(6)代入双线性方程(3)中，我们得到了一个包含自变量 x,y,z 和 t 的非线性方程组。将这些方程的系数分别取零后得到一个关于待定系数的超定代数方程组，利用 Maple 求解如下：

$$\left\{a_2=2a_3, a_4=\frac{\left(a_3^2-a_1^4\right)\left[3\left(a_1^4+a_3^2\right)^2+16a_1^4\right]}{a_1\left(a_1^4+a_3^2\right)^2}, a_6=0, a_7=2a_1^2, a_8=\frac{1}{2}a_7, a_9=\frac{2a_1a_3\left[3\left(a_1^4+a_3^2\right)^2-16a_1^4\right]}{\left(a_1^4+a_3^2\right)^2},\right.$$
$$\left. b_0=\frac{b_3\left(a_3^2-3a_1^4\right)}{a_1^4+a_3^2}, b_1=kc_1, b_2=kc_2, b_4=b_3, c_0=-kb_0, c_1=-\frac{4a_3b_3a_1^2}{a_1^4+a_3^2}, c_2=\frac{4a_1^4b_3}{a_1^4+a_3^2}, c_3=-kb_3, c_4=c_3\right\} \tag{7}$$

这里 k 是附加的实常数，需满足下列条件：

$$b_3^2\left(1+k^2\right)=1 \tag{8}$$

由此可以得到(3+1)维 Mel'nikov 方程的一类 Lump 解：

$$\begin{cases} u=\dfrac{4a_1^2\left(1-s^2+h^2\right)}{\left(1+s^2+h^2\right)^2}, \\ \phi=\dfrac{g_R+ig_I}{1+s^2+h^2}, \end{cases} \tag{9}$$

其中

$$\begin{aligned} g_R&=\frac{b_3\left(a_3^2-3a_1^4\right)}{a_1^4+a_3^2}-k\frac{4a_3b_3a_1^2}{a_1^4+a_3^2}s+k\frac{4a_1^4b_3}{a_1^4+a_3^2}h+b_3\left(s^2+h^2\right), \\ g_I&=-k\frac{b_3\left(a_3^2-3a_1^4\right)}{a_1^4+a_3^2}-\frac{4a_3b_3a_1^2}{a_1^4+a_3^2}s+\frac{4a_1^4b_3}{a_1^4+a_3^2}h-kb_3\left(s^2+h^2\right), \end{aligned} \tag{10}$$

且

$$\begin{aligned} s&=a_1x+2a_3y+a_3z+\frac{\left(a_3^2-a_1^4\right)\left[3\left(a_1^4+a_3^2\right)^2+16a_1^4\right]}{a_1\left(a_1^4+a_3^2\right)^2}t+a_5, \\ h&=2a_1^2y+a_1^2z+\frac{2a_1a_3\left[3\left(a_1^4+a_3^2\right)^2-16a_1^4\right]}{\left(a_1^4+a_3^2\right)^2}t+a_{10}. \end{aligned} \tag{11}$$

在该 Lump 解中包含六个任意系数 $a_1, a_3, b_3, k, a_5, a_{10}$。不失一般性，依据平移不变性我们可以设 $a_5 = 0$ 或 $a_{10} = 0$。此外，对于任意给定时间 t，在上述 Lump 解中 $u \to 0$，$|\phi|^2 \to b_3^2(1+k^2) = 1$ 当且仅当对应的 $s^2 + h^2 \to 0$，或者换句话说，当 $a_1 \neq 0$ 时，$x^2 + y^2 + z^2 \to \infty$。因此，条件 $a_1 \neq 0$ 既保证了 Lump 解的解析性，又保证了 Lump 解的局域性。因此本文得到的解在空间所有方向上都是有理局域化的。为了获得更多的动力学性质，我们给定了一些参数值，并作出它们的二维和三维图像，如图 1~4 所示。

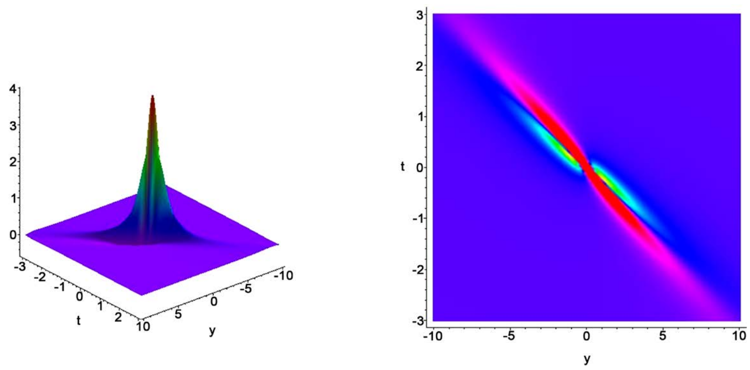

Figure 1. Plots of lump solution for u with $a_1 = 1, a_3 = 2, b_3 = -0.8, a_5 = a_{10} = 0, k = 0.75$ when $x = 0, z = 0$

图 1. 当 $x = 0, z = 0$，$a_1 = 1, a_3 = 2, b_3 = -0.8, a_5 = a_{10} = 0, k = 0.75$ 时 u 的图像

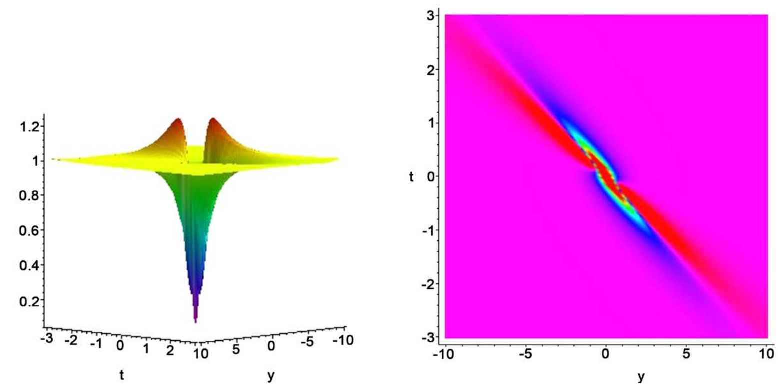

Figure 2. Plots of lump solution for $|\phi|^2$ with $a_1 = 1, a_3 = 2, b_3 = -0.8, a_5 = a_{10} = 0, k = 0.75$ when $x = 0, z = 0$

图 2. 当 $x = 0, z = 0$，$a_1 = 1, a_3 = 2, b_3 = -0.8, a_5 = a_{10} = 0, k = 0.75$ 时 $|\phi|^2$ 的图像

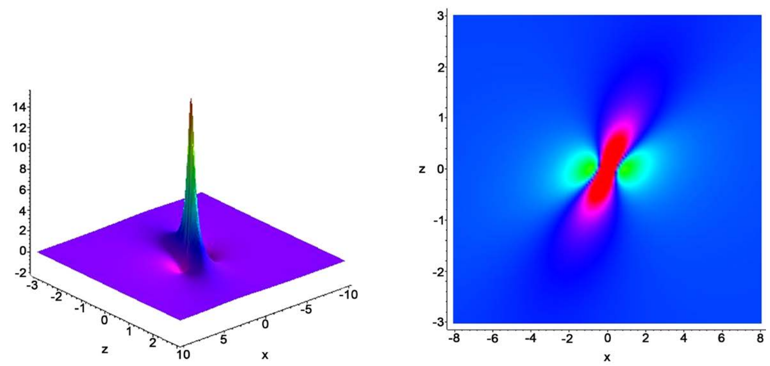

Figure 3. Plots of lump solution for u with $a_1 = 2, a_3 = -2, b_3 = 0.8, a_5 = a_{10} = 0, k = 0.75$ when $y = 0, t = 0$

图 3. 当 $y = 0, t = 0$，$a_1 = 2, a_3 = -2, b_3 = 0.8, a_5 = a_{10} = 0, k = 0.75$ 时 u 的图像

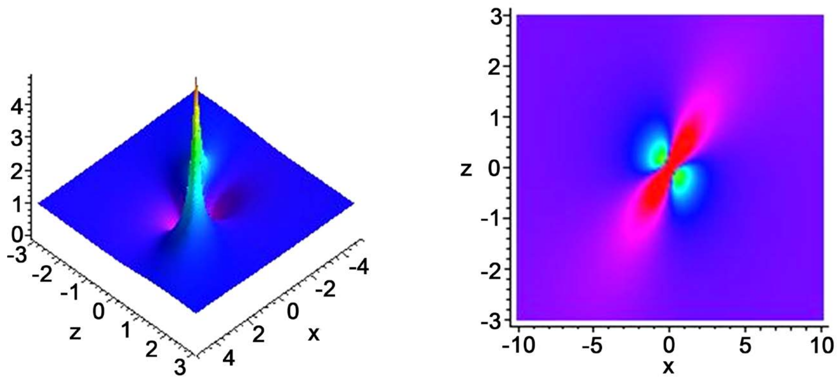

Figure 4. Plots of lump solution for $|\phi|^2$ with $a_1=2, a_3=-2, b_3=0.8, a_5=a_{10}=0, k=0.75$ when $y=0, t=0$

图 4. 当 $y=0, t=0$，$a_1=2, a_3=-2, b_3=0.8, a_5=a_{10}=0, k=0.75$ 时 $|\phi|^2$ 的图像

3. 结论

本文我们利用 Hirota 双线性方法给出了(3+1)维 Mel'nikov 方程的 Lump 解。通过作图我们观察到了方程更多的动力学性质。但是本文我们通过符号计算法仅得到了一阶 Lump 解，未来对于更高阶的解的研究将会是一个非常有意义的问题。

基金项目

本文工作受到了中央高校基本科研业务费(2019MS050)的资助。

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