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脉冲控制下变系数时变时滞模糊细胞神经网络的同步

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摘 要

通过设计脉冲控制，研究了变系数时变时滞模糊细胞神经网络的同步问题。采用**Lyapunov**泛函方法和矩阵不等式方法，给出了保证系统同步的线性矩阵不等式条件。同时，给出了指数同步条件和未知参数的渐近行为。最后，通过仿真实例验证了该方法的有效性。

关键词

模糊细胞神经网络，脉冲控制，同步，时变时滞，变系数

Synchronization of Fuzzy Cellular Neural Networks with Variable Coefficients and Time Delays under Impulse Control

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Abstract

This paper investigates the synchronization of fuzzy cellular neural networks with variable coefficients and time-varying delay by designing a impulsive control. By taking Lyapunov functional method and the matrix inequality method, the linear matrix inequality conditions are given to ensure the synchronization of the system. Meanwhile, exponential synchronization conditions and asymptotic behavior of unknown parameters are derived. Finally, a simulation example is given to verify the effectiveness of the proposed method.

Keywords

Fuzzy Cellular Networks, Impulsive Control, Synchronization, Time-Varying, Variable Coefficients

1. 引用

模糊细胞神经网络是杨 [1,2]等人在细胞神经网络与模糊逻辑相结合的基础上提出的一种神经网络模型。在过去的几十年中，对于模糊细胞神经网络的同步问题有许多不同的研究方法, 如自适应控制 [3]、脉冲控制 [4–7]、周期间歇控制 [8]。众所周知, 脉冲控制只在离散时刻起作用。因此, 与连续控制相比, 脉冲控制在降低成本和信息传递方面具有很大的优势。

在文献 [3]研究的模型基础上，本文研究了在脉冲影响下的具有扩散项的变系数模糊细胞神经网络同步问题。主要工作概括如下: 1)本文模糊细胞神经网络模型综合考虑了不确定参数和时变时滞; 2) 通过一个线性矩阵不等式来确保主从系统的同步性; 3) 构造Lyapunov-Krasovskii 泛函，得出了模糊细胞神经网络同步的若干判据, 并给出了未知系数的渐近性态。

2. 模型建立

本文主要研究了一类时变时滞且参数未知的模糊细胞神经网络基于脉冲控制下同步问题。

$$\begin{cases} \dot{u}_i(t,x) = \sum_{k=1}^{m} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}) - \omega_i u_i(t,x) + \sum_{j=1}^{n} b_{ij}\tilde{f}_j(u_j(t,x)) + \sum_{j=1}^{n} a_{ij}v_j + I_i \\ \qquad + \bigwedge_{j=1}^{n} \eta_{ij}\tilde{f}_j[u_j(t-\tau(t),x)] + \bigwedge_{j=1}^{n} T_{ij}v_j \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij}\tilde{f}_j[u_j(t-\tau(t),x)] + \bigvee_{j=1}^{n} S_{ij}v_j, i=1,2,\cdots,n, t\geq 0, t\neq t_k, \\ \triangle u_i(t_k,x) = u_i(t_k^+,x) - u_i(t_k^-,x) = -\theta_{ik}u_i(t_k,x), k\in\mathbb{Z}_+, t=t_k, \\ u_i(t,x) = 0, (t,x)\in[-\tau,+\infty]\times\partial\Omega, \quad i=1,2,...,n \\ u_i(t,x) = \psi_i(t,x), (t,x)\in[-\tau,0]\times\Omega, \quad i=1,2,...,n. \end{cases} \tag{2.1}$$

其中 $u_i(t,x)$ 对应于时刻t 和空间x 的神经单元的状态; n 表示神经元的个数; $x=(x_1,...x_m)^T\in\Omega\subset\mathbb{R}^m$ 且$\Omega=\{x=(x_1,...,x_m)^T :\mid x\mid< w_k, k=1,2,...,m\}$ 是具有光滑边界$\partial\Omega$ 的有界紧集且$mes\Omega>0$; $D_{ik}=D_{ik}(t,x)\geq 0$ 是扩散系数; $\omega_i>0$ 表示当神经网络未连接且没有外部附加电压差时, 第i 个神经元返回孤立静息状态的速率; $\eta_{ij}(\beta_{ij})$, $T_{ij}(S_{ij})$ 分别是模糊反馈最小（大）模板和模糊前馈最小（大）模板的元素. a_{ij} 和b_{ij} 是前馈模板和反馈模板的元素. $\bigwedge$ ($\bigvee$) 表示模糊和(或). I_i 和v_i 分别表示第i 个神经元的输入和偏置. $0<\tau(t)<\tau$ 是时变延迟, $\tilde{f}$ 是激励函数; $\psi_i(s,x)(i=1,2,...,n)$ 表示在$[-\tau,0]\times\Omega$ 是有界的和连续的. θ_{ik} $(i=1,2,...,n.\ k\in\mathbb{Z}_+)$ 是脉冲强度. $\{t_k|k=1,2,...\}$ 满足 $0\leq t_0<t_1<t_2<...<t_k<...,\lim_{k\to\infty}t_k=+\infty$, $\lim_{t\to t_k^+}u_i(t,x)=u_i(t_k^+,x)$, $u_i(t_k^-,x)=u_i(t_k,x)$.

模型(2.1)的从系统为

$$\begin{cases} \dot{\tilde{u}}_i(t,x) = \sum_{k=1}^{m} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial \tilde{u}_i(t,x)}{\partial x_k}) - \bar{\omega}_i(t)\tilde{u}_i(t,x) + \sum_{j=1}^{n} \bar{b}_{ij}(t)\tilde{f}_j(\tilde{u}_j(t,x)) + \sum_{j=1}^{n} a_{ij}v_j \\ \qquad + I_i + \bigwedge_{j=1}^{n} \bar{\eta}_{ij}(t)\tilde{f}_j[\tilde{u}_j(t-\tau(t),x)] + \bigwedge_{j=1}^{n} T_{ij}v_j + \bigvee_{j=1}^{n} \bar{\beta}_{ij}(t)\tilde{f}_j[\tilde{u}_j(t-\tau(t),x)] \\ \qquad + \bigvee_{j=1}^{n} S_{ij}v_j + \varepsilon_i(t)(\tilde{u}_i(t,x)-u_i(t,x)), i=1,2,\cdots,n, t\geq 0, t\neq t_k, \\ \triangle\tilde{u}_i(t_k,x) = \tilde{u}_i(t_k^+,x) - \tilde{u}_i(t_k^-,x) = -\theta_{ik}\tilde{u}_i(t_k,x), \quad i=1,2,...,n, k\in\mathbb{Z}_+, t=t_k, \\ \tilde{u}_i(t,x) = 0, (t,x)\in[-\tau,+\infty]\times\partial\Omega, \quad i=1,2,...,n, \\ \tilde{u}_i(t,x) = \phi_i(t,x), (t,x)\in[-\tau,0]\times\Omega, \quad i=1,2,...,n. \end{cases} \tag{2.2}$$

其中$\varepsilon(t)=(\varepsilon_1(t),\varepsilon_2(t),\cdots,\varepsilon_n(t))\in\mathbb{R}^n(i=1,2,\cdots,n)$ 表示控制器增益值. 将同步误差定义

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为$e_i(t,x)=\tilde{u}_i(t,x)-u_i(t,x)$, 我们有

$$\begin{cases}\dot{e}_i(t,x)=\sum\limits_{k=1}^{m}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k})-\omega_i e_i(t,x)+\sum\limits_{j=1}^{n}b_{ij}f_j(e_j(t,x))\\ \qquad +\bigwedge\limits_{j=1}^{n}\eta_{ij}f_j^*[e_j(t-\tau(t),x)]+\bigvee\limits_{j=1}^{n}\beta_{ij}f_j^{**}[e_j(t-\tau(t),x)]\\ \qquad -(\bar{\omega}_i(t)-\omega_i)\tilde{u}_i(t,x)+\sum\limits_{j=1}^{n}(\bar{b}_{ij}(t)-b_{ij})\tilde{f}_j(\tilde{u}_j(t,x))\\ \qquad +\bigwedge\limits_{j=1}^{n}(\bar{\eta}_{ij}(t)-\eta_{ij})\tilde{f}_j[\tilde{u}_j(t-\tau(t),x)]+\bigvee\limits_{j=1}^{n}(\bar{\beta}_{ij}(t)-\beta_{ij})\tilde{f}_j[\tilde{u}_j(t-\tau(t),x)]\\ \qquad +\varepsilon_i(t)e_i(t,x), i=1,2,\cdots,n, t\geq 0, t\neq t_k,\\ \triangle e_i(t_k,x)=e_i(t_k^+,x)-e_i(t_k^-,x)=-\theta_{ik}e_i(t_k,x),\quad i=1,2,...,n, k\in\mathbb{Z}_+, t=t_k,\\ e_i(t,x)=0,(t,x)\in[-\tau,+\infty]\times\partial\Omega,\quad i=1,2,...,n,\\ e_i(t,x)=\psi_i(t,x)-\phi_i(t,x),(t,x)\in[-\tau,0]\times\Omega,\quad i=1,2,...,n.\end{cases}\tag{2.3}$$

其中

$$\begin{aligned}&f_j(e_j(t,x))=\tilde{f}_j(\tilde{u}_j(t,x))-\tilde{f}_j(u_j(t,x)),\\ &\bigwedge_{j=1}^{n}\eta_{ij}f_j^*[e_j(t-\tau(t),x)]=\bigwedge_{j=1}^{n}\eta_{ij}\tilde{f}_j[\tilde{u}_j(t-\tau(t),x)]-\bigwedge_{j=1}^{n}\eta_{ij}\tilde{f}_j[u_j(t-\tau(t),x)],\\ &\bigvee_{j=1}^{n}\beta_{ij}f_j^{**}[e_j(t-\tau(t),x)]=\bigvee_{j=1}^{n}\beta_{ij}\tilde{f}_j[\tilde{u}_j(t-\tau(t),x)]-\bigvee_{j=1}^{n}\beta_{ij}\tilde{f}_j[u_j(t-\tau(t),x)].\end{aligned}$$

定义2.1. 对于$\psi(t,x)=(\psi_1(t,x),...\psi_n(t,x))^T\in C([-\tau,0]\times\mathbb{R}^m,\mathbb{R}^n)$ 是从$[-\tau,0]\times\mathbb{R}^m$ 到$\mathbb{R}^n$上的连续函数映射的Banach空间, 定义

$$\parallel\psi(t,x)\parallel_2=\sqrt{\sum_{i=1}^{n}\parallel\psi_i(t,x)\parallel_2^2},$$

其中$\parallel\psi_i(t,x)\parallel_2=\left(\int_\Omega\sup\limits_{-\tau\leq t\leq 0,x\in\Omega}\mid\psi_i(t,x)\mid^2dx\right)^{\frac{1}{2}}$.

定义2.2. 如果存在$L>1$, $\varepsilon>0$ 使得

$$\parallel\tilde{u}_i(t,x)-u_i(t,x)\parallel_2=\parallel e(t,x)\parallel_2\leq L\parallel\psi-\phi\parallel_2 e^{-\varepsilon t},(t,x)\in[0,+\infty)\times\Omega.$$

则称系统(2.1)和(2.2)是全局指数同步的。其中

$$\parallel\tilde{u}_i(t,x)-u_i(t,x)\parallel_2=\left(\int_\Omega\sum_{i=1}^{n}\mid\tilde{u}_i(t,x)-u_i(t,x)\mid^2dx\right)^{\frac{1}{2}}.$$

引理2.1. 若$h(x) \in C^1(\Omega, \mathbb{R})$, $h(x) \mid_{\partial\Omega} = 0, \Omega$ 是有界紧集, $\mid x_k \mid \leq w_k (k = 1, 2, ..., m)$. 则有

$$\int_\Omega h^2(x)dx \leq w_k^2 \int_\Omega \left(\frac{\partial h}{\partial x_k}\right)^2 dx.$$

引理2.2.([2]). 对于$\forall\, \eta_{ij}, \beta_{ij} \in \mathbb{R}$, 有

$$\begin{aligned} |\bigwedge_{j=1}^{n} \eta_{ij}\tilde{f}_j(u_j) - \bigwedge_{j=1}^{n} \eta_{ij}\tilde{f}_j(\tilde{u}_j)| &\leq \sum_{j=1}^{n} |\eta_{ij}||\tilde{f}_j(u_j) - \tilde{f}_j(\tilde{u}_j)|, \\ |\bigvee_{j=1}^{n} \beta_{ij}\tilde{f}_j(u_j) - \bigvee_{j=1}^{n} \beta_{ij}\tilde{f}_j(\tilde{u}_j)| &\leq \sum_{j=1}^{n} |\beta_{ij}||\tilde{f}_j(u_j) - \tilde{f}_j(\tilde{u}_j)|. \end{aligned}$$

引理2.3. 对于 $\forall\, x, y \in \mathbb{R}^n$ 和正定矩阵 $Q \in \mathbb{R}^{n\times n}$, 有

$$2x^T y \leq x^T Q x + y^T Q^{-1} y.$$

引理2.4. ($Schur$补引理) 对于分块矩阵

$$X = \begin{pmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{pmatrix} > 0,$$

其中X_{11}, X_{22} 是方阵, 下列条件是等价的:

1) $X_{11} > 0,\ \ X_{22} - X_{12}^T X_{11}^{-1} X_{12} > 0,$

2) $X_{22} > 0,\ \ X_{11} - X_{12} X_{22}^{-1} X_{12}^T > 0.$

3. 主要结果

为了确保系统(2.1) 和(2.2)同步, 假设以下条件成立:

(H_1). $\tilde{f}_i(x)$ 是Lipschitz 连续的, 存在 $h_i > 0$ 使得

$$|\tilde{f}_i(x) - \tilde{f}_i(y)| \leq h_i|x - y|,$$

$\forall x, y \in \mathbb{R}$, $x \neq y, i = 1, 2, \cdots, n$. 且$h = \max\limits_{1\leq i\leq n}(h_i^2)$.

(H_2). $\tau(t) > 0, 0 \leq \dot{\tau}(t) \leq \mu \leq 1$, 对任意的$t$, 这里$\mu$ 是常数.

(H_3). $\theta_{ik} \in \mathbb{R}$ 和$\theta_{ik} \in [0, 2]$, $i = 1, 2, ..., n$. $k \in \mathbb{Z}_+$.

(H_4). 下列线性矩阵不等式

$$\begin{pmatrix} \Gamma & Q(|\eta| + |\beta|) \\ (|\eta| + |\beta|)^T Q & E \end{pmatrix} > 0, \tag{3.1}$$

其中$Q=diag(q_i), W=diag(\omega_i), B=(b_{ij})_{n\times n}, |\eta|=(\eta_{ij})_{n\times n}, |\beta|=(|\beta_{ij}|)_{n\times n}, H=diag(h_i)$, $\Gamma=QD+QW-\lambda Q-\frac{1}{2}(Q|B|H+H|B^T|Q)-\frac{1}{1-\mu}H^TH$, $\lambda>0$, E 是单位矩阵.

定理3.1. 基于$(H_1)-(H_4)$. 假设$\bar{\omega}_i(t), \bar{b}_{ij}(t), \bar{\eta}_{ij}(t), \bar{\beta}_{ij}(t), \varepsilon_i(t)(i,j=1,2,\cdots,n)$ 满足:

$$\begin{array}{rcl}\dot{\bar{\omega}}_i(t) &=& \frac{1}{\gamma_i}e^{\lambda t}e_i(t,x)\tilde{u}_i(t,x),\\ \dot{\bar{b}}_{ij}(t) &=& -\frac{1}{\alpha_{ij}}e^{\lambda t}e_i(t,x)\tilde{g}_j(\tilde{u}_j(t,x)),\\ \dot{\bar{\eta}}_{ij}(t) &=& -\frac{1}{\varrho_{ij}}e^{\lambda t}|e_i(t,x)\tilde{g}_j[\tilde{u}_j(t-\tau(t),x)]|sgn(\bar{\eta}_{ij}(t)-\eta_{ij}),\\ \dot{\bar{\beta}}_{ij}(t) &=& -\frac{1}{\sigma_{ij}}e^{\lambda t}|e_i(t,x)\tilde{g}_j[\tilde{u}_j(t-\tau(t),x)]|sgn(\bar{\beta}_{ij}(t)-\beta_{ij}),\\ \dot{\varepsilon}_i(t) &=& -\frac{1}{\delta_i}e^{\lambda t}e_i^2(t,x).\end{array} \tag{3.2}$$

并且$\varepsilon_i(t_k)=\varepsilon_i(t_k^+)$, $\bar{\omega}_i(t_k)=\bar{\omega}_i(t_k^+)$, $\bar{b}_{ij}(t_k)=\bar{b}_{ij}(t_k^+)$, $\bar{\eta}_{ij}(t_k)=\bar{\eta}_{ij}(t_k^+)$, $\bar{\beta}_{ij}(t_k)=\bar{\beta}_{ij}(t_k^+)$. q_i, ω_i, $\delta_i, \gamma_i, \alpha_{ij}, \varrho_{ij}, \sigma_{ij}$ 是任意的正常数, 则系统(2.1) 和(2.2) 同步, 并且

$$\lim_{t\to\infty}(\bar{\omega}_i(t))=\omega_i, \lim_{t\to\infty}(\bar{b}_{ij}(t))=b_{ij}, \lim_{t\to\infty}(\bar{\eta}_{ij}(t))=\eta_{ij}, \lim_{t\to\infty}(\bar{\beta}_{ij}(t))=\beta_{ij}. \tag{3.3}$$

证明. 构造Lyapunov-krasovskii 函数

$$\begin{array}{rcl}V(t) &=& \int_\Omega\{\frac{1}{2}\sum_{i=1}^{n}q_i\{e^{\lambda t}e_i^2(t,x)+\gamma_i(\bar{\omega}_i(t)-\omega_i)^2+\sum_{j=1}^{n}\alpha_{ij}(\bar{b}_{ij}(t)-b_{ij})^2\\ &+& \sum_{j=1}^{n}\varrho_{ij}(\bar{\eta}_{ij}(t)-\eta_{ij})^2+\sum_{j=1}^{n}\sigma_{ij}(\bar{\beta}_{ij}(t)-\beta_{ij})^2+\delta_i\varepsilon_i^2(t)\}\\ &+& \int_{t-\tau(t)}^{t}\frac{1}{1-\mu}e^{\lambda(s+\tau)}|f^T[e(s,x)]||f[e(s,x)]|ds\}dx.\end{array} \tag{3.4}$$

对于$t\geq 0, t\neq t_k$, $k\in\mathbb{Z}_+$, 计算(3.4)的导数,

$$\begin{array}{rcl}\dot{V}(t) &=& \int_\Omega\{\sum_{i=1}^{n}q_i\{e^{\lambda t}e_i(t,x)\frac{\partial e_i(t,x)}{\partial t}+\frac{1}{2}\lambda e^{\lambda t}e_i^2(t,x)+\gamma_i(\bar{\omega}_i(t)-\omega_i)\dot{\bar{\omega}}_i(t)\\ &+& \sum_{j=1}^{n}\alpha_{ij}(\bar{b}_{ij}(t)-b_{ij})\dot{\bar{b}}_{ij}(t)+\sum_{j=1}^{n}\varrho_{ij}(\bar{\eta}_{ij}(t)-\eta_{ij})\dot{\bar{\eta}}_{ij}(t)+\sum_{j=1}^{n}\sigma_{ij}(\bar{\beta}_{ij}(t)-\beta_{ij})\dot{\bar{\beta}}_{ij}(t)\\ &+& \delta_i\varepsilon_i(t)\dot{\varepsilon}_i(t)\}+\frac{1}{1-\mu}e^{\lambda(t+\tau)}|f^T[e(t,x)]||f[e(t,x)]|\\ &-& \frac{1-\dot{\tau}(t)}{1-\mu}e^{\lambda(t-\tau(t)+\tau)}|f^T[e(t-\tau(t),x)]||f[e(t-\tau(t),x)]|\}dx\\ &\leq& \int_\Omega\{\sum_{i=1}^{n}q_i\{e^{\lambda t}e_i(t,x)\sum_{k=1}^{m}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k})-\omega_i e^{\lambda t}e_i^2(t,x)\\ &+& \sum_{j=1}^{n}b_{ij}e^{\lambda t}e_i(t,x)f_j(e_j(t,x))+\lambda e^{\lambda t}e_i^2(t,x)+\sum_{j=1}^{n}|\eta_{ij}|e^{\lambda t}e_i(t,x)|f_j[e_j(t-\tau(t),x)]|\\ &+& \sum_{j=1}^{n}|\beta_{ij}|e^{\lambda t}e_i(t,x)|f_j[e_j(t-\tau(t),x)]|\}+\frac{1}{1-\mu}e^{\lambda(t+\tau)}|f^T[e(t,x)]||f[e(t,x)]|\\ &-& \frac{1-\dot{\tau}(t)}{1-\mu}e^{\lambda(t-\tau(t)+\tau)}|f^T[e(t-\tau(t),x)]||f[e(t-\tau(t),x)]|\}dx,\end{array}$$

由引理 2.1 及边值条件得

$$
\begin{aligned}
&\int_\Omega e_i(t,x)\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right)dx\\
&=\int_\Omega e_i(t,x)\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right)dx\\
&=\int_\Omega e_i(t,x)\nabla\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right)_{k=1}^{m}dx\\
&=\left(\int_{\partial\Omega} e_i(t,x)\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right)_{k=1}^{m}dx-\int_\Omega\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right)_{k=1}^{m}\nabla e_i(t,x)dx\right)\\
&=-\int_\Omega\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right)_{k=1}^{m}\nabla e_i(t,x)dx\\
&=-\sum_{k=1}^{m}\int_\Omega D_{ik}\left(\frac{\partial e_i(t,x)}{\partial x_k}\right)^2dx\\
&\leq-\sum_{k=1}^{m}\int_\Omega \frac{D_{ik}}{w_k^2}e_i^2(t,x)dx,
\end{aligned}
$$

其中$\nabla=(\frac{\partial}{\partial x_1},\frac{\partial}{\partial x_2},...,\frac{\partial}{\partial x_m})^T$ 是梯度算子, 并且

$$
(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k})_{k=1}^{m}=(D_{i1}\frac{\partial e_i(t,x)}{\partial x_1},D_{i2}\frac{\partial e_i(t,x)}{\partial x_2},...D_{im}\frac{\partial e_i(t,x)}{\partial x_m}).
$$

根据(H_1) 和引理 2.2, 我们有

$$
\begin{aligned}
\dot{V}(t)\ \leq\ &\int_\Omega\{-\sum_{i=1}^{n}q_i(\sum_{k=1}^{m}\frac{D_{ik}}{w_k^2}+\omega_i)e^{\lambda t}e_i^2(t,x)+\sum_{i=1}^{n}\sum_{j=1}^{n}e^{\lambda t}q_i|e_i(t,x)||b_{ij}||f_j(e_j(t,x))|\\
+\ &\sum_{i=1}^{n}q_i\lambda e^{\lambda t}e_i^2(t,x)+\sum_{i=1}^{n}\sum_{j=1}^{n}e^{\lambda t}q_i|e_i(t,x)||\eta_{ij}||f_j[e_j(t-\tau(t),x)]|\\
+\ &\sum_{i=1}^{n}\sum_{j=1}^{n}e^{\lambda t}q_i|e_i(t,x)||\beta_{ij}||f_j[e_j(t-\tau(t),x)]|+\tfrac{1}{1-\mu}e^{\lambda(t+\tau)}|f^T[e(t,x)]||f[e(t,x)]|\\
-\ &\tfrac{1-\dot{\tau}(t)}{1-\mu}e^{\lambda(t-\tau(t)+\tau)}|f^T[e(t-\tau(t),x)]||f[e(t-\tau(t),x)]|\}dx\\
\leq\ &\int_\Omega\{-e^{\lambda t}e^T(t,x)(QD+QW)e(t,x)+e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n}q_i|e_i(t,x)||b_{ij}||h_i||e_j(t,x)|\\
+\ &\lambda e^{\lambda t}e^T(t,x)Qe(t,x)+e^{\lambda t}|e^T(t,x)|Q(|\eta|+|\beta|)|f[e(t-\tau(t),x)]|\\
-\ &\tfrac{1-\dot{\tau}(t)}{1-\mu}e^{\lambda(t-\tau(t)+\tau)}|f^T[e(t-\tau(t),x)]||f[e(t-\tau(t),x)]|\\
+\ &\tfrac{1}{1-\mu}e^{\lambda(t+\tau)}|f^T[e(t,x)]||f[e(t,x)]|\}dx,
\end{aligned}
$$

由(H_2) 和引理 2.3, 可得

$$\begin{aligned}
\dot{V}(t) &\le \int_\Omega \{-e^{\lambda t} e^T(t,x)(QD+QW)e(t,x) + e^{\lambda t}|e^T(t,x)|\frac{1}{2}(Q|B|H+H|B^T|Q)|e(t,x)| \\
&+ \lambda e^{\lambda t} e^T(t,x) Q e(t,x) + \tfrac{1}{2} e^{\lambda t}|e^T(t,x)|Q(|\eta|+|\beta|)(|\eta|+|\beta|)^T Q^T|e(t,x)| \\
&+ \tfrac{1}{2} e^{\lambda t} f^T[e(t-\tau(t),x)]||f[e(t-\tau(t),x)]| \\
&- \tfrac{1-\dot{\tau}(t)}{1-\mu} e^{\lambda(t-\tau(t)+\tau)}|f^T[e(t-\tau(t),x)]||f[e(t-\tau(t),x)]| \\
&+ \tfrac{1}{1-\mu} e^{\lambda(t+\tau)}|f^T[e(t,x)]||f[e(t,x)]|\}dx \\
&\le \int_\Omega e^{\lambda t}\{-|e^T(t,x)|\{QD+QW-\lambda Q-\frac{1}{2}(Q|B|H+H|B^T|Q) \\
&- \tfrac{1}{2}Q(|\eta|+|\beta|)(|\eta|+|\beta|)^T Q^T - \tfrac{1}{1-\mu}H^T H\}|e(t,x)|\}dx \\
&= -Ke^{\lambda t} \parallel e_i(t,x) \parallel_2^2 \le 0,
\end{aligned}$$

其中$K = QD+QW-\lambda Q-\frac{1}{2}(Q|B|H+H|B^T|Q)-\frac{1}{2}Q(|\eta|+|\beta|)(|\eta|+|\beta|)^T Q^T - \frac{1}{1-\mu}H^T H$. 通过引理2.4得, $K>0$. 然后有$V(t) \le -Ke^{\lambda t} \parallel e_i(t,x) \parallel_2^2 \le 0$.因此$V(t) \le V(t_{k-1}^+)$对任意的$t \in (t_{k-1}, t_k]$, $k \in \mathbb{Z}_+$, 这里$V(0^+) = V(0)$. 由(2.3)(3.2)(3.4) 和(H_3), 可知

$$\begin{aligned}
V(t_k^+) &= \int_\Omega \{\frac{1}{2}\sum_{i=1}^n q_i\{e^{\lambda t_k^+} e_i^2(t_k^+,x) + \gamma_i(\bar{\omega}_i(t_k^+)-\omega_i)^2 + \sum_{j=1}^n \alpha_{ij}(\bar{b}_{ij}(t_k^+)-b_{ij})^2 \\
&+ \sum_{j=1}^n \varrho_{ij}(\bar{\eta}_{ij}(t_k^+)-\eta_{ij})^2 + \sum_{j=1}^n \sigma_{ij}(\bar{\beta}_{ij}(t_k^+)-\beta_{ij})^2 + \delta_i \varepsilon_i^2(t_k^+)\} \\
&+ \int_{t_k^+-\tau(t_k)}^{t_k^+} \frac{1}{1-\mu} e^{\lambda(s+\tau)}|f^T[e(s,x)]||f[e(s,x)]|ds\}dx \\
&= \int_\Omega \{\frac{1}{2}\sum_{i=1}^n q_i\{e^{\lambda t_k}(1-\theta_{ik})^2 e_i^2(t_k,x) + \gamma_i(\bar{\omega}_i(t_k)-\omega_i)^2 + \sum_{j=1}^n \alpha_{ij}(\bar{b}_{ij}(t_k)-b_{ij})^2 \\
&+ \sum_{j=1}^n \varrho_{ij}(\bar{\eta}_{ij}(t_k)-\eta_{ij})^2 + \sum_{j=1}^n \sigma_{ij}(\bar{\beta}_{ij}(t_k)-\beta_{ij})^2 + \delta_i \varepsilon_i^2(t_k)\} \\
&+ \int_{t_k-\tau(t_k)}^{t_k} \frac{1}{1-\mu} e^{\lambda(s+\tau)}|f^T[e(s,x)]||f[e(s,x)]|ds \\
&- \int_{t_k-\tau(t_k)}^{t_k^+-\tau(t_k)} \frac{1}{1-\mu} e^{\lambda(s+\tau)}|f^T[e(s,x)]||f[e(s,x)]|ds \\
&+ \int_{t_k}^{t_k^+} \frac{1}{1-\mu} e^{\lambda(s+\tau)}|f^T[e(s,x)]||f[e(s,x)]|ds\}dx \\
&\le V(t_k).
\end{aligned}$$

因此, 系统(2.1) 和(2.2) 是同步的. 定理3.1证毕.

推论3.1. 基于定理2.1的条件, 如果$\varepsilon_i(0)=0$, $\bar{\omega}_i(0)=\omega_i$, $\bar{b}_{ij}(0)=b_{ij}$, $\bar{\eta}_{ij}(0)=\eta_{ij}$, $\bar{\beta}_{ij}(0)=\beta_{ij}$. 则系统(2.1) 和(2.2)全局指数同步.

注3.1. 当$\theta_{ik} \equiv 0, i=1,2,...,n.\ k \in \mathbb{Z}_+$, 系统(2.1) 和(2.2) 退化为文献 [3, 4]中的无脉冲效应的延迟细胞神经网络的情况.

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4. 数值模拟

例4.1. 在(2.1)式中取$D_{ik}=0$，$i=1,2$.

$$(\omega_{ij})_{2\times 2}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\ (b_{ij})_{2\times 2}=\begin{pmatrix}2.0 & -0.1\\ -5.0 & 2.8\end{pmatrix},$$

$$(\beta_{ij})_{2\times 2}=(\eta_{ij})_{2\times 2}=\begin{pmatrix}-1.5 & -0.1\\ -0.21 & -1.8\end{pmatrix},\tau(t)=\frac{e^t}{1+e^t},\tilde{g}_j(t)=tanh(t).$$

初始条件如下：$u_1(t)=-0.01,\ u_2(t)=0.05,\ \tilde{u}_1(t)=-0.5,\ \tilde{u}_2(t)=0.5,$
$\varepsilon_1(0)=-0.1,\ \varepsilon_2(0)=0.02;\quad b_1(0)=-0.02,\ b_2(0)=4.46,$
$\eta_1(0)=-0.02,\ \eta_2(0)=0.01;\beta_1(0)=0.2,\ \beta_2(0)=-0.2.\theta_{ik}=0.1.$

系统参数渐近规律

$$\begin{aligned}
\dot{\bar{b}}_1(t) &= -9.43\times(\tilde{u}_1-u_1)tanh(\tilde{u}_1),\quad \dot{\bar{b}}_2(t)=-2.173\times(\tilde{u}_2-u_2)tanh(\tilde{u}_2).\\
\dot{\bar{\eta}}_1(t) &= -6.65\times sgn(\bar{\eta}_1+1.5)|(\tilde{u}_1-u_1)tanh(\tilde{u}_1(t-\tau))|,\\
\dot{\bar{\eta}}_2(t) &= -1.45\times sgn(\bar{\eta}_2+1.8)|(\tilde{u}_2-u_2)tanh(\tilde{u}_2(t-\tau))|.\\
\dot{\bar{\beta}}_1(t) &= -9.22\times sgn(\bar{\beta}_1+1.5)|(\tilde{u}_1-u_1)tanh(\tilde{u}_1(t-\tau))|,\\
\dot{\bar{\beta}}_2(t) &= -2.5\times sgn(\bar{\beta}_2+1.8)|(\tilde{u}_2-u_2)tanh(\tilde{u}_2(t-\tau))|.\\
\dot{\varepsilon}_1(t) &= -0.5\times(\tilde{u}_1-u_1)^2,\quad \dot{\varepsilon}_2(t)=-0.35\times(\tilde{u}_2-u_2)^2.
\end{aligned}$$

通过图 1和图 2可知, 系统(2.1)和(2.2) 同步, 并且系统(2.2)的系数, $\bar{b}_1,\bar{b}_2$,$\bar{\eta}_1,\bar{\eta}_2$,$\bar{\beta}_1,\bar{\beta}_2$,分别渐近趋向于2.0, 2.8, -1.5, -1.8, -1.5, -1.8.

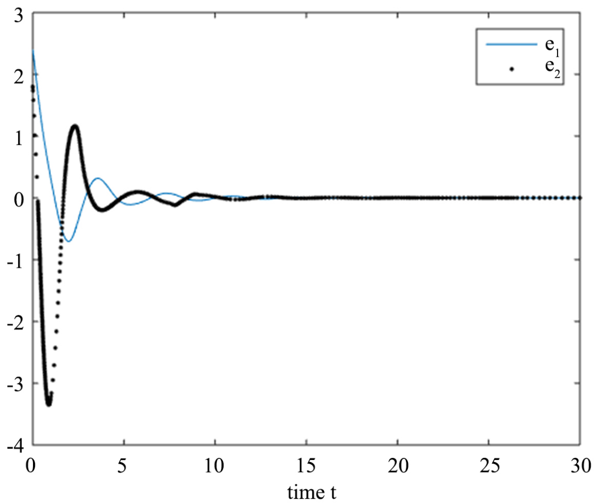

Figure 1. Synchronization errors $e_1(t)$ and $e_2(t)$
图 1. 误差同步$e_1(t)$ 和$e_2(t)$

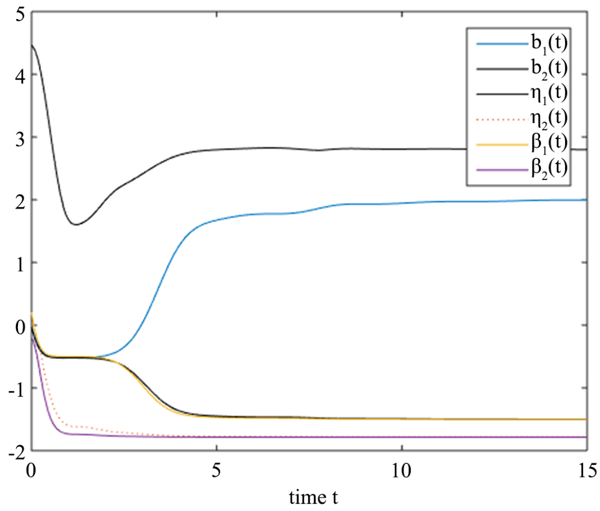

Figure 2. Asymptotic behavior of system (2.2) parameters

图 2. 系统(2.2) 系数的渐近图像

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