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基于不动点理论的分数阶模糊细胞神经网络的稳定性分析

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摘　要

针对具有时变时滞的分数阶模糊细胞神经网络，采用Banach不动点理论和方法，直接得到了分数阶神经网络的一个新的稳定性判据。该方法较为新颖，得到比已有结果更简单的结论。最后，给出了一个数值例子来说明所提方法的有效性。

关键词

分数阶神经网络，压缩映射理论，全局指数稳定性

Stability Analysis of Fractional Order Fuzzy Cellular Neural Networks via Fixed Point Approach

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Abstract

For fractional-order fuzzy cellular neural networks with time-varying delays, Banach fixed point

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theory and technique are employed to derive a new stability criterion of fractional-order neural networks. This method is relatively novel, and a simpler conclusion is obtained than the existing results. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Keywords

Fractional Order Neural Networks, Contraction Mapping Theory, Global Exponential Stability

1. 引言

近年来，许多研究者提出并研究了细胞神经网络(CNN) [1] [2]。到目前为止，已经提出了三种基本的细胞神经网络(CNN)结构。第一个是传统的 CNN [3]；第二种是延迟型 CNN [4]；最后一个是离散时间 CNN [5]。与传统的 CNN 结构不同，模糊细胞神经网络(FCNN)除了产品运算的总和外，还在其模板输入和/或输出之间实现了模糊逻辑。与细胞神经网络不同，模糊细胞神经网络将模糊逻辑集成到传统细胞神经系统的结构中，并保持细胞之间的邻近连接。模糊细胞神经网络(FCNNs)，由 Yang 等人[6]提出，并具有广泛应用，如思维和推理、白细胞检测和模式识别[7] [8] [9]。

神经网络模型的设计和实际适用性在很大程度上取决于其稳定性特性。与传统 CNN 类似，系统的稳定性在 FCNN 的设计中非常重要，前人已经提出了 FCNN 的几个条件[10] [11] [12]。分数阶神经网络在时序预测、图像处理、自然语言处理方面[13] [14] [15]有重要应用。稳定性是不同类别的分数阶模型中研究最多的定性之一，包括分数阶神经网络模型，现已有很多研究分数阶神经网络的解的存在唯一性和稳定性分析，但基本使用的是 Lyapunov 函数方法[16]-[21]。然而，使用不动点理论直接得出稳定性的，基本是对于整数阶神经网络[22] [23] [24]。

因此，本文应用不动点理论研究了分数阶模糊细胞神经网络的全局指数稳定性问题。利用 Caputo 分数阶积分导数等定义进行计算，在乘积空间上构造映射，并证明了这个映射是连续的、自射的、压缩的，从而直接得到了该系统解的稳定性判据。方法不同，应用对象不同，结论也就不同于以往文献。

2. 预备知识

模型是在 n 维欧式空间 R^n 中，Ω 有界开集，光滑边界 $\partial\Omega$，测度 $mes\Omega>0$。考虑下列分数阶模糊细胞神经网络系统

$$\begin{aligned}{}_{\Omega}^{C}D_t^{\alpha}w_i(t,x)=&-c_iw_i(t,x)+\sum_{j=1}^{m}a_{ij}g_j(w_j(t,x))+\sum_{j=1}^{m}b_{ij}g_j(w_j(t-\tau_j(t),x))+I_i\\&+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}u_{ij}g_j(w_j(t-\tau_j(t),x))+\bigwedge_{j=1}^{m}R_{ij}\mu_j+\bigvee_{j=1}^{m}v_{ij}g_j(w_j(t-\tau_j(t),x))+\bigvee_{j=1}^{m}Q_{ij}\mu\end{aligned}\tag{1}$$

狄利克雷(Dirichlet)边界和初始条件：

$$\begin{gathered}w_i(s,x)=\xi_i(s,x),\ s\in[-\tau,0],\ x\in\Omega_j\\w_i(t,x)=0,\ t\in[-\tau,\infty),\ x\in\partial\Omega\end{gathered}\tag{2}$$

其中，$t>0$，$i=1,2,\cdots,n$，$n\ge 2$，对应于神经网络中的单元数；$w=(w_1,w_2\cdots,w_n)^{\mathrm{T}}\in R^n$，$w_i(t,x)$对应于第 i 个单元在时间 t 和空间 x 的状态；$g_j(w_j)$表示第 j 个单元的激活函数。进一步地，a_{ij},b_{ij},I_i,c_i是常数；a_{ij}表示在时间 t 和空间 x 中第 i 个单元上第 j 个单元的强度；b_{ij}表示在时间$t-\tau_j(t)$和空间 x 中第 i 个单元上第 j 个单元的强度；I_i表示第 i 个单元上的外部偏置，c_i表示当与网络和外部输入断开连接时，第 i 个单元将其电位重置为隔离状态的速率。此外，$\tau_j(t)$对应于沿第 j 个单元的轴突的传输延迟。h_{ij}是前馈模板的元素。$u_{ij},v_{ij},R_{ij},Q_{ij}$分别是模糊反馈最小模板、模糊反馈最大模板的元素。$\wedge$和$\vee$分别表示模糊或运算和模糊和运算。

注 1：虽然文献[18]中也应用了 Banach 不动点定理，但本文所用的方法与文献[18]不同，我们直接用 Banach 不动点定理导出了稳定性判据。

定义 1 [25]对于任意$t>0$，函数$l\in C^1\left[[0,b]\times\Omega,R\right],b>0$，$0<\alpha<1$的下限为 0，则$\alpha$阶的时间 Caputo 分数阶导数定义为：

$$ {}_0^C D_t^\alpha l(t,x)=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{\partial l(s,x)}{\partial s}\frac{\mathrm{d}s}{(t-s)^\alpha}. $$

Γ是伽马函数。

定义 2 [24] 对于任意的初始条件$(\xi(s))\in C\left([-\tau,0],R^{n+m}\right)$，系统(1)和(2)被称为全局指数稳定的，如果存在$a>0,b>0$使得

$$ \left\|\left(w(t;s,\xi)\right)\right\|\le be^{-at},t>0 $$

定义的范数为$\|w(t)\|=\left(\sum_{i=1}^n|w_i(t)|^2\right)^{1/2}$，$\xi=(\xi_1,\cdots,\xi_n)^{\mathrm{T}}$，$w=(w_1,\cdots,w_n)^{\mathrm{T}}$。

引理 1 [26] 设区间$[0,t]$被分为 M 份，记为$0=t_0<t_1<\cdots<t_M=t$，$\Delta t_i=t_i-t_{i-1}$，$i=1,\cdots,M$，于是有

$$ {}_0^C D_t^\alpha w(t_M)=\frac{1}{\Gamma(2-\alpha)}\sum_{i=1}^M d_i w(t_i). $$

其中，$i=0$时，$d_0=\dfrac{t_M^{1-\alpha}-(t_M-t_1)^{1-\alpha}}{t_1}$；$i\in(1,M-1)$时，

$d_i=\dfrac{(t_M-t_{i-1})^{1-\alpha}-(t_M-t_i)^{1-\alpha}}{\Delta t_i}-\dfrac{(t_M-t_i)^{1-\alpha}-(t_M-t_{i+1})^{1-\alpha}}{\Delta t_{i+1}}$；$i=M$ 时，$d_M=\dfrac{(t_M-t_{M-1})^{1-\alpha}}{t_M-t_{M-1}}$。

引理 2 [6] x 与 x' 是模糊细胞神经网络的两种状态，于是我们有

$$ \left|\mathop{\wedge}_{j=1}^{mn}u_{ij}f_j(x_j)-\mathop{\wedge}_{j=1}^{mn}u_{ij}f_j(x_j')\right|\le\sum_{j=1}^{mn}|u_{ij}|\left|f_j(x_j)-f_j(x_j')\right|; $$

$$ \left|\mathop{\vee}_{j=1}^{mn}u_{ij}f_j(x_j)-\mathop{\vee}_{j=1}^{mn}u_{ij}f_j(x_j')\right|\le\sum_{j=1}^{mn}|u_{ij}|\left|f_j(x_j)-f_j(x_j')\right|. $$

为了确保接下来的计算和证明，我们提出了以下假设：

1. (Lipschitz 连续)激活函数g_j是 Lipschitz 连续的，如果存在常数$L_j>0\ (j=1,2,\cdots,m)$使得

$$ |g_j(z)-g_j(w)|\le L_j|z-w| $$

并且有$g_j(0)=0$，$z,w\in R$，$z\ne w$。

3. 主要结果

在本节中，我们假设 δ 是一个足够小的正数，并且假设 1 成立。

定理 1 系统(1)和(2)是全局指数稳定的，如果存在一个正标量 $\kappa<1$

$$\kappa=\max_{i\in\mathrm{N}}\left\{\left|\frac{1}{\Gamma(\alpha)}\right|\cdot\left[c_i+\sum_{j=1}^{m}\left(\left|a_{ij}\right|+\left|b_{ij}\right|+\left|u_{ij}\right|+\left|v_{ij}\right|\right)L_j\right]\right\}. \tag{3}$$

证明：所有的证明可以分为四个步骤：

第一步，首先要构造映射即需要构造一个算子，那么根据引理 1 以及指数函数一些性质，我们需要对于 Caputo 分数阶导数利用线性插值进行形式变换，将指数函数代入构造所需算子。

$$\begin{aligned}{}_0^CD_t^\alpha\left(e^t\cdot w(t)\right)&=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{\partial\left(e^s\cdot w(s)\right)}{\partial s}\frac{\mathrm{d}s}{(t-s)^\alpha}=\frac{1}{\Gamma(1-\alpha)}\sum_{i=1}^{M}d_ie^{t_i}w(t_i)\\&\le e^t\frac{1}{\Gamma(1-\alpha)}\sum_{i=1}^{M}d_iw(t_i)=e^t\,{}_0^CD_t^\alpha w(t)\\&=e^t\left[-c_iw_i(t,x)+\sum_{j=1}^{m}a_{ij}g_j\left(w_j(t,x)\right)+\sum_{j=1}^{m}b_{ij}g_j\left(w_j\left(t-\tau_j(t),x\right)\right)\right.\\&\quad+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}u_{ij}g_j\left(w_j\left(t-\tau_j(t),x\right)\right)+I_i\\&\quad\left.+\bigwedge_{j=1}^{m}R_{ij}\mu_j+\bigvee_{j=1}^{m}v_{ij}g_j\left(w_j\left(t-\tau_j(t),x\right)\right)+\bigvee_{j=1}^{m}Q_{ij}\mu_j\right]\end{aligned}$$

则 ${}_\Omega^CD_t^\alpha\left(\mathrm{e}^t\cdot w(t)\right)$ 取上界，再两边同时分数阶积分

$$e^t\cdot w_i(t)-\xi_i(0)=J^\alpha\left[e^t\cdot{}_\Omega^CD_t^\alpha w_i(t)\right]$$

$$w_i(t)=e^{-t}\cdot\left\{\xi_i(0)+J^\alpha\left[e^t\cdot{}_\Omega^CD_t^\alpha w_i(t)\right]\right\}$$

从而，我们就构造了一个算子 Θ，$\Theta=\left(\Theta_1,\Theta_2,\Theta_3,\cdots,\Theta_n\right)$，$\Theta_i:\chi_i\to\chi_i$ 如下：

$$\begin{cases}\Theta_iw_i(t,x)=e^{-t}\cdot\left\{\xi_i(0)+\frac{1}{\Gamma(\alpha)}\int_0^te^t(t-s)^{\alpha-1}\cdot\left[-c_iw_i(s,x)+\sum_{j=1}^{m}a_{ij}g_j\left(w_j(s,x)\right)\right.\right.\\\qquad+\sum_{j=1}^{m}b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+I_i+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigwedge_{j=1}^{m}R_{ij}\mu_j\\\qquad\left.\left.+\bigvee_{j=1}^{m}v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigvee_{j=1}^{m}Q_{ij}\mu_j\right]\mathrm{d}s\right\},\ t\ge0,i\in\mathrm{N}\\\Theta_iw_i(s,x)=\xi_i(s,x),s\in[-\tau,0],x\in\Omega\\\Theta_iw_i(t,x)=0,s\in[-\tau,\infty],x\in\partial\Omega\end{cases} \tag{4}$$

定义的距离为：

$$dist(w,z)=\max_{i\in\mathrm{N}}\left(\sup_{t\ge-\tau}\left|w_i(t,x)-z_i(t,x)\right|\right),\forall w,z\in\chi_1\times\chi_2\times\cdots\times\chi_n \tag{5}$$

可见，$\chi_1\times\chi_2\times\cdots\times\chi_n$ 是一个完备的度量空间。

第二步，要证明(4)对于 $t\in[0,+\infty)$ 上是连续的，那么令 δ 是足够小的标量。

$$
\begin{aligned}
&\left|\Theta_i w_i(t+\delta,x)-\Theta_i w_i(t,x)\right| \\
&=\Bigg|e^{-(t+\delta)}\cdot\Bigg\{\xi_i(0,x)+\frac{1}{\Gamma(\alpha)}\int_0^{t+\delta}e^s(t-s)^{\alpha-1}\cdot\Bigg[-c_iw_i(s,x)+\sum_{j=1}^m a_{ij}g_j(w_j(s,x)) \\
&\quad+\sum_{j=1}^m b_{ij}g_j(w_j(s-\tau_j(s),x))+I_i+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j(w_j(s-\tau_j(s),x))+\bigwedge_{j=1}^m R_{ij}\mu_j \\
&\quad+\bigvee_{j=1}^m v_{ij}g_j(w_j(s-\tau_j(s),x))+\bigvee_{j=1}^m Q_{ij}\mu_j\Bigg]\mathrm{d}s\Bigg\} \\
&\quad-e_i^{-t}\cdot\Bigg\{\xi_i(0,x)+\frac{1}{\Gamma(\alpha)}\int_0^t e_i^s(t-s)^{\alpha-1}\cdot\Bigg[-c_iw_i(s,x)+\sum_{j=1}^m a_{ij}g_j(w_j(s,x)) \\
&\quad+\sum_{j=1}^m b_{ij}g_j(w_j(s-\tau_j(s),x))+I_i+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j(w_j(s-\tau_j(s),x))+\bigwedge_{j=1}^m R_{ij}\mu_j \\
&\quad+\bigvee_{j=1}^m v_{ij}g_j(w_j(s-\tau_j(s),x))+\bigvee_{j=1}^m Q_{ij}\mu_j\Bigg]\mathrm{d}s\Bigg\}\Bigg| \\
&\le\left|e^{-(t+\delta)}\cdot\xi_i(0,x)-e^{-t}\cdot\xi_i(0,x)\right|+\Bigg|\frac{e^{-(t+\delta)}}{\Gamma(\alpha)}\int_0^{t+\delta}e^s(t-s)^{\alpha-1}\cdot\Bigg[-c_iw_i(s,x)+\sum_{j=1}^m a_{ij}g_j(w_j(s,x)) \\
&\quad+\sum_{j=1}^m b_{ij}g_j(w_j(s-\tau_j(s),x))+I_i+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j(w_j(s-\tau_j(s),x))+\bigwedge_{j=1}^m R_{ij}\mu_j \\
&\quad+\bigvee_{j=1}^m v_{ij}g_j(w_j(s-\tau_j(s),x))+\bigvee_{j=1}^m Q_{ij}\mu_j\Bigg]\mathrm{d}s-\frac{e_i^{-t}}{\Gamma(\alpha)}\int_0^t e_i^s(t-s)^{\alpha-1}\cdot\Bigg[-c_iw_i(s,x)+\sum_{j=1}^m a_{ij}g_j(w_j(s,x)) \\
&\quad+\sum_{j=1}^m b_{ij}g_j(w_j(s-\tau_j(s),x))+I_i+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j(w_j(s-\tau_j(s),x))+\bigwedge_{j=1}^m R_{ij}\mu_j \\
&\quad+\bigvee_{j=1}^m v_{ij}g_j(w_j(s-\tau_j(s),x))+\bigvee_{j=1}^m Q_{ij}\mu_j\Bigg]\mathrm{d}s\Bigg|
\end{aligned}
$$

将上式不等号右边分成以下两式分别求解：

显然有

$$
\left|e^{-(t+\delta)}\cdot\xi_i(0,x)-e^{-t}\cdot\xi_i(0,x)\right|=\left|\left[e^{-(t+\delta)}-e^{-t}\right]\cdot\xi_i(0,x)\right|\to 0,\ \delta\to 0;
$$

接下来可得：

$$
\begin{aligned}
&\Bigg|\frac{e^{-(t+\delta)}}{\Gamma(\alpha)}\int_0^{t+\delta}e^s(t-s)^{\alpha-1}\cdot\Bigg[-c_iw_i(s,x)+\sum_{j=1}^m a_{ij}g_j(w_j(s,x))+\sum_{j=1}^m b_{ij}g_j(w_j(s-\tau_j(t),x))+I_i \\
&+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j(w_j(s-\tau_j(s),x))+\bigwedge_{j=1}^m R_{ij}\mu_j+\bigvee_{j=1}^m v_{ij}g_j(w_j(s-\tau_j(s),x))+\bigvee_{j=1}^m Q_{ij}\mu_j\Bigg]\mathrm{d}s \\
&-\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\Bigg[-c_iw_i(s,x)+\sum_{j=1}^m a_{ij}g_j(w_j(s,x))+\sum_{j=1}^m b_{ij}g_j(w_j(s-\tau_j(s),x))+I_i \\
&+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j(w_j(s-\tau_j(s),x))+\bigwedge_{j=1}^m R_{ij}\mu_j+\bigvee_{j=1}^m v_{ij}g_j(w_j(s-\tau_j(s),x))+\bigvee_{j=1}^m Q_{ij}\mu_j\Bigg]\mathrm{d}s\Bigg|
\end{aligned}
$$

$$\le\left|\frac{e^{-(t+\delta)}}{\Gamma(\alpha)}\int_t^{t+\delta}e^{\varpi_i(s)}(t-s)^{\alpha-1}\cdot\left[-c_iw_i(s,x)+\sum_{j=1}^{m}a_{ij}g_j\left(w_j(s,x)\right)+\sum_{j=1}^{m}b_{ij}g_j\left(w_j\left(s-\tau_j(t),x\right)\right)+I_i\right.\right.$$
$$\left.\left.+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigwedge_{j=1}^{m}R_{ij}\mu_j+\bigvee_{j=1}^{m}v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigvee_{j=1}^{m}Q_{ij}\mu_j\right]\mathrm{d}s\right|$$
$$+\left|\left(\frac{e_i^{-(t+\delta)}}{\Gamma(\alpha)}-\frac{e_i^{-t}}{\Gamma(\alpha)}\right)\int_0^t e_i^{s}(t-s)^{\alpha-1}\cdot\left[-c_iw_i(s,x)+\sum_{j=1}^{m}a_{ij}g_j\left(w_j(s,x)\right)+\sum_{j=1}^{m}b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+I_i\right.\right.$$
$$\left.\left.+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigwedge_{j=1}^{m}R_{ij}\mu_j+\bigvee_{j=1}^{m}v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigvee_{j=1}^{m}Q_{ij}\mu_j\right]\mathrm{d}s\right|\to 0,\delta\to 0;$$

因此，证明了系统在$t\in[0,+\infty)$上是连续的。

第三步，要证明$\left|e^{\gamma t}\Theta_iw_i(t,x)\right|\to 0,t\to+\infty,\gamma\in(0,1)$，基于$\left|e^{\gamma t}X_i(t,x)\right|\to 0$，有$\left|e^{\gamma t}X_i(t,x)(\Delta t)^{\alpha-1}\right|\to 0$，$\Delta t=t-s$，$s\in[0,t]$，则$\forall\varepsilon>0,t^*\ge\tau$使得$\left|e^{\gamma t}X_i(t,x)(\Delta t)^{\alpha-1}\right|<\varepsilon\mu,t\ge t^*,\mu=(1,1,\cdots,1)^{\mathrm{T}}$。

$$\begin{aligned}
&\left|e^{\gamma t}\Theta_iw_i(t,x)\right|\\
&=e^{\gamma t}\cdot\left|e^{-t}\cdot\left\{\xi_i(0,x)+\frac{1}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\left[-c_iw_i(s,x)+\sum_{j=1}^{m}a_{ij}g_j\left(w_j(s,x)\right)\right.\right.\right.\\
&\quad+\sum_{j=1}^{m}b_{ij}g_j\left(w_j\left(s-\tau_j(t),x\right)\right)+I_i+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\\
&\quad\left.\left.\left.+\bigwedge_{j=1}^{m}R_{ij}\mu_j+\bigvee_{j=1}^{m}v_{ij}g_j\left(w_j\left(s-\tau_j(t),x\right)\right)+\bigvee_{j=1}^{m}Q_{ij}\mu_j\right]\mathrm{d}s\right\}\right|\\
&\le e^{\gamma t}\left|e_i^{-t}\cdot\xi_i(0,x)\right|+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\left(-c_iw_i(s,x)\right)\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\sum_{j=1}^{m}a_{ij}g_j\left(w_j(s,x)\right)\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\sum_{j=1}^{m}b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\left[I_i+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}R_{ij}\mu_j+\bigvee_{j=1}^{m}Q_{ij}\mu_j\right]\mathrm{d}s\right|\\
&\le e^{\gamma t}\left|e_i^{-t}\cdot\xi_i(0,x)\right|+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\left(-c_iw_i(s,x)\right)\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\sum_{j=1}^{m}a_{ij}g_j\left(w_j(s,x)\right)\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\sum_{j=1}^{m}b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\left[I_i+\sum_{j=1}^{m}h_{ij}\mu_j+\bigwedge_{j=1}^{m}R_{ij}\mu_j+\bigvee_{j=1}^{m}Q_{ij}\mu_j\right]\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\left[\bigwedge_{j=1}^{m}u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right|\\
&\quad+e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^{s}(t-s)^{\alpha-1}\cdot\left[\bigvee_{j=1}^{m}v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right|
\end{aligned}$$

将上式分成以下分式，具体计算过程如下：

显然地，有

$$e^{\gamma t}\left|e^{-t}\cdot\xi_i(0,x)\right|\to 0, t\to+\infty;$$

$$e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\left[I_i+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m R_{ij}\mu_j+\bigvee_{j=1}^m Q_{ij}\mu_j\right]\mathrm{d}s\right|\to 0, t\to+\infty$$

$$\begin{aligned}&e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot(-c_iw_i(s,x))\mathrm{d}s\right|\\ &\le\left|\frac{c_ie^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^{t^*}e^s(t-s)^{\alpha-1}\cdot|w_i(s,x)|\mathrm{d}s+\left|\frac{c_ie^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_{t^*}^t e^s(t-s)^{\alpha-1}\cdot|w_i(s,x)|\mathrm{d}s\end{aligned}$$

从而，根据计算有

$$\begin{aligned}&\left|\frac{c_ie^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^{t^*}e^s(t-s)^{\alpha-1}\cdot|w_i(s,x)|\mathrm{d}s\\ &\le t^*\left|\frac{c_ie^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|e^{t^*}\cdot\max_i\left[\sup_{-\tau\le s\le t^*}\left|w_i(s,x)(t-s)^{\alpha-1}\right|\right]\mu\to 0, t\to+\infty,\end{aligned}\tag{6}$$

$$\left|\frac{c_ie^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_{t^*}^t e^s(t-s)^{\alpha-1}\cdot|w_i(s,x)|\mathrm{d}s\le\varepsilon\left|\frac{c_ie^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\cdot\frac{e^{\lambda\tau}}{1-\gamma}\mu,\tag{7}$$

由于 ε 的任意性

$$\left|e^{\gamma t}\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot(-c_iw_i(s,x))\mathrm{d}s\right|\to 0, t\to+\infty;$$

接下来，根据假设 1 以及(6)和(7)

$$\begin{aligned}&e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\sum_{j=1}^m a_{ij}g_j(w_j(s,x))\mathrm{d}s\right|\\ &\le\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^t\left|e^s(t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m|a_{ij}|L_j|w_j(s,x)|\mathrm{d}s\\ &=\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^{t^*}\left|e^s(t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m|a_{ij}|L_j|w_j(s,x)|\mathrm{d}s+\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_{t^*}^t\left|e^s(t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m|a_{ij}|L_j|w_j(s,x)|\mathrm{d}s\\ &\le t^*\cdot\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\cdot e^{t^*}\cdot\sum_{j=1}^m|a_{ij}|L_j\cdot\left[\max_j\sup_{-\tau\le s\le t^*}\left|w_j(s,x)(t-s)^{\alpha-1}\right|\right]\mu+\varepsilon\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\sum_{j=1}^m|a_{ij}|L_j\cdot\frac{e^{\gamma\tau}}{1-\gamma}\mu\end{aligned}$$

由于 ε 的任意性

$$\left|e^{\gamma t}\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\sum_{j=1}^m a_{ij}g_j(w_j(s,x))\mathrm{d}s\right|\to 0, t\to+\infty$$

同理可得

$$\left|e^{\gamma t}\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\sum_{j=1}^m b_{ij}g_j(w_j(s-\tau_j(s),x))\mathrm{d}s\right|\to 0, t\to+\infty$$

根据引理 2 以及(6)和(7)

$$
\begin{aligned}
& e^{\gamma t}\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\left[\bigwedge_{j=1}^m u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right| \\
& \le\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^t\left|e^s(t-s)^{\alpha-1}\right|\cdot\left|\bigwedge_{j=1}^m u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\right|\mathrm{d}s \\
& \le\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^t\left|e^s(t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m\left|u_{ij}\right|\left|g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\right|\mathrm{d}s \\
& \le\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^t\left|e^s(t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m\left|u_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)\right|\mathrm{d}s \\
& =\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_0^{t^*}\left|e_i^s(t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m\left|u_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)\right|\mathrm{d}s \\
& \quad+\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\int_{t^*}^t\left|e^s(t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m\left|u_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)\right|\mathrm{d}s \\
& \le t^*\cdot\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\cdot e^{t^*}\cdot\sum_{j=1}^m\left|u_{ij}\right|L_j\cdot\left[\max_j\sup_{-\tau\le s\le t^*}\left|w_j(s,x)(t-s)^{\alpha-1}\right|\right]\mu+\varepsilon\left|\frac{e^{-(1-\gamma)t}}{\Gamma(\alpha)}\right|\sum_{j=1}^m\left|u_{ij}\right|L_j\cdot\frac{e^{\gamma\tau}}{1-\gamma}\mu
\end{aligned}
$$

由于 ε 的任意性

$$
\left|e^{\gamma t}\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\left[\bigwedge_{j=1}^m u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right|\to 0,\ t\to\infty
$$

同理可得

$$
\left|e^{\gamma t}\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\left[\bigvee_{j=1}^m v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right|\to 0,\ t\to\infty
$$

从上述三步可以得出 $\Theta(\chi_1\times\chi_2\times\cdots\times\chi_n)\subset\chi_1\times\chi_2\times\cdots\times\chi_n$。

第四步，要证明 Θ 在 $\chi_1\times\chi_2\times\cdots\times\chi_n$ 上是一个压缩映射。

$$
\begin{aligned}
& \left|\Theta\left(w_i(t)\right)-\Theta\left(z_i(t)\right)\right| \\
& =\left|e^{-t}\cdot\left\{\xi_i(0)+\frac{1}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\left[-c_iw_i(s,x)+\sum_{j=1}^m a_{ij}g_j\left(w_j(s,x)\right)+\sum_{j=1}^m b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+I_i\right.\right.\right. \\
& \quad\left.\left.+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigwedge_{j=1}^m R_{ij}\mu_j+\bigvee_{j=1}^m v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigvee_{j=1}^m Q_{ij}\mu_j\right]\mathrm{d}s\right\} \\
& \quad-e^{-t}\cdot\left\{\xi_i(0)+\frac{1}{\Gamma(\alpha)}\int_0^t e^s(t-s)^{\alpha-1}\cdot\left[-c_iz_i(s,x)+\sum_{j=1}^m a_{ij}g_j\left(z_j(s,x)\right)+\sum_{j=1}^m b_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)+I_i\right.\right. \\
& \quad\left.\left.\left.+\sum_{j=1}^m h_{ij}\mu_j+\bigwedge_{j=1}^m u_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)+\bigwedge_{j=1}^m R_{ij}\mu_j+\bigvee_{j=1}^m v_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)+\bigvee_{j=1}^m Q_{ij}\mu_j\right]\mathrm{d}s\right\}\right|
\end{aligned}
$$

$$
\begin{aligned}
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot c_i\left[w_i(s,x)-z_i(s,x)\right]\mathrm{d}s\right| \\
&\quad+\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\sum_{j=1}^m a_{ij}g_j\left(w_j(s,x)\right)-\sum_{j=1}^m a_{ij}g_j\left(z_j(s,x)\right)\right]\mathrm{d}s\right| \\
&\quad+\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\sum_{j=1}^m b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-\sum_{j=1}^m b_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right| \\
&\quad+\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\bigwedge_{j=1}^m u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-\bigwedge_{j=1}^m u_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right| \\
&\quad+\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\bigvee_{j=1}^m v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-\bigvee_{j=1}^m v_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right|
\end{aligned}
$$

上述不等式右端分别可以计算如下：

$$
\begin{aligned}
&\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot c_i\left[w_i(s,x)-z_i(s,x)\right]\mathrm{d}s\right| \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\left|c_i\right|\cdot\left|w_i(s,x)-z_i(s,x)\right|\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\sum_{j=1}^m a_{ij}g_j\left(w_j(s,x)\right)-\sum_{j=1}^m a_{ij}g_j\left(z_j(s,x)\right)\right]\mathrm{d}s\right| \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m \left|a_{ij}\right|\left|g_j\left(w_j(s,x)\right)-g_j\left(z_j(s,x)\right)\right|\mathrm{d}s \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m \left|a_{ij}\right|L_j\left|w_j(s,x)-z_j(s,x)\right|\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\sum_{j=1}^m b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-\sum_{j=1}^m b_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right| \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m \left|b_{ij}\right|L_J\left|w_j\left(s-\tau_j(s),x\right)-z_j\left(s-\tau_j(s),x\right)\right|\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\bigwedge_{j=1}^m u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-\bigwedge_{j=1}^m u_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right| \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\left|\bigwedge_{j=1}^m u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-\bigwedge_{j=1}^m u_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right|\mathrm{d}s \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m \left|u_{ij}\right|\left|g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right|\mathrm{d}s \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m \left|u_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)-z_j\left(s-\tau_j(s),x\right)\right|\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&\left|\frac{e^{-t}}{\Gamma(\alpha)}\int_0^t e^s (t-s)^{\alpha-1}\cdot\left[\bigvee_{j=1}^m v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)-\bigvee_{j=1}^m v_{ij}g_j\left(z_j\left(s-\tau_j(s),x\right)\right)\right]\mathrm{d}s\right| \\
&\le \left|\frac{e^{-t}}{\Gamma(\alpha)}\right|\int_0^t \left|e^s (t-s)^{\alpha-1}\right|\cdot\sum_{j=1}^m \left|v_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)-z_j\left(s-\tau_j(s),x\right)\right|\mathrm{d}s
\end{aligned}
$$

结合以上计算，则可以写成如下形式

$$
\begin{aligned}
&\left|\Theta\left(w_i(t)\right)-\Theta\left(z_i(t)\right)\right| \\
&\le e^{-t}\left|\frac{1}{\Gamma(\alpha)}\right|\int_0^t e^s(t-s)^{\alpha-1}\cdot\left[\sum_{j=1}^{m}\left|a_{ij}\right|L_j\left|w_j(s,x)-z_j(s,x)\right|\right. \\
&\quad+\sum_{j=1}^{m}\left|b_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)-z_j\left(s-\tau_j(s),x\right)\right| \\
&\quad+\sum_{j=1}^{m}\left|u_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)-z_j\left(s-\tau_j(s),x\right)\right| \\
&\quad\left.+\sum_{j=1}^{m}\left|v_{ij}\right|L_j\left|w_j\left(s-\tau_j(s),x\right)-z_j\left(s-\tau_j(s),x\right)\right|\right]\mathrm{d}s \\
&\le\left|\frac{1}{\Gamma(\alpha)}\right|\cdot\left[c_i+\sum_{j=1}^{m}\left(\left|a_{ij}\right|+\left|b_{ij}\right|+\left|u_{ij}\right|+\left|v_{ij}\right|\right)L_j\right]\cdot dist\left(w_i(t,x),v_i(t,x)\right) \\
&\le\max_{i\in N}\left\{\left|\frac{1}{\Gamma(\alpha)}\right|\cdot\left[c_i+\sum_{j=1}^{m}\left(\left|a_{ij}\right|+\left|b_{ij}\right|+\left|u_{ij}\right|+\left|v_{ij}\right|\right)L_j\right]\right\}\cdot dist\left(w_i(t,x),v_i(t,x)\right)
\end{aligned}
$$

结合(3)以及以上计算，得到

$$dist\left(\Theta\left(w_i(t)\right),\Theta\left(z_i(t)\right)\right)\le\kappa\cdot dist\left(w_i(t),z_i(t)\right)$$

因此，存在压缩映射 $\Theta:\chi_1\times\chi_2\times\cdots\times\chi_n\to\chi_1\times\chi_2\times\cdots\times\chi_n$ 的不动点 $w(t)$ 是系统(1)和(2)的解，其满足 $e^{\gamma t}\left\|\left(w(t)\right)\right\|\to 0, t\to+\infty$。即证。

注 2：首次利用不动点定理得到了分数阶模糊细胞神经网络的稳定性判据。在以往的许多相关文献中，只能用 Lyapunov 函数方法来处理这样的复杂系统。然而，不同的方法带来了不同的条件和结论，这显然意味着定理 1 比以前更简洁、广泛。

4. 数值模拟

在这一部分，将给出一个例子来验证模型的计算，考虑下面分数阶系统：

$$
\begin{cases}
{}_{\Omega}^{C}D_t^{\alpha}w_i(t,x)=-c_iw_i(s,x)+\sum\limits_{j=1}^{2}a_{ij}g_j\left(w_j(s,x)\right)+\sum\limits_{j=1}^{2}b_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right) \\
\qquad +I_i+\sum\limits_{j=1}^{2}h_{ij}\mu_j+\bigwedge\limits_{j=1}^{2}u_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigwedge\limits_{j=1}^{2}R_{ij}\mu_j \\
\qquad +\bigvee\limits_{j=1}^{2}v_{ij}g_j\left(w_j\left(s-\tau_j(s),x\right)\right)+\bigvee\limits_{j=1}^{2}Q_{ij}\mu_j,\quad t\ge 0, i\in 1,2 \\
w_i(s,x)=\xi_i(s,x),\ s\in[-\tau,0],\ x\in\Omega \\
w_i(t,x)=0,\ s\in[-\tau,\infty],\ x\in\partial\Omega
\end{cases}
\tag{8}
$$

其中 $t>0$，$0<\alpha<1$，$m=n=2$，$I_1=I_2=0$，$c_1=c_2=1$，$g_j\left(w_j(t,x)\right)=\tanh\left(w_j(t,x)\right)$，$\tau_j=2t$，$j=1,2$，$x\in\Omega=(-1,1)$，$L_1=L_2=1/15$。

$$\left(a_{ij}\right)_{2\times 2}=\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}=\begin{pmatrix}2 & -1.4\\ -0.6 & 1\end{pmatrix}$$

$$\left(b_{ij}\right)_{2\times 2}=\begin{pmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{pmatrix}=\begin{pmatrix}-1.5 & 0.7\\ -0.6 & -0.5\end{pmatrix}$$

$$\left(u_{ij}\right)_{2\times 2} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} = \begin{pmatrix} 0.5 & 0.3 \\ 1.2 & 2 \end{pmatrix}$$

$$\left(v_{ij}\right)_{2\times 2} = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix} = \begin{pmatrix} -0.5 & 0.2 \\ -5.2 & -2 \end{pmatrix}$$

于是，根据条件和如上各系数值，利用 MATLAB 绘图得出图 1。

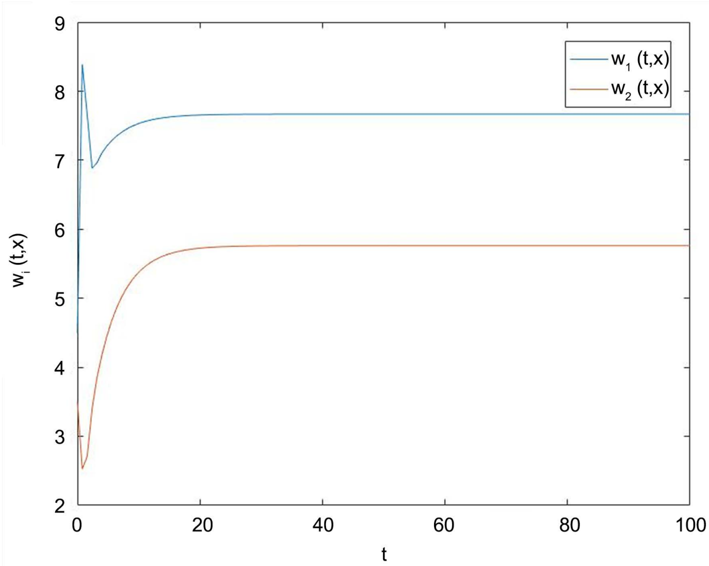

Figure 1. State track of the system (8)
图 1. 系统(8)的状态轨迹

5. 小结

本文利用压缩映射理论研究了一类具有时变时滞的分数阶模糊细胞神经网络稳定性问题。模糊细胞神经网络在图像识别领域有着其独特的存在地位，我们在基本的时滞模糊细胞神经网络上，引入分数阶导数的定义，利用不同于构造 Lyapunov 函数的方法得到其稳定性判据，使所得结果比以往更为简洁和广泛。该模型的分析可以应用于工程、生物学和医学的不同类别分数阶神经网络模型的定性理论。

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