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Estimation of Solutions for Stochastic Control Systems with Time Delays

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Abstract

Estimation of solutions for stochastic time delay systems is an important basis for the problem of optimal control systems with time delay. In this paper, we estimate the solution of the state equation of time delay control systems for the general case by using Cauchy-Schwarz and Gronwall inequalities. We use two methods to prove our conclusions, and lay a theoretical foundation for further study of time-delay control problems. And we hope to lay a theoretical foundation for further research on time-delay control.

Keywords

Time Delay System, Estimation, Cauchy-Schwarz Inequality, Gronwall Inequality

时滞随机控制系统解的估计

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摘　要

对于时滞随机系统解的估计问题是研究时滞系统控制问题的重要基础，本文利用Cauchy-Schwarz不等式、Gronwall不等式对一般情况下时滞控制系统状态方程的解进行估计，用两种方法证明了我们的结论，为进一步研究时滞系统控制问题提供理论基础。

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关键词

时滞系统，估计，Cauchy-Schwarz不等式，Gronwall不等式

1. 引言

最优控制问题是指在给定的约束条件下，寻求一个控制，使给定的系统性能指标达到极大值(或极小值)。然而在很多实际问题中，最优控制不存在或者即使存在却不易求得，于是关于最优控制问题的近似最优解问题得到了人们的广泛关注[1] [2] [3] [4] [5]。在近似最优控制问题的研究中，关于时滞控制系统状态方程解的估计在进一步研究最优控制和近似最优控制相应结论中具有非常重要的作用[6] [7] [8]。文献[2]中讨论了线性时滞随机系统中状态变量和控制变量含有不同时滞变量的状态方程解的估计问题，文献[3]中探讨了一般情况的时滞系统近似最优控制问题，但是在文献[3]的讨论中，仅讨论了状态变量和控制变量含有相同时滞变量的情况。在此类文章研究的基础上，我们对更一般情况的时滞随机系统，即状态变量和控制变量含有不同时滞变量的一般时滞系统进行研究，利用两种方法给出此类系统状态方程解的估计。从而为进一步研究时滞系统近似最优控制问题的最大值原理奠定理论基础。

2. 符号介绍

设(Ω,F,P)是完备的概率空间，$\{F_t\}_{t\ge0}$是由标准布朗运动$W(t)$生成的域流。δ_1、δ_2和T是已知常数。$L_F^2(0,T;R^m)$表示R^m值F_t适应过程$\{X(t),0\le t\le T\}$的空间，其中$E\left[\int_s^r |X(t)|^2 \mathrm{d}t\right]<\infty$；$\zeta:[-\delta_1,0]\to R^n$是连续函数。

我们研究一般的时滞随机系统，其状态方程为：

$$\begin{cases} \mathrm{d}x(t)=b(t,x(t),x(t-\delta_1),u(t),u(t-\delta_2))\mathrm{d}t+\sigma(t,x(t),x(t-\delta_1),u(t),u(t-\delta_2))\mathrm{d}W(t), \\ x(t)=\zeta(t),\ t\in[-\delta_1,0] \end{cases} \tag{1}$$

代价泛函(性能指标)

$$J(u(\cdot))=E\left\{\int_0^T \{L(t),x(t),u(t)\}\mathrm{d}t+\Phi(x(T))\right\} \tag{2}$$

其中$b:[0,T]\times R^m\times R^m\times R^k\times R^k\to R^m$，$\sigma:[0,T]\times R^m\times R^m\times R^k\times R^k\to R^{m\times d}$为已知函数，且$b,\sigma$满足以下假设条件：

(A1) 存在常数$C>0$使得对任意$x_1(t),x_2(t),x_1(t-\delta),x_2(t-\delta),u_1(t),u_2(t),u_1(t-\delta),u_2(t-\delta)$下面不等式成立：

$$\begin{aligned} &\left|\rho(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2))-b(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2))\right|^2 \\ &\le C\left[|x_1(t)-x_2(t)|^2+|x_1(t-\delta_2)-x_2(t-\delta_2)|^2+|u_1(t)-u_2(t)|^2+|u_1(t-\delta_2)-u_2(t-\delta_2)|^2\right] \end{aligned}$$

其中$\rho=b,\sigma$。

Γ 是非空的凸控制域，$U[0,T]\subset\Gamma$ 表示可容控制集合，$u(t)$ 定义为

$$u(t)=\begin{cases}0, & t\in[-\delta_2,0]\\ u(t)\in L_F^2(0,T;R^m)\text{且}u(t)\in U, & t\in[0,T]\end{cases}$$

3. 主要内容

定义 1：设 $u_1(t),u_2(t)\in U[0,T]$，定义 $d(u_1(t),u_2(t))=\left[E\int_0^T|u_1(t)-u_2(t)|^2\right]^{\frac{1}{2}}$。

时滞系统状态方程解的估计在最优控制理论中具有重要意义，下面我们用两种方法给出关于时滞系统解的估计的证明。

定理 1：在假设条件(A1)下，设 $u_1(t),u_2(t)\in U[0,T]$，其对应的状态轨迹分别设为 $x_1(t),x_2(t)$，则存在常数 C 使得 $E\left[\sup_{0\le t\le T}|x_1(t)-x_2(t)|^2\right]\le Cd(u_1(t),u_2(t))^2$。

证明: (方法一) 利用 Cauchy-Schwarz 不等式及假设条件(A1)可得

$$\begin{aligned}&E\left[\sup_{0\le t\le T}|x_1(t)-x_2(t)|^2\right]\\&\le C\Big\{E\int_0^T\left|b(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2))-b(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2))\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|\sigma(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2))-\sigma(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2))\right|^2\mathrm{d}t\Big\}\end{aligned}\tag{3}$$

$$\begin{aligned}&E\int_0^T\left|b(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2))-b(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2))\right|^2\mathrm{d}t\\&\le E\int_0^T\left|b(t,x_1(t),x_1(t-\delta_1),u_1(t),u_2(t-\delta_2))-b(t,x_1(t),x_1(t-\delta_1),u_2(t),u_2(t-\delta_2))\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|b(t,x_1(t),x_1(t-\delta_1),u_2(t),u_2(t-\delta_2))-b(t,x_1(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2))\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|b(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2))-b(t,x_1(t),x_1(t-\delta_1),u_1(t),u_2(t-\delta_2))\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|b(t,x_1(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2))-b(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2))\right|^2\mathrm{d}t\end{aligned}$$

$$\begin{aligned}&\le E\int_0^T|u_1(t)-u_2(t)|^2\mathrm{d}t+E\int_0^T|u_1(t-\delta_2)-u_2(t-\delta_2)|^2\mathrm{d}t\\&\quad+E\int_0^T|x_1(t)-x_2(t)|^2\mathrm{d}t+E\int_0^T|x_1(t-\delta_1)-x_2(t-\delta_1)|^2\mathrm{d}t\end{aligned}\tag{4}$$

利用变量代换

$$\begin{aligned}&E\int_0^T|u_1(t-\delta_2)-u_2(t-\delta_2)|^2\mathrm{d}t\\&=E\int_{-\delta_2}^{T-\delta_2}|u_1(t)-u_2(t)|^2\mathrm{d}t\\&=E\int_{-\delta_2}^{0}|u_1(t)-u_2(t)|^2\mathrm{d}t+E\int_0^{T-\delta_2}|u_1(t)-u_2(t)|^2\mathrm{d}t\\&\le E\int_0^T|u_1(t)-u_2(t)|^2\mathrm{d}t\\&=d(u_1(t),u_2(t))^2\end{aligned}\tag{5}$$

同理

$$\begin{aligned}&E\int_0^T\left|x_1(t-\delta_2)-x_2(t-\delta_2)\right|^2\mathrm{d}t\\&=E\int_{-\delta_2}^{T-\delta_2}\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t\\&=E\int_{-\delta_2}^{0}\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t+E\int_0^{T-\delta_2}\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t\\&\le E\int_0^T\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t\end{aligned}\tag{6}$$

将(5) (6)代入(4)得

$$\begin{aligned}&E\int_0^T\left|b\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-b\left(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\\&\le E\int_0^T\left|u_1(t)-u_2(t)\right|^2\mathrm{d}t+E\int_0^T\left|u_1(t-\delta_2)-u_2(t-\delta_2)\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t+E\int_0^T\left|x_1(t-\delta_2)-x_2(t-\delta_2)\right|^2\mathrm{d}t\\&\le d\left(u_1(t),u_2(t)\right)+E\int_0^T\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t\end{aligned}\tag{7}$$

上述结论中当 b 换成 σ 时结论显然成立，则有

$$\begin{aligned}&E\int_0^T\left|\sigma\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-\sigma\left(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\\&\le E\int_0^T\left|\sigma\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_2(t-\delta_2)\right)-\sigma\left(t,x_1(t),x_1(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|\sigma\left(t,x_1(t),x_1(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)-\sigma\left(t,x_1(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|\sigma\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-\sigma\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|\sigma\left(t,x_1(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)-\sigma\left(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\end{aligned}$$

$$\begin{aligned}&\le E\int_0^T\left|u_1(t)-u_2(t)\right|^2\mathrm{d}t+E\int_0^T\left|u_1(t-\delta_2)-u_2(t-\delta_2)\right|^2\mathrm{d}t\\&\quad+E\int_0^T\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t+E\int_0^T\left|x_1(t-\delta_2)-x_2(t-\delta_2)\right|^2\mathrm{d}t\\&\le d\left(u_1(t),u_2(t)\right)+E\int_0^T\left|x_1(t)-x_2(t)\right|^2\mathrm{d}t\end{aligned}\tag{8}$$

将(7) (8)代入(3)

$$\begin{aligned}&E\left[\sup_{0\le t\le T}\left|x_1(t)-x_2(t)\right|^2\right]\\&\le C\left\{E\int_0^T\left|b\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-b\left(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\right.\\&\quad\left.+E\int_0^T\left|\sigma\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-\sigma\left(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\mathrm{d}t\right\}\\&\le C\left\{d\left(u_1(t),u_2(t)\right)^2+\int_0^T E\left[\sup_{0\le t\le\theta}\left|x_1(t)-x_2(t)\right|^2\right]\mathrm{d}\theta\right\}\end{aligned}$$

利用 Gronwall 不等式，可得 $E\left[\sup_{0\le t\le T}\left|x_1(t)-x_2(t)\right|^2\right]\le Cd\left(u_1(t),u_2(t)\right)^2$，证毕。

证明(方法二)：利用文献[1]的估计 3 或文献[2]中命题 2 可得

$$\begin{aligned}&\left|b\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-b\left(t,x_2(t),x_2(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2\\&\le C\left[\left|u_1(t)-u_2(t)\right|^2+\left|u_1(t-\delta_2)-u_2(t-\delta_2)\right|^2\right]\end{aligned}$$

再由假设条件(A1)有

$$
\begin{aligned}
&E\int_0^T \left|b\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-b\left(t,x_1(t),x_1(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2 \mathrm{d}t \\
&\le CE\int_0^T \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t + CE\int_0^T \left|u_1(t-\delta_2)-u_2(t-\delta_2)\right|^2 \mathrm{d}t \\
&= CE\int_0^T \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t + CE\int_{-\delta_2}^{T-\delta_2} \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t \\
&= CE\int_0^T \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t + CE\int_{-\delta_2}^{0} \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t + CE\int_0^{T-\delta_2} \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t \\
&\le CE\int_0^T \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t + CE\int_0^T \left|u_1(t)-u_2(t)\right|^2 \mathrm{d}t \\
&\le Cd\left(u_1(t),u_2(t)\right)^2
\end{aligned}
$$

同理可证

$$
\begin{aligned}
&E\int_0^T \left|\sigma\left(t,x_1(t),x_1(t-\delta_1),u_1(t),u_1(t-\delta_2)\right)-\sigma\left(t,x_1(t),x_1(t-\delta_1),u_2(t),u_2(t-\delta_2)\right)\right|^2 \mathrm{d}t \\
&\le Cd\left(u_1(t),u_2(t)\right)^2
\end{aligned}
$$

因此 $E\left[\sup\limits_{0\le t\le T}\left|x_1(t)-x_2(t)\right|^2\right]\le Cd\left(u_1(t),u_2(t)\right)^2$ 证毕。

本文对时滞系统一般情况的状态方程的解进行估计，推广了文章[2] [3]中的部分结果，为进一步研究时滞系统的最优问题或近似最优问题提供一定理论基础。

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