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Study of Even-Even Nuclei ^{100}Zr by Interacting Boson Model

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Abstract

Even-even nuclei ^{100}Zr were studied within the framework of the interacting boson model. The E-Gamma Over Spin (E-GOS) was drawn, and the analysis of the dynamic symmetry limit found that ^{100}Zr is a transition nuclei from U(5) vibrational limit to SU(3) rotational limit , close to O(6) dynamic symmetry limit. At the same time, the energy spectrum of low-lying states of ^{100}Zr was fitted, the components of the wave function were analyzed, and the B(E2) values of transitions between low-lying states of ^{100}Zr were analyzed respectively. The results show good agreement with the available experimental data.

Keywords

Even-Even Nuclei, Interacting Boson Model, Energy Level, E-GOS Curve, Electromagnetic Transitions

IBM模型对偶－偶核^{100}Zr的理论研究

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摘　要

本文在相互作用玻色子模型框架下对偶－偶核^{100}Zr进行了理论研究。绘制了基态带的E-GOS曲线，讨论其动力学对称性极限性质，数据分析表明^{100}Zr是具有U(5)振动极限到SU(3)转动极限之间的过渡核，趋近于O(6)极限。同时文中也拟合了^{100}Zr核的低能谱的谱带，并对波函数结构进行了理论研究，计算了^{100}Zr核的低能谱部分的电磁跃迁，计算结果表明理论计算与实验值符合较好。

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关键词

偶－偶核，相互作用玻色子模型，能谱，E-GOS曲线，电磁跃迁

1. 引言

采用唯象的理论模型是原子核结构研究的重要手段。相互作用玻色子模型(interacting boson model，简称为 IBM)就是一个十分成功的研究原子核集体运动的代数模型，利用该模型，人们成功地描述了原子核低激发能谱、电磁跃迁以及相变等性质[1]。在 IBM 中，假设原子核有一个稳定的双幻核芯，价核子配对成角动量是 0 或 2 的核子对，这些核子都被看作是玻色子。角动量 L 为 0 的 s 玻色子和 L 为 2 的 d 玻色子共有六种，这六种玻色子算符构成了 IBM 模型的哈密顿量，即能谱的生成代数是 U(6)。从 U(6)开始约化，有 U(5)、SU(3)和 O(6)三种约化方式。约化的三个子群链为：

$$\begin{aligned}&\mathrm{U}(6)\supset\mathrm{U}(5)\supset\mathrm{O}(5)\supset\mathrm{O}(3)\supset\mathrm{O}(2)\\&\mathrm{U}(6)\supset\mathrm{SU}(5)\supset\mathrm{O}(3)\supset\mathrm{O}(2)\\&\mathrm{U}(6)\supset\mathrm{O}(6)\supset\mathrm{O}(5)\supset\mathrm{O}(3)\supset\mathrm{O}(2)\end{aligned}\tag{1}$$

这三个子群链分别对应于不同类型的动力学对称性，用来描述原子核的三种集体运动极限：振动、转动和 γ-不稳定特性[2]-[7]。

三个极限的晕态能谱和能级衰变能分别为：

$$E_I=\frac{I}{2}\hbar\omega,\quad E_\gamma(I\to I-2)=\hbar\omega,$$

$$E_I=\frac{\hbar^2}{2J}I(I+1),\quad E_\gamma(I\to I-2)=\frac{\hbar^2}{2J}(4I-2),$$

$$E_I=\frac{I(I+6)}{16}E(2^+),\quad E_\gamma(I\to I-2)=\frac{E(2^+)}{4}(I+2)。$$

令 $R=\dfrac{E_\gamma(I\to I-2)}{I}$ [8]，做 I-R 曲线既为 E-GOS 曲线。

将基向量表示为 $|\Psi\rangle=|n_d,n_\beta,n_\Delta,L_d,L\rangle$，哈密顿量可写成多极展开形式为：

$$\begin{aligned}H=&\mathrm{EPS}n_d+\frac{1}{2}\mathrm{ELL}(\bar{L}\cdot\bar{L})+\frac{1}{2}\mathrm{QQ}(\bar{Q}\cdot\bar{Q})\\&-5\sqrt{7}\mathrm{OCT}\left[\left(d^\dagger\tilde{d}\right)^{(3)}\times\left(d^\dagger\tilde{d}\right)^{(3)}\right]_0^{(0)}\\&+15\mathrm{HEX}\left[\left(d^\dagger\tilde{d}\right)^{(4)}\times\left(d^\dagger\tilde{d}\right)^{(4)}\right]_0^{(0)}\end{aligned}\tag{2}$$

其中：

$$\vec{L}\cdot\vec{L}=-10\sqrt{3}\left[\left(d^{\dagger}\tilde{d}\right)^{(1)}\times\left(d^{\dagger}\tilde{d}\right)^{(1)}\right]_{0}^{(0)}$$

$$\vec{Q}\cdot\vec{Q}=\sqrt{5}\left[\left\{\left(s^{\dagger}\tilde{d}+d^{\dagger}s\right)^{(2)}+\frac{\text{CHQ}}{\sqrt{5}}\left(d^{\dagger}\tilde{d}\right)^{(2)}\right\}\times\left\{\left(s^{\dagger}\tilde{d}+d^{\dagger}s\right)^{(2)}+\frac{\text{CHQ}}{\sqrt{5}}\left(d^{\dagger}\tilde{d}\right)^{(2)}\right\}\right]_{0}^{(0)}$$

式中的 EPS、ELL、QQ、OCT、HEX、CHQ 为模型的可调参数[9]。本文工作是在合理的范围内调节参数值，使计算结果符合实验数据。

2. 计算结果

本文研究的是 ^{100}Zr，它有 10 个价质子(空穴)和 10 个价中子，共组成 10 个玻色子。

2.1. 各级限值和 E-GOS 曲线

根据其实验能谱可以计算出 R 值并做 E-GOS 曲线，R 值见表 1。相应的 E-GOS 曲线见图 1。本文选取了实验数据 20_1^+ 以下的角动量为偶数的能级。

Table 1. Experimental data and the dynamic symmetry limit of ^{100}Zr
表 1. ^{100}Zr 核的实验及各动力学极限值

I	2	4	6	8	10	12	14	16	18	20
实验	106.25	88.00	82.85	78.20	73.92	70.14	66.90	64.52	63.01	62.19
U(5)	106.25	53.13	35.42	26.56	21.25	17.71	15.18	13.28	11.81	10.63
SU(3)	106.25	123.96	129.86	132.81	134.58	135.76	136.61	136.98	137.24	138.13
O(6)	106.25	79.69	70.83	66.41	63.75	61.98	60.71	59.77	59.03	58.44

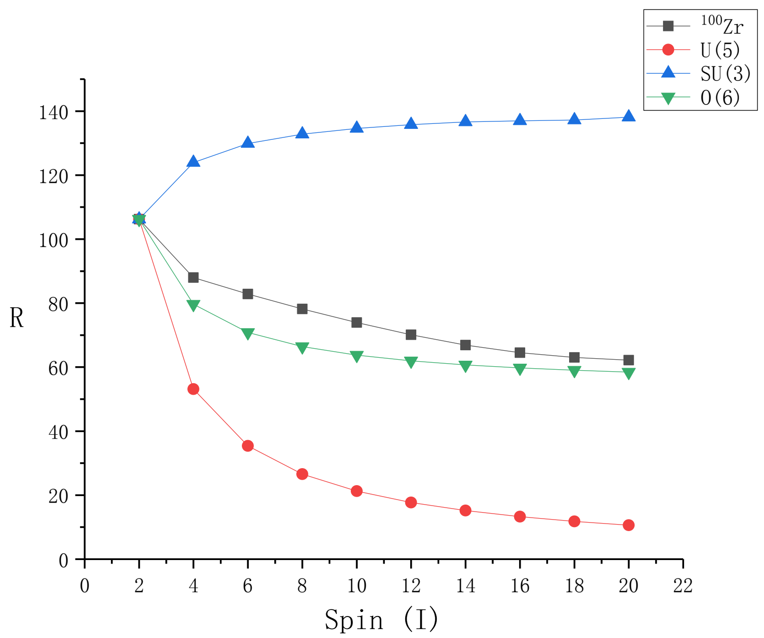

Figure 1. Curve: the E-Gamma over spin of ^{100}Zr
图 1. ^{100}Zr 核的 E-GOS 曲线

2.2. 模型参数

通过拟合实验的能级，确定了模型的参数，见表 2。

Table 2. Hamiltonian matrix of ^{100}Zr
表 2. ^{100}Zr 的哈密顿参数

EPS	ELL	QQ	OCT	HEX	CHQ
−0.3201	0.0237	−0.00116	0.0365	0.0252	−2.9580

2.3. 能谱结果

在选定的参数下，理论计算的能级与实验能级的对比图见图 2。可见所选参数较好地拟合了低激发态能谱，其中 Band 1 和 Band 2 的符合程度均很好，只是在较合理的范围内存在一定的误差。

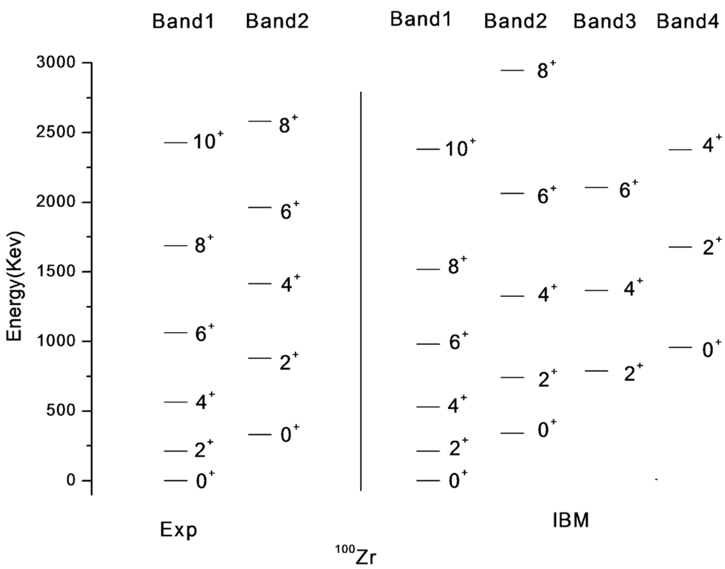

Figure 2. Experimental energy states and theoretical energy states of ^{100}Zr
图 2. ^{100}Zr 的实验能谱与理论能谱

2.4. 波函数

确定了模型参数，我们就可以给出每条能级具体的波函数，本文主要用到的波函数的结构为：

$$\left|0_1^+\right\rangle \approx 0.5248\left|s^{10}d^0\right\rangle + 0.4457\left|s^8d^2\right\rangle + 0.0292\left|s^6d^4\right\rangle + 0.0003\left|s^4d^6\right\rangle$$

$$\left|\Psi_0^1\right\rangle \approx 0.724\left|0,0,0,0,0^+\right\rangle + 0.668\left|2,1,0,0,0^+\right\rangle + 0.171\left|4,2,0,0,0^+\right\rangle$$
$$+0.018\left|6,3,0,0,0^+\right\rangle + 0.001\left|8,4,0,0,0^+\right\rangle$$

$$\left|0_2^+\right\rangle \approx 0.4704\left|s^{10}d^0\right\rangle + 0.4528\left|s^8d^2\right\rangle + 0.0756\left|s^6d^4\right\rangle + 0.0012\left|s^4d^6\right\rangle$$

$$\left|\Psi_0^2\right\rangle \approx 0.686\left|0,0,0,0,0^+\right\rangle - 0.673\left|2,1,0,0,0^+\right\rangle - 0.275\left|4,2,0,0,0^+\right\rangle$$
$$-0.035\left|6,3,0,0,0^+\right\rangle - 0.002\left|8,4,0,0,0^+\right\rangle$$

$$\left|0_3^+\right\rangle \approx 0.8624\left|s^6d^4\right\rangle + 0.1010\left|s^8d^2\right\rangle + 0.0317\left|s^4d^6\right\rangle + 0.0048\left|s^{10}d^0\right\rangle$$

$$\begin{aligned}\left|\Psi_0^3\right\rangle \approx & -0.069\left|0,0,0,0,0^+\right\rangle + 0.318\left|2,1,0,0,0^+\right\rangle - 0.929\left|4,2,0,0,0^+\right\rangle \\ & -0.178\left|6,3,0,0,0^+\right\rangle - 0.011\left|8,4,0,0,0^+\right\rangle\end{aligned}$$

$$\left|2_1^+\right\rangle \approx 0.8335\left|s^9d^1\right\rangle + 0.1519\left|s^7d^3\right\rangle + 0.0105\left|s^8d^2\right\rangle + 0.0002\left|s^6d^4\right\rangle$$

$$\begin{aligned}\left|\Psi_2^1\right\rangle \approx & 0.913\left|1,0,0,2,2^+\right\rangle - 0.103\left|2,0,0,2,2^+\right\rangle + 0.390\left|3,1,0,2,2^+\right\rangle - 0.015\left|4,1,0,2,2^+\right\rangle \\ & +0.062\left|5,2,0,2,2^+\right\rangle - 0.001\left|6,2,0,2,2^+\right\rangle + 0.004\left|7,3,0,2,2^+\right\rangle\end{aligned}$$

$$\begin{aligned}\left|2_2^+\right\rangle \approx & 0.7410\left|s^7d^3\right\rangle + 0.1203\left|s^9d^1\right\rangle + 0.0899\left|s^8d^2\right\rangle + 0.0434\left|s^5d^5\right\rangle \\ & +0.0051\left|s^6d^4\right\rangle + 0.0003\left|s^2d^8\right\rangle + 0.0001\left|s^3d^7\right\rangle\end{aligned}$$

$$\begin{aligned}\left|\Psi_2^2\right\rangle \approx & -0.347\left|1,0,0,2,2^+\right\rangle + 0.300\left|2,0,0,2,2^+\right\rangle + 0.861\left|3,1,0,2,2^+\right\rangle + 0.071\left|4,1,0,2,2^+\right\rangle \\ & +0.208\left|5,2,0,2,2^+\right\rangle + 0.007\left|6,2,0,2,2^+\right\rangle + 0.018\left|7,3,0,2,2^+\right\rangle + 0.001\left|9,4,0,2,2^+\right\rangle\end{aligned}$$

$$\begin{aligned}\left|2_3^+\right\rangle \approx & 0.8338\left|s^8d^2\right\rangle + 0.0599\left|s^5d^5\right\rangle + 0.0564\left|s^7d^3\right\rangle + 0.0451\left|s^9d^1\right\rangle \\ & +0.0040\left|s^6d^4\right\rangle + 0.0007\left|s^3d^7\right\rangle\end{aligned}$$

$$\begin{aligned}\left|\Psi_2^3\right\rangle \approx & -0.212\left|1,0,0,2,2^+\right\rangle - 0.913\left|2,0,0,2,2^+\right\rangle + 0.238\left|3,1,0,2,2^+\right\rangle - 0.245\left|4,1,0,2,2^+\right\rangle \\ & +0.063\left|5,2,0,2,2^+\right\rangle - 0.026\left|6,2,0,2,2^+\right\rangle - 0.001\left|7,3,0,2,2^+\right\rangle\end{aligned}$$

$$\left|2_4^+\right\rangle \approx 0.9079\left|s^5d^5\right\rangle + 0.0656\left|s^8d^2\right\rangle + 0.0262\left|s^7d^3\right\rangle + 0.0002\left|s^9d^1\right\rangle + 0.0001\left|s^2d^8\right\rangle$$

$$\begin{aligned}\left|\Psi_2^4\right\rangle \approx & 0.014\left|1,0,0,2,2^+\right\rangle + 0.256\left|2,0,0,2,2^+\right\rangle - 0.002\left|3,1,0,2,2^+\right\rangle - 0.953\left|4,1,0,2,2^+\right\rangle \\ & -0.004\left|5,2,0,2,2^+\right\rangle - 0.162\left|6,2,0,2,2^+\right\rangle - 0.009\left|7,3,0,2,2^+\right\rangle\end{aligned}$$

$$\left|4_1^+\right\rangle \approx 0.9348\left|s^8d^2\right\rangle + 0.0646\left|s^6d^4\right\rangle + 0.0007\left|s^4d^4\right\rangle$$

$$\left|\Psi_4^1\right\rangle \approx 0.967\left|2,0,0,4,4^+\right\rangle + 0.254\left|4,1,0,4,4^+\right\rangle + 0.027\left|6,2,0,4,4^+\right\rangle + 0.001\left|8,3,0,4,4^+\right\rangle$$

$$\left|4_2^+\right\rangle \approx 0.9087\left|s^6d^4\right\rangle + 0.0650\left|s^8d^2\right\rangle + 0.0263\left|s^4d^6\right\rangle + 0.0001\left|s^2d^8\right\rangle$$

$$\left|\Psi_4^2\right\rangle \approx -0.225\left|2,0,0,4,4^+\right\rangle + 0.953\left|4,1,0,4,4^+\right\rangle + 0.162\left|6,2,0,4,4^+\right\rangle + 0.009\left|8,3,0,4,4^+\right\rangle$$

$$\left|4_3^+\right\rangle \approx 0.9689\left|s^7d^3\right\rangle + 0.0309\left|s^5d^5\right\rangle + 0.0002\left|s^3d^7\right\rangle$$

$$\left|\Psi_4^3\right\rangle \approx 0.984\left|3,0,0,4,4^+\right\rangle + 0.176\left|5,1,0,4,4^+\right\rangle + 0.013\left|7,2,0,4,4^+\right\rangle$$

$$\left|4_4^+\right\rangle \approx 0.9549\left|s^5d^5\right\rangle + 0.0310\left|s^7d^3\right\rangle + 0.0140\left|s^3d^7\right\rangle$$

$$\left|\Psi_4^4\right\rangle \approx -0.176\left|3,0,0,4,4^+\right\rangle + 0.977\left|5,1,0,4,4^+\right\rangle + 0.118\left|7,2,0,4,4^+\right\rangle + 0.004\left|9,3,0,4,4^+\right\rangle$$

2.5. 电磁跃迁

利用波函数我们可以进一步研究原子核的电磁性质，本文计算了低激发能级的 B(E2)值，见表 3。

Table 3. The B(E2) of electromagnetic transitions of ^{100}Zr
表 3. ^{100}Zr 电磁跃迁的 B(E2) 值

	B(E2)		B(E2)		B(E2)
$2_1^+ \to 0_1^+$	19.3592	$2_2^+ \to 0_2^+$	7.5032	$2_4^+ \to 0_3^+$	3.3401
$2_3^+ \to 2_1^+$	21.3204	$2_4^+ \to 2_2^+$	16.2634	$3_1^+ \to 2_3^+$	15.8039
$4_1^+ \to 2_1^+$	25.9874	$4_2^+ \to 2_2^+$	16.2476	$4_3^+ \to 2_3^+$	16.2775
$4_4^+ \to 2_4^+$	11.6057	$4_1^+ \to 3_1^+$	6.5414	$4_3^+ \to 4_1^+$	14.0172
$4_4^+ \to 4_2^+$	10.6001	$5_1^+ \to 3_1^+$	16.7196	$5_1^+ \to 4_3^+$	7.5998
$6_1^+ \to 4_1^+$	29.4361	$6_2^+ \to 4_2^+$	22.2603	$6_3^+ \to 4_3^+$	21.7138
$6_4^+ \to 4_4^+$	16.2353	$6_3^+ \to 6_1^+$	10.1331	$6_4^+ \to 6_2^+$	7.5764
$7_1^+ \to 5_1^+$	22.3819	$8_1^+ \to 6_1^+$	31.8469	$8_2^+ \to 6_2^+$	23.8116
$8_3^+ \to 6_3^+$	24.8066	$8_4^+ \to 6_4^+$	17.5420	$8_3^+ \to 8_1^+$	7.8337
$8_4^+ \to 8_2^+$	5.5396	$9_1^+ \to 7_1^+$	24.2268	$10_1^+ \to 8_1^+$	32.6402
$10_2^+ \to 8_2^+$	23.0816	$10_3^+ \to 8_3^+$	25.5915	$10_4^+ \to 8_4^+$	16.3520

3. 结论

本文用 IBM 模型对偶－偶核 ^{100}Zr 进行了研究，在模型所选的参数下拟合了低激发能级，计算结果在一定的误差允许范围内是合理的。同时也用能级的对应的波函数计算了约化跃迁几率。^{100}Zr 核素的 E-GOS 曲线结果表明 ^{100}Zr 是具有 U(5)振动极限到 SU(3)转动极限之间的过渡核，趋近于 O(6)极限，具有较明显的 γ-不稳定特性。

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