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Reciprocally Convex Approach to Asymptotic Behavior of Stochastic Hopfield Neural Networks

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Abstract

In this paper, the problems of stochastic uniformly ultimate boundedness, the existence of stochastic attractor and mean square exponential stability for stochastic Hopfield neural networks are studied by constructing proper Lyapunov-Krasovskii functional, utilizing the reciprocally convex approach, the free-weighting matrix method, some stochastic analysis techniques and linear matrix inequalities technique (LMI). We deduce novel results and delay-dependent sufficient conditions for stochastic Hopfield neural networks. Finally, numerical simulations are given on MATLAB to verify the effectiveness of the gained results.

Keywords

**Hopfield Neural Networks, Boundedness, Mean Square Exponential Stability,
Reciprocally Convex Approach**

随机Hopfield神经网络渐进行为的
变互式凸组合方法

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摘　要

本文通过构造合适Lyapunov-Krasovskii泛函，采用变互式凸组合方法、自由权矩阵方法、随机分析技巧、线性矩阵不等式(LMI)方法，研究了一类随机Hopfield神经网络的随机一致最终有界性、随机吸引子的存在性以及均方指数的稳定性。我们得到了随机Hopfield神经网络的新的结果和时滞依靠的充分条件。最后，应用MATLAB数值模拟检验来得到了理论的有效性。

关键词

Hopfield神经网络，有界性，均方指数稳定，变互式凸组合方法

1. 引言

　　人工神经网络是一种模仿生物神经网络的结构和功能的数学模型或计算模型，是一个普遍存在的非线性系统。近年来，由于其良好的信息处理能力和非线性映射能力，被广泛的应用到模式识别、图像处理、语音处理、DNA 片段选择、数据挖掘、自动目标识别和优化等各个领域，并取得了令人瞩目的成果[1]-[3]。自从 1982 年，Hopfield 第一次提出了 Hopfiele 神经网络并且用 LC 电子电路进行了模拟，与此同时与 Hopfield 神经网络有关的动力学行为得到了学者们的广泛关注。

　　由于时滞在工程和现实生活中不可避免，因此研究时滞 Hopfield 神经网络具有一定的学术价值和实际应用。通常神经网络的很多应用都建立在神经网络的动力学行为上，包括稳定性、有界性、吸引子的存在性、周期解、收敛性、同步、混沌等[4] [5]。我们知道当非线性系统有一个平衡点的时候研究是相对容易的，但是如果神经网络的激活函数无界可能没有平衡点或者有多个平衡点时，研究系统的平衡点的稳定性将比较困难。虽然在[6]中，作者利用 Lasalle 不变理论研究了细胞神经网络的随机一致有界，而一致最终有界性和吸引子的存在性作为动态系统的一种基本性质，值得我们研究。因此在本文中，我们利用变互式凸组合的方法研究了随机 Hopfield 神经网络的随机一致最终有界性和随机吸引子的存在性、以及特殊情况下的均方指数稳定性。

2. 模型建立与解决

　　随机 Hopfield 神经网络描述为下面的非线性积分型微分方程：

$$\begin{cases} \mathrm{d}x(t) = \left[-Cx(t) + AG(x(t)) + BG(x(t-\tau(t))) + J \right]\mathrm{d}t + \sigma(t, x(t), x(t-\tau(t)))\mathrm{d}\omega(t) \\ x(v) = \varphi(v), v \in (-\tau_D, 0] \end{cases}, \tag{1}$$

其中，连接权矩阵 A，B，C 为适当维数的已知常数矩阵，$x(t) = \left[x_1(t), \cdots, x_n(t) \right]^T \in R^n$ 是神经元的状态函数：$G(x(t)) = \left[G_1(x_1(t)), \cdots G_n(x_n(t)) \right]^T$ 是神经元的激励函数向量，$J = \left[J_1, \cdots, J_n \right]^T$ 是神经网络的常值外部输入，$\sigma(t) = \left(\sigma_{ij}(t) \right)_{n \times n}$ 是噪声强度矩阵，$\omega(x(t)) = \left[\omega_1(x_1(t)), \cdots, \omega_n(x_n(t)) \right]^T$ 是定义在由 $\left\{ \omega(s) : 0 \le s \le t \right\}$ 形成的自然流 $\left\{ F_t \right\}_{t \ge o}$ 的完备空间 (Ω, F, P) 上的 n 维布朗运动，并且满足 $E\{\mathrm{d}\omega(t)\} = 0$，$E\{\mathrm{d}\omega^2(t)\} = \mathrm{d}t$。时变时滞 $\tau(t) = \left[\tau_1(t), \cdots, \tau_n(t) \right]^T$ 是有界的，且满足：$0 \le \tau_d \le \tau(t) \le \tau_D$。对于系统的初始条件 $x(v) = \varphi(v)$，$-\tau_D < v \le 0$，初始值函数为 $\varphi \in L^b_{F_0}([-\tau_D, 0], R^n)$，其中 $L^b_{F_0}([-\tau_D, 0], R^n)$ 表示定义在 $[-\tau_D, 0]$ 上取值于 R^n 的随机过程 $\varphi(v)$ 的全体，并且 $\varphi(v)$ 是 F_0 可测的，满足 $\sup_{-\tau_D \le s \le 0} E\left\| \varphi(s) \right\|^2 < \infty$，$E$ 定义

为概率 P 的期望.

假设 2.1： 对任意的 $i = 1, 2, \cdots, n$，存在常值标量 δ_j^- 和 δ_j^+，使得激励函数 $G(\cdot)$ 满足 $G(0) \neq 0$ 及

$$\delta_j^- \leq \frac{G_j(x) - G_j(y)}{x - y} \leq \delta_j^+, \forall x, y \in R, x \neq y \tag{2}$$

记

$$\Omega_1 = \mathrm{diag}\left\{\delta_1^- \delta_1^+, \delta_2^- \delta_2^+, \cdots, \delta_n^- \delta_n^+\right\}, \Omega_2 = \mathrm{diag}\left\{\frac{\delta_1^- + \delta_1^+}{2}, \frac{\delta_2^- + \delta_2^+}{2}, \cdots, \frac{\delta_n^- + \delta_n^+}{2}\right\}.$$

假设 2.2： 若 $\sigma(t, \cdot, \cdot): R^+ \times R^n \times R^n \to R^{n \times m}, (\sigma(t, 0, 0)) = 0$ 满足 Lipschtiz 条件，并且存在适当维数的任意矩阵 $\Pi_i, (i = 1, 2)$，使得

$$\mathrm{trace}\left[\sigma^{\mathrm{T}}(t, x, y) \sigma(t, x, y)\right] \leq x^{\mathrm{T}} \Pi_1^{\mathrm{T}} \Pi_1 x + y^{\mathrm{T}} \Pi_2^{\mathrm{T}} \Pi_2 y \tag{3}$$

定义 2.3 [7]： 系统(1)是随机一致最终有界的，如果对任意的 $\rho > 0$，有 $t' = t'(\rho) > 0$，存在常值 $\tilde{B} > 0$，当 $t \geq t_0 + t'$，$t_0 > 0$，$\|\varphi\| < \rho$，系统(1)的解 $x(t, t_0, \varphi)$ 满足

$$E\|x(t, t_0, \varphi)\| < \tilde{B},$$

其中，$E\|x(t, t_0, \varphi)\| = \sup_{-\tau_M \leq s \leq 0} E\|x_i(t + s, t_0, \varphi)\|$，$\varphi \in L_{F_0}^b([-\tau_D, 0]; R^n)$。

定义 2.4 [6]： 非空闭集 $A \subset R^n$，$\Lambda = \left\{\varphi \in L_{F_0}^b \mid E\|\varphi(s)\| \leq \tilde{B}\right\}$ 称为系统(1)的解 $x(t; \varphi)$ 的随机吸引子，使得

$$\lim_{t \to \infty} \sup \inf_{y \in A} \|x - y\| = 0,$$

其中 $\varphi \in C([-\tau_D, 0], R^n)$

引理 2.5 [8]： 若 $f_1, f_2, \cdots, f_N : R^m \mapsto R$ 是开集 $D \in R^m$ 中的正值，则开集 D 中的变互式凸组合 f_i 满足：

$$\min_{\left\{\alpha_i \mid \alpha_i > 0, \sum_{i=1} \alpha_i = 1\right\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t), \tag{4}$$

从而有

$$\left\{g_{i,j} : R^m \mapsto R, g_{j,i}(t) \overset{\Delta}{=} g_{i,j}(t) \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0\right\}. \tag{5}$$

定理 2.6： 对于常值 $a > 0$，如果存在正定矩阵 $P = \mathrm{diag}(p_1, p_2, \cdots, p_n)$，

$D_i = \mathrm{diag}(D_{i1}, D_{i2}, \cdots, D_{in})$，$i = 1, 2, \cdots, Q_1, Q_2, R_1, R_2, M_1, M_2, S$，使得下面的线性矩阵不等式成立，

$$\Delta = \begin{pmatrix} \Phi_{11} & 0 & 0 & 0 & \Phi_{15} & 0 & 0 & PB & -ZC \\ * & \Phi_{22} & \mathrm{e}^{-\beta \tau_D} U & \mathrm{e}^{-\beta \tau_D} S & -\tau_D R_1 & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & \mathrm{e}^{-\beta \tau_D} S & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & ZA \\ * & * & * & * & * & \Phi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 \\ * & * & * & * & * & * & 0 & \Phi_{88} & ZB \\ * & * & * & * & * & * & 0 & 0 & \Phi_{99} \end{pmatrix} < 0 \tag{6}$$

$$P < \lambda I$$

其中

$$\Phi_{11} = \beta P - PC + P + \tau_D \mathrm{e}^{\beta\tau_D} Q_1 + \tau_d \mathrm{e}^{\beta\tau_d} Q_2 - \Omega_1 D_1 + \frac{1}{4a^2} I + \lambda \Pi_1^{\mathrm{T}} \Pi_1$$

$$\Phi_{15} = PA + \tau_D \mathrm{e}^{\beta\tau_D} R_1 + \tau_d \mathrm{e}^{\beta\tau_d} R_2 + \Omega_2 D_1$$

$$\Phi_{22} = -\tau_D Q_1 - \mathrm{e}^{\beta\tau_D} S, \Phi_{33} = -\tau_d R_2 - \mathrm{e}^{\beta\tau_d} S$$

$$\Phi_{44} = \lambda \Pi_2^{\mathrm{T}} \Pi_2 - D_1 + \frac{1}{4a^2} I - \Omega_1 D_2 - 2\mathrm{e}^{\beta\tau_D} S, \Phi_{55} = -D_1 + \frac{1}{a^2} I + \mathrm{e}^{\beta\tau_D} M_1 + \tau_d \mathrm{e}^{\beta\tau_d} M_2$$

$$\Phi_{66} = -\tau_D M_1, \Phi_{77} = -\tau_d M_2, \Phi_{88} = -D_2 + \frac{1}{a^2} I, \Phi_{99} = (\tau_D - \tau_d) S - Z^{\mathrm{T}} - Z$$

"$*$"表示矩阵的对称项，则 Hopfield 神经网络(1)是随机一致最终有界的。

证明：考虑下面的 Lyapunov-Krasovskii，

$$V(t) = V_1(t) + V_2(t) + V_3(t) , \tag{7}$$

其中，

$$V_1(t) = \mathrm{e}^{\beta t} x^{\mathrm{T}}(t) P x(t) ,$$

$$V_2(t) = \tau_D \int_{t-\tau_D}^{t} e^{\beta(s+\tau_D)} \begin{bmatrix} x(s) \\ G(x(s)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_1 & R_1 \\ * & M_1 \end{bmatrix} \begin{bmatrix} x(s) \\ G(x(s)) \end{bmatrix} \mathrm{d}s$$

$$+ \tau_d \int_{t-\tau_d}^{t} \mathrm{e}^{\beta(s+\tau_d)} \begin{bmatrix} x(s) \\ G(x(s)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_2 & R_2 \\ * & M_2 \end{bmatrix} \begin{bmatrix} x(s) \\ G(x(s)) \end{bmatrix} \mathrm{d}s$$

$$V_3(t) = (\tau_D - \tau_d) \int_{-\tau_D}^{-\tau_d} \int_{t+\theta}^{t} \mathrm{e}^{\beta s} y^T(s) S y(s) \mathrm{d}s \mathrm{d}\theta .$$

其中，$y(t)\mathrm{d}t = \mathrm{d}x(t)$

我们开始计算 $V_1(t)$ 沿系统(1)轨迹的时间导数，得到

$$\mathrm{d}V(t) = LV\mathrm{d}t + 2\mathrm{e}^{\beta t} x^{\mathrm{T}}(t) P \sigma(t, x(t), x(t-\tau(t))) \mathrm{d}\omega(t) \tag{8}$$

根据伊藤微分公式，得

$$\begin{aligned} LV_1 &= \beta \mathrm{e}^{\beta t} x^{\mathrm{T}}(t) P x(t) + \mathrm{e}^{\beta t} x^{\mathrm{T}}(t) P \left[-Cx(t) + AG(x(t)) + BG(x(t-\tau(t))) \right] \\ &+ \mathrm{e}^{\beta t} \frac{1}{2} tra\left[\sigma(t, x(t), x(t-\tau(t)))^{\mathrm{T}} V_{1xx}(x,t) \sigma(t, x(t), x(t-\tau(t))) \right] \end{aligned}, \tag{9}$$

根据假设 2.2，下列不等式成立

$$\begin{aligned} &\frac{1}{2} \mathrm{e}^{\beta t} tra\left[\sigma(t, x(t), x(t-\tau(t)))^{\mathrm{T}} V_{1xx}(x,t) \sigma(t, x(t), x(t-\tau(t))) \right] \\ &= \mathrm{e}^{\beta t} \lambda_{\max}(P) tra\left[\sigma(t, x(t), x(t-\tau(t)))^{\mathrm{T}} (x,t) \sigma(t, x(t), x(t-\tau(t))) \right] \\ &\le \lambda \mathrm{e}^{\beta t} x^{\mathrm{T}}(t) \Pi_1^{\mathrm{T}} \Pi_1 x(t) + \lambda \mathrm{e}^{\beta t} x^{\mathrm{T}}(t-\tau(t)) \Pi_2^{\mathrm{T}} \Pi_2 x(t-\tau(t)), \end{aligned} \tag{10}$$

结合(9)和(10)式，

$$\begin{aligned} LV_1 &\le \beta \mathrm{e}^{\beta t} x^{\mathrm{T}}(t) P x(t) \\ &+ \mathrm{e}^{\beta t} \left[-x^{\mathrm{T}}(t) P C x(t) + x^{\mathrm{T}}(t) P A G(x(t)) + x^{\mathrm{T}}(t) P B G(x(t-\tau(t))) + x^{\mathrm{T}}(t) P x(t) + J^{\mathrm{T}} P J \right] \\ &+ \lambda \mathrm{e}^{\beta t} x^{\mathrm{T}}(t) \Pi_1^{\mathrm{T}} \Pi_1 x(t) + \lambda \mathrm{e}^{\beta t} x^{\mathrm{T}}(t-\tau(t)) \Pi_2^{\mathrm{T}} \Pi_2 x(t-\tau(t)) \end{aligned} \tag{11}$$

另外，

$$
\begin{aligned}
LV_2 &= \tau_D e^{\beta(t+\tau_D)} x^{\mathrm{T}}(t) Q_1 x(t) - \tau_D e^{\beta t} x^{\mathrm{T}}(t-\tau_D) Q_1 x(t-\tau_D) \\
&\quad + \tau_d e^{\beta(t+\tau_d)} x^{\mathrm{T}}(t) Q_2 x(t) - \tau_d e^{\beta t} x^{\mathrm{T}}(t-\tau_d) Q_2 x(t-\tau_d) \\
&\quad + \tau_D e^{\beta(t+\tau_D)} x^{\mathrm{T}}(t) R_1 G(x(t)) - \tau_D e^{\beta t} x^{\mathrm{T}}(t-\tau_D) R_1 G(x(t-\tau_D)) \\
&\quad + \tau_d e^{\beta(t+\tau_d)} x^{\mathrm{T}}(t) R_2 G(x(t)) - \tau_d e^{\beta t} x^{\mathrm{T}}(t-\tau_d) R_2 G(x(t-\tau_d)) \\
&\quad + e^{\beta(t+\tau_D)} G^{\mathrm{T}}(t) M_1 G(t) - \tau_D e^{\beta t} G^{\mathrm{T}}(x(t-\tau_D)) M_1 G(x(t-\tau_D)) + \tau_d e^{\beta(t+\tau_d)} G^{\mathrm{T}} x(t) M_2 G(x(t)) \\
&\quad - \tau_d e^{\beta t} G^{\mathrm{T}}(x(t-\tau_d)) M_2 G(x(t-\tau_d))
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
LV_3 &= (\tau_D - \tau_d)^2 e^{\beta t} y^{\mathrm{T}}(t) S y(t) - \int_{t-\tau_D}^{t-\tau_d} e^{\beta s} y^{\mathrm{T}}(s) S y(s) \mathrm{d}s \\
&\leq (\tau_D - \tau_d)^2 e^{\beta t} y^{\mathrm{T}}(t) S y(t) - e^{\beta(t-\tau_D)}(\tau_D - \tau_d) \int_{t-\tau_D}^{t-\tau_d} y^{T}(s) S y(s) \mathrm{d}s
\end{aligned}
\tag{13}
$$

根据引理 2.5，变互式凸组合的方法有较低的保守型，可得到

$$
\begin{aligned}
&-(\tau_D - \tau_d) \int_{t-\tau_D}^{t-\tau_d} y^{\mathrm{T}}(s) S y(s) \mathrm{d}s \\
&= -(\tau_D - \tau_d) \left[\int_{t-\tau_D}^{t-\tau(t)} y^{\mathrm{T}}(s) S y(s) \mathrm{d}s + \int_{t-\tau(t)}^{t-\tau_d} y^{\mathrm{T}}(s) S y(s) \mathrm{d}s \right] \\
&= -\frac{\tau(t)-\tau_d}{\tau_D - \tau(t)} [x(t-\tau(t)) - x(t-\tau_D)]^{\mathrm{T}} S [x(t-\tau(t)) - x(t-\tau_D)] - [x(t-\tau(t)) - x(t-\tau_D)]^{\mathrm{T}} \\
&\quad \times S [x(t-\tau(t)) - x(t-\tau_D)] - \frac{\tau_D - \tau(t)}{\tau(t)-\tau_d} [x(t-\tau_d) - x(t-\tau(t))]^{\mathrm{T}} S [x(t-\tau_d) - x(t-\tau(t))] \\
&\quad - [x(t-\tau_d) - x(t-\tau(t))]^{\mathrm{T}} S [x(t-\tau_d) - x(t-\tau(t))] \\
&= -\begin{bmatrix} \sqrt{\dfrac{\tau_D - \tau(t)}{\tau(t)-\tau_d}} [x(t-\tau_d) - x(t-\tau(t))] \\ -\sqrt{\dfrac{\tau(t)-\tau_d}{\tau_D - \tau(t)}} [x(t-\tau(t)) - x(t-\tau_D)] \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S & U \\ * & S \end{bmatrix} \begin{bmatrix} \sqrt{\dfrac{\tau_D - \tau(t)}{\tau(t)-\tau_d}} [x(t-\tau_d) - x(t-\tau(t))] \\ -\sqrt{\dfrac{\tau(t)-\tau_d}{\tau_D - \tau(t)}} [x(t-\tau(t)) - x(t-\tau_D)] \end{bmatrix} \\
&\quad - \begin{bmatrix} x(t-\tau_d) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-\tau_D) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S & U \\ * & S \end{bmatrix} \begin{bmatrix} x(t-\tau_d) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-\tau_D) \end{bmatrix} \\
&\leq -\begin{bmatrix} x(t-\tau_d) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-\tau_D) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S & U \\ * & S \end{bmatrix} \begin{bmatrix} x(t-\tau_d) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-\tau_D) \end{bmatrix}
\end{aligned}
\tag{14}
$$

即

$$
\begin{aligned}
&-e^{\beta(t-\tau_D)}(\tau_D - \tau_d) \int_{t-\tau_D}^{t-\tau_d} y^{\mathrm{T}}(s) S y(s) \mathrm{d}s \\
&\leq -e^{\beta(t-\tau_D)} \begin{bmatrix} x(t-\tau_d) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-\tau_D) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S & U \\ * & S \end{bmatrix} \begin{bmatrix} x(t-\tau_d) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-\tau_D) \end{bmatrix}
\end{aligned}
\tag{15}
$$

对任意的 $j = 1, 2, \cdots, n$，根据假设 2.1，易得

$$
[F_j(x_j(t)) - F_j(0) - l_j x_j(t)] \times [F_j(x_j(t)) - F_j(0) - L_j x_j(t)] \leq 0
\tag{16}
$$

和

$$\left[F_j\left(x_j\left(t-\tau(t) \right) \right) - F_j(0) - l_j x_j\left(t-\tau(t) \right) \right] \times \left[F_j\left(x_j\left(t-\tau(t) \right) \right) - F_j(0) - L_j x_j\left(t-\tau(t) \right) \right] \le 0 \tag{17}$$

从而有

$$
\begin{aligned}
Y_1 &= -\sum_{j=1}^{n} D_{1j}\left[F_j\left(x_j(t) \right) - F_j(0) - \delta_j^- x_j(t) \right] \times \left[F_j\left(x_j(t) \right) - F_j(0) - \delta_j^+ x_j(t) \right] \\
&\quad -\sum_{j=1}^{n} D_{2j}\left[F_j\left(x_j(t) \right) - F_j(0) - \sigma_j^- x_j\left(t-\tau(t) \right) \right] \times \left[F_j\left(x_j(t) \right) - F_j(0) - \sigma_j^+ x_j\left(t-\tau(t) \right) \right] \\
&\le -\sum_{j=1}^{n} D_{1j}\left[F_j\left(x_j(t) \right) - \delta_j^- x_j(t) \right] \times \left[F_j\left(x_j(t) \right) - \delta_j^+ x_j(t) \right] \\
&\quad -\sum_{j=1}^{n} D_{2j}\left[F_j\left(x_j(t) \right) - \delta_j^- x_j\left(t-\tau(t) \right) \right] \times \left[F_j\left(x_j(t) \right) - \delta_j^+ x_j\left(t-\tau(t) \right) \right] \\
&\quad +\sum_{j=1}^{n}\left[\frac{1}{a^2} F_j^{\,2}\left(x_j(t) \right) + a^2 D_{1j}^2 F_j^{\,2}(0) + \frac{1}{4a^2} x_j^{\,2}(t) + a^2 D_{1j}^2 F_j^{\,2}(0)\left(\delta_j^+ + \delta_j^- \right)^2 \right] \\
&\quad +\sum_{j=1}^{n}\left[\frac{1}{a^2} F_j^{\,2}\left(x_j\left(t-\tau(t) \right) \right) + a^2 D_{1j}^2 F_j^{\,2}(0) + \frac{1}{4a^2} x_j^{\,2}\left(t-\tau(t) \right) + a^2 D_{1j}^2 F_j^{\,2}(0)\left(\delta_j^+ + \delta_j^- \right)^2 \right]
\end{aligned}
\tag{18}
$$

对任意的自由权矩阵 Z，得到，

$$
\begin{aligned}
Y_2 &= 2\mathrm{e}^{\beta t}\left[y^{\mathrm{T}}(t)Z \right] \times \left\{ \left[-Cx(t) + AG\left(x(t) \right) + BG\left(x\left(t-\tau(t) \right) \right) - y(t) \right]\mathrm{d}t \right. \\
&\quad \left. +\sigma\left(t,x(t),x\left(t-\tau(t) \right) \right)\mathrm{d}\omega(t) \right\} = 0
\end{aligned}
\tag{19}
$$

定义

$$
\begin{aligned}
\varsigma^{\mathrm{T}}(t) &= \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}\left(t-\tau_D \right), x^{\mathrm{T}}\left(t-\tau_d \right), x^{\mathrm{T}}\left(t-\tau(t) \right), G^{\mathrm{T}}\left(x(t) \right), \right. \\
&\quad \left. G^{\mathrm{T}}\left(x\left(t-\tau_D \right) \right), G^{\mathrm{T}}\left(x\left(t-\tau_D \right) \right), G^{\mathrm{T}}\left(x\left(t-\tau(t) \right) \right), y(t)^{\mathrm{T}} \right]
\end{aligned}
$$

根据(8)~(19)，得到

$$
\begin{aligned}
\mathrm{d}V(t) &\le LV\mathrm{d}t + 2\mathrm{e}^{\beta t} x^{\mathrm{T}}(t) P\sigma\left(t,x(t),x\left(t-\tau(t) \right) \right)\mathrm{d}\omega(t) + \mathrm{e}^{\beta t}\left(Y_1 + Y_2 \right) \\
&\le \mathrm{e}^{\beta t}\varsigma^{\mathrm{T}}(t)\Delta\varsigma(t) + \mathrm{e}^{\beta t} J^{\mathrm{T}}PJ + 2\mathrm{e}^{\beta t}\left[x^{\mathrm{T}}(t)P + y^{\mathrm{T}}(t)Z \right]\sigma\left(t,x(t),x\left(t-\tau(t) \right) \right)\mathrm{d}\omega(t) \\
&\quad +\mathrm{e}^{\beta t}\sum_{j=1}^{n}\left[a^2 D_{1j}^2 F_j^{\,2}(0) + a^2 D_{1j}^2 F_j^{\,2}(0)\left(\delta_j^+ + \delta_j^- \right)^2 + a^2 D_{2j}^2 F_j^{\,2}(0) + a^2 D_{2j}^2 F_j^{\,2}(0)\left(\delta_j^+ + \delta_j^- \right)^2 \right]
\end{aligned}
\tag{20}
$$

令 $H = \sum_{j=1}^{n}\left[aD_{1j}^2 F_j^{\,2}(0) + aD_{1j}^2 F_j^{\,2}(0)\left(\delta_j^+ + \delta_j^- \right)^2 + aD_{2j}^2 F_j^{\,2}(0) + aD_{2j}^2 F_j^{\,2}(0)\left(\delta_j^+ + \delta_j^- \right)^2 \right]$

两边同时对不等式（20）从 0 到 t 积分，并且取期望

$$E\left\{ V\left(x(t) \right) \right\} \le E\left\{ V(0) \right\} + E\left\{ \mathrm{e}^{\beta t} J^{\mathrm{T}}PJ + H \right\}, \tag{21}$$

由(7)式得到

$$E\left\{ V\left(x(t) \right) \right\} \ge YE\left\| x(t) \right\|^2 \mathrm{e}^{\beta t}, \tag{22}$$

使得

$$E\left\| x(t) \right\|^2 \le \frac{\mathrm{e}^{-\beta t} E\left\{ V\left(x(0) \right) \right\} + \beta^{-1} J^{\mathrm{T}}PJ + H}{Y}, \tag{23}$$

其中，$Y = \lambda_{\min}(P)$。

对任意的常数 $\rho > 0$ 和 $\|\varphi\| < \rho$，有 $t' = t'(\rho) > 0$，若取 $\tilde{B} = \sqrt{\dfrac{1 + \beta^{-1}J^{\mathrm{T}}PJ + H}{Y}} > 0$，对所有的 $t \geq t'$，使得 $\mathrm{e}^{-\beta t}E\{V(x(0))\} < 1$ 成立。

根据定义 2.3，对所有的 $t \geq t'$，使得 $E\|x(t,0,\varphi)\| < \tilde{B}$。也就是说，系统(1)是随机一致最终有界的。证毕。

定理 2.7： 假如定理 2.6 成立，系统(1)存在一个随机吸引子 $\Lambda_{\tilde{B}}$，使得 $\Lambda_{\tilde{B}} = \left\{\varphi \in L^b_{F_0} \middle| E\|\varphi(s)\| \leq \tilde{B}, t \geq t_0\right\}$.

证明：选取 $\tilde{B} = \sqrt{\dfrac{1 + \beta^{-1}\overline{a}^2 J^{\mathrm{T}}PJ + H}{Y}} > 0$，定理 2.6 表明对任意的 ϕ，有 $t' > 0$，当 $t \geq t'$，使得 $E\|x(t,0,\varphi)\| < \tilde{B}$. 让 $\Lambda_{\tilde{B}}$ 记为 $\Lambda_{\tilde{B}} = \left\{\varphi \in L^b_{F_0} \middle| E\|\varphi(s)\| \leq \tilde{B}, t \geq t_0\right\}$。显然，$\Lambda_{\tilde{B}}$ 是闭的，有界的，不变的. 从而有 $\limsup\limits_{t \to \infty} \inf\limits_{y \in \Lambda_{\tilde{B}}} \|x(t;0,\phi) - y\| = 0$。因此，$\Lambda_{\tilde{B}}$ 是系统(1)解的一个随机吸引子。证毕。

定理 2.8： 如果定理 2.6 对所有的情况都成立，当 $J = 0$，$F_j(0) = 0$ 时，系统(1)有一个初始解 $x(t) \equiv 0$，使得系统(1)的初始解是均方指数稳定的.

证明：当 $J = 0$，$F_j(0) = 0$ 时，显然 $x(t) \equiv 0$，是系统(1)的初始解.

根据定理 2.6，对任意 ϕ，我们推得 $E\|x(t;0,\phi)\|^2 \leq K\mathrm{e}^{-\beta t}$，其中，$K = \dfrac{E\|V(x(0))\|^2}{Y}$。

因此，系统(1)的初始解是均方指数稳定的.证毕.

备注 2.9： 本文主要运用线性矩阵不等式的理论，是一种构造性证明问题，通过构造负定的矩阵不等式(6)，利用 MATLAB 中的 LMI 工具箱寻找使(6)成立的正定未知矩阵 $P, Q_1, Q_2, R_1, R_2, M_1, M_2, S$，从而保证定理 2.6 的合理性。

3. 数值模拟

考虑下面随机 Hopfield 神经网络

$$\mathrm{d}x(t) = \left[-Cx(t) + AG(x(t)) + BG(x(t - \tau(t))) + J\right]\mathrm{d}t + \sigma(t, x(t), x(t - \tau(t)))\mathrm{d}\omega(t) \qquad (24)$$

参量 $\tau(t) = 0.2\cos(x(t)) + 0.3, G_i(x_i(t)) = 0.5\tanh(x_i(t))(i = 1,2)$，连接权矩阵满足

$$A = \begin{pmatrix} -0.2 & 0.1 \\ 0.6 & -0.8 \end{pmatrix}, \quad B = \begin{pmatrix} -0.5 & 0.7 \\ 1 & -0.9 \end{pmatrix}, \quad C = \begin{pmatrix} 2.1 & 0 \\ 0 & 1.2 \end{pmatrix}, \quad J = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$
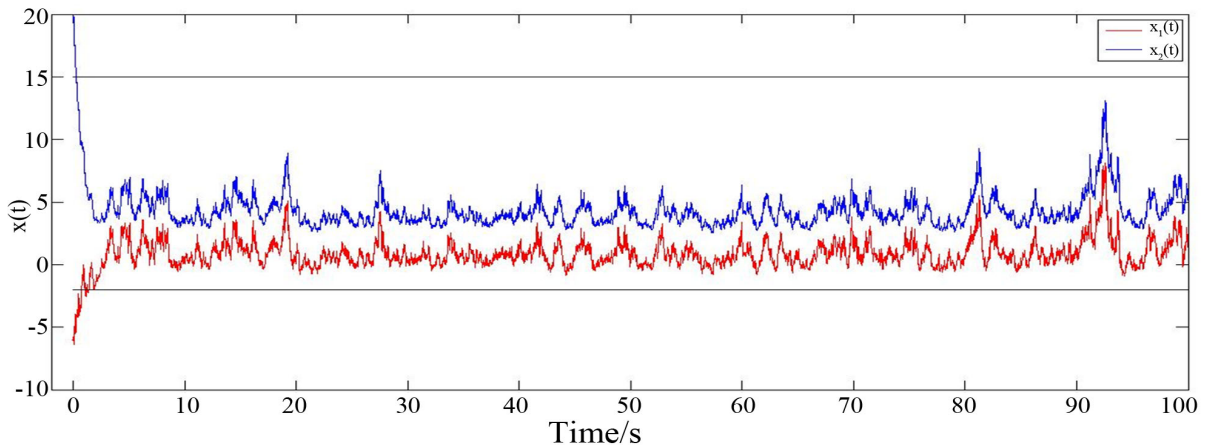

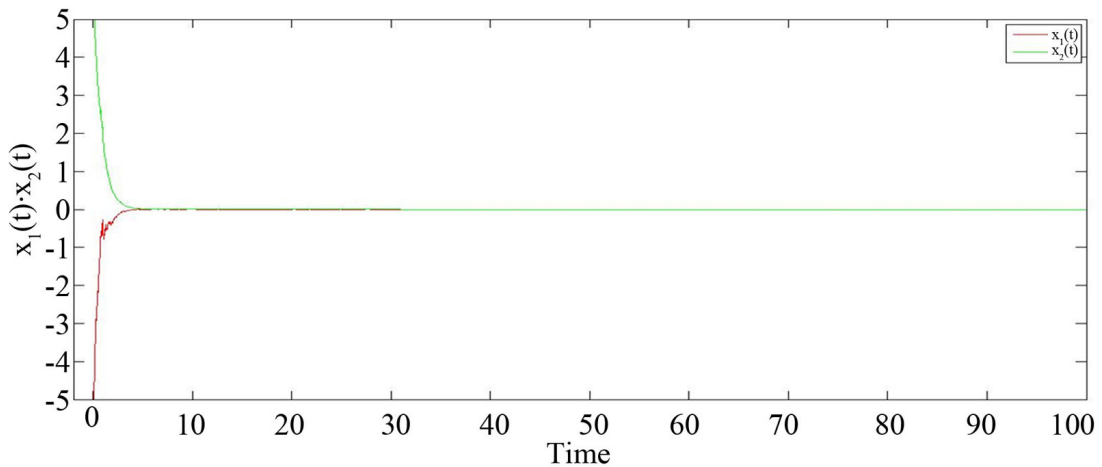**Figure 1.** Stochastic uniformly ultimate boundness of system
图 1. 系统的随机一致最终有界性

Figure 2. Mean square exponential stability with initial value [-5,5]
图 2. 初始值为[-5,5]下的均方指数稳定性

因此，当 $\beta = 1.5$，运用 MATLAB 中的 LMI 控制工具箱，计算 LMIS，(6)可得到可行解

$$P = \begin{pmatrix} 0.7710 & 0 \\ 0 & 0.7710 \end{pmatrix} Q_1 = \begin{pmatrix} 20.9422 & -0.1884 \\ -0.1884 & 20.7196 \end{pmatrix} R_1 = \begin{pmatrix} 13.4596 & 0.1870 \\ 0.1870 & 13.4344 \end{pmatrix}$$

$$M_1 = \begin{pmatrix} 2.3446 & 0.0257 \\ 0.0257 & 2.1958 \end{pmatrix} Q_2 = \begin{pmatrix} 15.2327 & 2.8740 \\ 2.8740 & 5.5712 \end{pmatrix} R_2 = \begin{pmatrix} 7.4214 & -0.3563 \\ -0.3563 & 7.6758 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 4.7800 & 0.0520 \\ 0.0520 & 4.4785 \end{pmatrix} S = \begin{pmatrix} 2.2200 & 0.7850 \\ 0.7850 & 1.8267 \end{pmatrix}, \quad \lambda = 2.61$$

则相应的图像如图 1，图 2。

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参考文献 (References)

[1] Ogura, H., Agata, H., Xie, M., *et al*. (1997) A study of learning splice sites of DNA sequence by neural networks. *Computers in Biology and Medicine*, **27**, 67-75.

[2] Waibel, A., Hanazawa, T., Hinton, G., *et al*. (1989) Phoneme recognition using time-delay neural networks. *IEEE Transactions on Acoustics*, *Speech and Signal Processing*, **37**, 328-339.

[3] Basu, J.K., Bhattacharyya, D. and Kim, T. (2010) Use of artificial neural network in pattern recognition. *International Journal of Software Engineering & Its Applications*, **4**.

[4] Lam, J., Xu, S., Ho, D.W.C., *et al*. (2012) On global asymptotic stability for a class of delayed neural networks. *International Journal of Circuit Theory and Applications*, **40**, 1165-1174.

[5] Lou, X., Ye, Q. and Cui, B. (2012) Stabilization analysis of stochastic Hopfield neural networks. 2012 12*th IEEE International Conference on Control*, *Automation and Systems* (*ICCAS*), 930-933.

[6] Wan, L. and Zhou, Q. (2011) Attractor and ultimate boundedness for stochastic cellular neural networks with delays. *Nonlinear Analysis*: *Real World Applications*, **12**, 2561-2566.

[7] Cao, J. and Liang, J. (2004) Boundedness and stability for Cohen-Grossberg neural network with time-varying delays. *Journal of Mathematical Analysis and Applications*, **296**, 665-685.

[8] Park, P.G., Ko, J.W. and Jeong, C. (2011) Reciprocally convex approach to stability of systems with time-varying delays. *Automatica*, **47**, 235-238.