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Existence and Uniqueness of Solution for Predator-Prey Population Model with Size-Structured in Polluted Environment

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Abstract

In recent years, people pay more and more attention to the problem of environmental pollution, and it is very important to study the law of population development in polluted environment. In this paper, a predator-prey population model with size-structured in polluted environment is proposed. The formal solution of the model is obtained by establishing the corresponding assumptions and using the characteristic line method. Then the existence and uniqueness of the solution are proved by inequality estimation and Banach fixed point theorem.

Keywords

Size-Structured, Existence and Uniqueness, Environmental Pollution, Banach Fixed Point Theorem

污染环境下具有尺度结构的捕食种群模型解的存在唯一性

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摘 要

近年来，环境污染问题受到人们的广泛关注，同时研究污染环境下种群的发展规律也显得尤为重要。本

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文提出一个污染环境下具有尺度结构的捕食种群模型，通过建立相应的假设并运用特征线法得到了系统的形式解，然后通过不等式估计和Banach不动点定理证明了模型解的存在唯一性。

关键词

尺度结构，存在唯一性，环境污染，Banach不动点定理

1. 建立模型

随着工业的发展，环境污染日趋严重，研究毒素对生物种群的影响显得尤重要。对这一问题的研究始于 T.G.Hallam 和他同事发表的论文[1] [2] [3]，随后国内外学者对这方面的问题进行了深入研究。2014 年，雒志学首次将毒素种群模型和具有年龄结构的种群模型结合进行了研究[4] [5]。基于上述文献启发，本文提出一类具有尺度结构的捕食-被捕食者种群模型并研究其解的存在唯一性。模型如下：

$$\begin{cases}\dfrac{\partial p(s,t)}{\partial t}+\dfrac{\partial\left[g(s)p(s,t)\right]}{\partial s}=-\left[\mu_1\left(s,c_{10}(t)\right)+\lambda_1(t)q(t)\right]p(s,t), & (s,t)\in Q,\\ \dfrac{\mathrm{d}q(t)}{\mathrm{d}t}=-\left[\mu_2\left(c_{20}(t)\right)+\lambda_2(t)P(t)\right]q(t), & t\in(0,+\infty),\\ \dfrac{\mathrm{d}c_{10}(t)}{\mathrm{d}t}=kc_e(t)-lc_{10}(t)-nc_{10}(t), & t\in(0,+\infty),\\ \dfrac{\mathrm{d}c_{20}(t)}{\mathrm{d}t}=kc_e(t)-lc_{20}(t)-nc_{20}(t), & t\in(0,+\infty),\\ \dfrac{\mathrm{d}c_e(t)}{\mathrm{d}t}=-k_1c_e(t)\left[P(t)+q(t)\right]+l_1\left[c_{10}(t)P(t)+c_{20}(t)q(t)\right]-hc_e(t)+v(t), & \\ g(0)p(0,t)=\lambda_3(t)q(t)\int_0^m\beta_1\left(s,c_{10}(t)\right)p(s,t)\mathrm{d}s, & \\ p(s,0)=p_0(s),P(t)=\int_0^m p(s,t)\mathrm{d}s,q(0)=q_0, & s\in(0,m),\\ 0\le c_0(0)\le 1,0\le c_e(0)\le 1. & \end{cases}\tag{1}$$

其中 $Q=(0,m)\times(0,+\infty)$，m 为捕食者种群个体的最大尺度。模型中其他参数表示含义如下:

$p_1(s,t)$：t 时刻尺度为 s 的捕食者种群个体密度；

$q(t)$：t 时刻尺度为 s 的食饵种群个体密度；

$g(s)$：捕食者种群个体的尺度增长函数；

$c_{10}(t),c_{20}(t)$：分别表示 t 时刻捕食者种群个体、食饵种群个体体内的毒素浓度；

$c_e(t)$：t 时刻环境中的毒素浓度；

$v(t)$：t 时刻外界向环境中输入的毒素浓度；

$\lambda_1(t),\lambda_2(t)$：$t$ 时刻捕食者、食饵的相互作用因子；

$\beta_1\left(s,c_{10}(t)\right),\mu_1\left(s,c_{10}(t)\right)$：分别表示尺度为 s，体内毒素浓度为 $c_{10}(t)$ 的捕食者种群的出生率和死亡率；

$\mu_2\left(c_{20}(t)\right)$：体内毒素浓度为 $c_{20}(t)$ 的食饵种群的死亡率；

本文作如下假设：

(H$_1$) $\mu_1(s,c_{10}(t))\in L^1_{loc}(Q)$， $0<\mu_1(s,c_{10}(t))\le\mu^0$， $\int_0^m\mu_1(s,c_{10}(t))\mathrm{d}s=+\infty$；

(H$_2$) $\beta_1(s,c_{10}(t))\in L^1_{loc}(Q)$， $0<\beta_1(s,c_{10}(t))\le\beta^0$；

(H$_3$) $\lambda_i(t)\in L^\infty(0,T)$， $0<\lambda_i(t)<\lambda_i^0$， $i=1,2,3$；

(H$_4$) $g\in C^1(0,m)$， $0<g(s)\le g^*$， $s\in(0,m)$；

(H$_5$) $v(\cdot)\in L^2(0,T)$， $0<v(t)<v^0<+\infty$；

(H$_6$) $|\beta_1(s,x_1)-\beta_1(s,x_2)|\le L_{\beta_1}|x_1-x_2|$，$|\mu_1(s,x_1)-\mu_1(s,x_2)|\le L_{\mu_1}|x_1-x_2|$，$|\beta_2(x_1)-\beta_2(x_2)|\le L_{\beta_2}|x_1-x_2|$，$|\mu_2(x_1)-\mu_2(x_2)|\le L_{\mu_2}|x_1-x_2|$；

(H$_7$) $0<p(s,t)<p^*$， $0<q(t)<q^*$；

(H$_8$) $l<k<l+n$， $v^0<h$ [6]。

2. 模型解的存在唯一性

定义 1.1：称向量$(p(s,t),q(t),c_{10}(t),c_{20}(t),c_e(t))$为模型(1)沿特征线$\Gamma(s)=\int_0^s 1/g(r)\mathrm{d}r$的解如果满足如下方程：

$$\begin{cases}\dfrac{\partial p(s,t)}{\partial t}+\dfrac{\partial[g(s)p(s,t)]}{\partial s}=-[\mu_1(s,c_{10}(t))+\lambda_1(t)q(t)]p(s,t), & \text{a.e. } (s,t)\in Q,\\ \dfrac{\mathrm{d}q(t)}{\mathrm{d}t}=-[\mu_2(c_{20}(t))+\lambda_2(t)P(t)+u_2(t)]q(t), & \text{a.e. } t\in(0,+\infty),\\ \dfrac{\mathrm{d}c_{10}(t)}{\mathrm{d}t}=kc_e(t)-lc_{10}(t)-nc_{10}(t), & \text{a.e. } t\in(0,+\infty),\\ \dfrac{\mathrm{d}c_{20}(t)}{\mathrm{d}t}=kc_e(t)-lc_{20}(t)-nc_{20}(t), & \text{a.e. } t\in(0,+\infty),\\ \dfrac{\mathrm{d}c_e(t)}{\mathrm{d}t}=-k_1c_e(t)[P(t)+q(t)]+l_1[c_{10}(t)P(t)+c_{20}(t)q(t)]-hc_e(t)+v(t), \\ \lim\limits_{\varepsilon\to0^+}g(0)p(\Gamma^{-1}(\varepsilon),t+\varepsilon)=\lambda_3(t)q(t)\int_0^m\beta_1(s,c_{10}(t))p(s,t)\mathrm{d}s, \\ \lim\limits_{\varepsilon\to0^+}p(\Gamma^{-1}(\Gamma(s)+\varepsilon),\varepsilon)=p_0(s),\ P(t)=\int_0^m p(s,t)\mathrm{d}s,\ q(0)=q_0, & \text{a.e. } s\in(0,m),\\ 0\le c_{10}(0)\le1, 0\le c_{20}(0)\le1, 0\le c_e(0)\le1. \end{cases}\tag{2}$$

定义 1.2：模型(1)的解空间为：

$$X=\Big\{(p,q,c_{10},c_{20},c_e)\in L^\infty(0,T;L^1(0,m))\times[L^\infty(0,T)]^4\,\big|\,p(s,t)>0,q(t)>0,$$
$$0\le\int_0^m p(s,t)\mathrm{d}s\le M,\ \text{a.e. } (s,t)\in Q\Big\}.$$

定理 1.1：模型(1)解的形式为

$$p(s,t)=\begin{cases}\exp\Big\{-\int_0^t\big[\mu_1(\Gamma^{-1}(\Gamma(s)+\tau-t),c_{10}(\tau))+g_s(\Gamma^{-1}(\Gamma(s)+\tau-t))\\ \quad+\lambda_1(\tau)q(\tau)\big]\mathrm{d}\tau\Big\}p_0(\Gamma^{-1}(\Gamma(s)-t)), & \Gamma(s)>t,\\ \exp\Big\{-\int_0^{\Gamma(s)}\big[\mu_1(\Gamma^{-1}(\tau),c_{10}(t+\tau-\Gamma(s)))+\lambda_2(t+\tau-\Gamma(s))P(t+\tau-\Gamma(s))\\ \quad+g_s(\Gamma^{-1}(\tau))\big]\mathrm{d}\tau\Big\}B(t-\Gamma(s)), & \Gamma(s)<t,\end{cases}$$

其中 $B(t)=p(0,t)$。

$$q(t)=q_0\cdot\exp\left\{-\int_0^t\left[\mu_2\left(c_{20}(\tau)\right)+\lambda_2(\tau)P(\tau)\right]\mathrm{d}\tau\right\};$$

$$c_{i0}(t)=c_{i0}(0)\exp\left\{-(l+n)t\right\}+k\int_0^t c_e(\tau)\exp\left\{(\tau-t)(l+n)\right\}\mathrm{d}\tau,\quad i=1,2;$$

$$c_e(t)=\int_0^t\left[l_1c_{10}(\tau)P(\tau)+l_1c_{20}(\tau)q(\tau)+v(\tau)\right]\exp\left\{\int_t^\tau\left[k_1P(r)+k_1q(r)+h\right]\mathrm{d}r\right\}\mathrm{d}\tau$$
$$+c_e(0)\exp\left\{-\int_0^t\left(k_1P(r)+k_1q(r)+h\right)\mathrm{d}r\right\}.$$

证明：模型(1)的解由特征线法和常数变易法即可导出[7] [8] [9]。

定理 1.2：如果假设(H_1)~(H_8)成立，模型(1)存在唯一解

$$\left(p(s,t),q(t),c_{10}(t),c_{20}(t),c_e(t)\right)\in X。$$

证明：首先，定义映射 $F:X\to X$，

$$F\left(p,q,c_{10},c_{20},c_e\right)=\left(F_1\left(p,q,c_{10},c_{20},c_e\right),F_2\left(p,q,c_{10},c_{20},c_e\right),\cdots,F_5\left(p,q,c_{10},c_{20},c_e\right)\right),$$

其中

$$F_1\left(p,q,c_{10},c_{20},c_e\right)=\begin{cases}\exp\left\{-\int_0^t\left[\mu_1\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right),c_{10}(\tau)\right)+g_s\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right)\right)\right.\right.\\ \left.\left.+\lambda_1(\tau)q(\tau)\right]\mathrm{d}\tau\right\}p_0\left(\Gamma^{-1}\left(\Gamma(s)-t\right)\right), & \Gamma(s)>t,\\ \exp\left\{-\int_0^{\Gamma(s)}\left[\mu_1\left(\Gamma^{-1}(\tau),c_{10}\left(t+\tau-\Gamma(s)\right)\right)+\lambda_2\left(t+\tau-\Gamma(s)\right)P\left(t+\tau-\Gamma(s)\right)\right.\right.\\ \left.\left.+g_s\left(\Gamma^{-1}(\tau)\right)\right]\mathrm{d}\tau\right\}B\left(t-\Gamma(s)\right), & \Gamma(s)<t,\end{cases}$$

$$F_2\left(p,q,c_{10},c_{20},c_e\right)=q_0\cdot\exp\left\{-\int_0^t\left[\mu_2\left(c_{20}(\tau)\right)+\lambda_2(\tau)P(\tau)\right]\mathrm{d}\tau\right\},$$

$$F_3\left(p,q,c_{10},c_{20},c_e\right)=c_{10}(0)\exp\left\{-(l+n)t\right\}+k\int_0^t c_e(\tau)\exp\left\{(\tau-t)(l+n)\right\}\mathrm{d}\tau,$$

$$F_4\left(p,q,c_{10},c_{20},c_e\right)=c_{20}(0)\exp\left\{-(l+n)t\right\}+k\int_0^t c_e(\tau)\exp\left\{(\tau-t)(l+n)\right\}\mathrm{d}\tau,$$

$$F_5\left(p,q,c_{10},c_{20},c_e\right)=\int_0^t\left[l_1c_{10}(\tau)P(\tau)+l_1c_{20}(\tau)q(\tau)+v(\tau)\right]\exp\left\{\int_t^\tau\left[k_1P(r)+k_1q(r)+h\right]\mathrm{d}r\right\}\mathrm{d}\tau$$
$$+c_e(0)\exp\left\{-\int_0^t\left(k_1P(r)+k_1q(r)+h\right)\mathrm{d}r\right\}.$$

显然，$F\left(p,q,c_{10},c_{20},c_e\right)\in X$。

另外，

$$\begin{aligned}&\int_0^m p(s,t)\mathrm{d}s=\int_0^{\Gamma^{-1}(t)}p(s,t)\mathrm{d}s+\int_{\Gamma^{-1}(t)}^m p(s,t)\mathrm{d}s\\&=\int_0^{\Gamma^{-1}(t)}\exp\left\{-\int_0^{\Gamma(s)}\left[\mu_1\left(\Gamma^{-1}(\tau),c_{10}\left(t+\tau-\Gamma(s)\right)\right)+\lambda_2\left(t+\tau-\Gamma(s)\right)P\left(t+\tau-\Gamma(s)\right)\right.\right.\\&\quad\left.\left.+g_s\left(\Gamma^{-1}(\tau)\right)\right]\mathrm{d}\tau\right\}B\left(t-\Gamma(s)\right)\mathrm{d}s+\int_{\Gamma^{-1}(t)}^m\exp\left\{-\int_0^t\left[\mu_1\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right),c_{10}(\tau)\right)\right.\right.\\&\quad\left.\left.+g_s\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right)\right)+\lambda_1(\tau)q(\tau)\right]\mathrm{d}\tau\right\}p_0\left(\Gamma^{-1}\left(\Gamma(s)-t\right)\right)\mathrm{d}s\\&\le\int_0^{\Gamma^{-1}(t)}B\left(t-\Gamma(s)\right)\mathrm{d}s+\int_{\Gamma^{-1}(t)}^m p_0\left(\Gamma^{-1}\left(\Gamma(s)-t\right)\right)\mathrm{d}s\\&\le g^{-1}(0)\lambda_3^0q^*\beta^0\int_0^t\int_0^m p(s,\tau)\mathrm{d}s\mathrm{d}\tau+mp^*\end{aligned}$$

则由 Gronwall 引理可得

$$\begin{aligned}&\int_0^m p(s,t)\mathrm{d}s\\ &\le mp^* \cdot \exp\left\{\int_0^t g^{-1}(0)\lambda_3^0 q^* \beta^0 \mathrm{d}t\right\}\\ &= M.\end{aligned}$$

接下来证明解对变量的连续依赖性。

设 $x^i = \left(p^i, q^i, c_{10}^i, c_{20}^i, c_e^i\right)$， $i = 1, 2$ 。当 $\Gamma(s) < t$ 时，有如下不等式成立：

$$\begin{aligned}&\int_0^m \left|p^1(s,t) - p^2(s,t)\right|\mathrm{d}s\\ &= \int_0^{\Gamma^{-1}(t)} \left|p^1(s,t) - p^2(s,t)\right|\mathrm{d}s + \int_{\Gamma^{-1}(t)}^m \left|p^1(s,t) - p^2(s,t)\right|\mathrm{d}s\\ &= \int_0^{\Gamma^{-1}(t)} \Big|B^1 \exp\Big\{-\int_0^{\Gamma(s)}\Big[\mu_1\left(\Gamma^{-1}(\tau), c_{10}^1\left(t+\tau-\Gamma(s)\right)\right)\\ &\quad + \lambda_2\left(t+\tau-\Gamma(s)\right)P^1\left(t+\tau-\Gamma(s)\right) + g_s\left(\Gamma^{-1}(\tau)\right)\Big]\mathrm{d}\tau\Big\}\\ &\quad - B^2 \exp\{-\int_0^{\Gamma(s)}[\mu_1\left(\Gamma^{-1}(\tau), c_{10}^2\left(t+\tau-\Gamma(s)\right)\right)\\ &+ g_s\left(\Gamma^{-1}(\tau)\right) + \lambda_2\left(t+\tau-\Gamma(s)\right)P^2\left(t+\tau-\Gamma(s)\right)\Big]\mathrm{d}\tau\Big\}\Big|\mathrm{d}s\\ &+ \int_{\Gamma^{-1}(t)}^m \Big|\exp\Big\{-\int_0^t\Big[\mu_1\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right), c_{10}^1(\tau)\right) + g_s\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right)\right)\\ &+ \lambda_1(\tau)q^1(\tau)\Big]\mathrm{d}\tau\Big\} - \exp\Big\{-\int_0^t\Big[\mu_1\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right), c_{10}^2(\tau)\right)\\ &+ g_s\left(\Gamma^{-1}\left(\Gamma(s)+\tau-t\right)\right) + \lambda_1(\tau)q^2(\tau)\Big]\mathrm{d}\tau\Big\}\Big| p_0\left(\Gamma^{-1}\left(\Gamma(s)-t\right)\right)\mathrm{d}s\\ &\le \int_0^{\Gamma^{-1}(t)} \left|B^1 - B^2\right|\mathrm{d}s + B^2 \cdot \int_0^{\Gamma^{-1}(t)}\int_0^{\Gamma(s)} \Big|\mu_1\left(\Gamma^{-1}(\tau), c_{10}^1\left(t+\tau-\Gamma(s)\right)\right)\\ &\quad - \mu_1\left(\Gamma^{-1}(\tau), c_{10}^2\left(t+\tau-\Gamma(s)\right)\right)\Big|\mathrm{d}\tau\mathrm{d}s + B^2 \cdot \int_0^{\Gamma^{-1}(t)}\int_0^{\Gamma(s)} \lambda_2\left(t+\tau-\Gamma(s)\right)\\ &\quad \cdot\left|P^1\left(t+\tau-\Gamma(s)\right) - P^2\left(t+\tau-\Gamma(s)\right)\right|\mathrm{d}\tau\mathrm{d}s\\ &\quad + \int_{\Gamma^{-1}(t)}^m p_0\left(\Gamma^{-1}\left(\Gamma(s)-t\right)\right)\cdot\int_0^t\Big|\mu_1\left(\Gamma^{-1}(\tau), c_{10}^1\left(t+\tau-\Gamma(s)\right)\right)\\ &\quad - \mu_1\left(\Gamma^{-1}(\tau), c_{10}^2\left(t+\tau-\Gamma(s)\right)\right)\Big|\mathrm{d}\tau\mathrm{d}s\\ &\quad + \int_{\Gamma^{-1}(t)}^m p_0\left(\Gamma^{-1}\left(\Gamma(s)-t\right)\right)\cdot\int_0^t \lambda_1(\tau)\left|q^1(\tau) - q^2(\tau)\right|\mathrm{d}\tau\mathrm{d}s\\ &\le \left(\frac{1}{g^*}\lambda_3^0\cdot\beta^0\cdot q^* + \lambda_2^0\cdot\lambda_3^0\cdot\beta^0\cdot q^*\cdot MT\right)\int_0^t\int_0^m \left|p^1(s,\tau) - p^2(s,\tau)\right|\mathrm{d}s\mathrm{d}\tau\\ &\quad + \left(\frac{1}{g^*}\lambda_3^0\cdot\beta^0\cdot M + m\cdot\lambda_1^0 p^*\right)\int_0^t\left|q^1(\tau) - q^2(\tau)\right|\mathrm{d}\tau\\ &\quad + \left(\frac{1}{g^*}\lambda_3^0\cdot q^*\cdot ML_{\beta_1} + T\cdot\lambda_3^0\cdot q^*\cdot ML_{\mu_1} + m\cdot p^*\cdot L_{\mu_1}\right)\int_0^t\left|c_{10}^1(\tau) - c_{10}^2(\tau)\right|\mathrm{d}\tau.\end{aligned}$$

因此，

$$\int_0^m \left|F_1\left(x^1\right) - F_1\left(x^2\right)\right|\mathrm{d}s \le M_1\left(\int_0^t\int_0^m \left|p^1(s,\tau) - p^2(s,\tau)\right|\mathrm{d}s\mathrm{d}\tau + \int_0^t\left|q^1(\tau) - q^2(\tau)\right|\mathrm{d}\tau + \int_0^t\left|c_{10}^1(\tau) - c_{10}^2(\tau)\right|\mathrm{d}\tau\right), \qquad (3)$$

且

$$M_1=\max\left\{\frac{1}{g^*}\lambda_3^0\cdot\beta^0\cdot q^*+\lambda_2^0\cdot\lambda_3^0\cdot\beta^0\cdot q^*\cdot MT,\frac{1}{g^*}\lambda_3^0\cdot\beta^0\cdot M+m\cdot\lambda_1^0 p^*,\right.$$
$$\left.\frac{1}{g^*}\lambda_3^0\cdot q^*\cdot ML_{\beta_1}+T\cdot\lambda_3^0\cdot q^*\cdot ML_{\mu_1}+m\cdot p^*\cdot L_{\mu_1}\right\}.$$

当$\Gamma(s)>t$时同理可得上述不等式成立。

$$\begin{aligned}\left|q^1-q^2\right|&=q_0\cdot\left|\exp\left\{-\int_0^t\left[\mu_2\left(c_{20}^1(\tau)\right)+\lambda_2(\tau)P^1(\tau)\right]\mathrm{d}\tau\right\}\right.\\&\quad\left.-\exp\left\{-\int_0^t\left[\mu_2\left(c_{20}^2(\tau)\right)+\lambda_2(\tau)P^2(\tau)\right]\mathrm{d}\tau\right\}\right|\\&\le q_0\cdot\int_0^t\left|\mu_2\left(c_{20}^1(\tau)\right)-\mu_2\left(c_{20}^2(\tau)\right)\right|\mathrm{d}\tau+q_0\cdot\int_0^t\lambda_2(\tau)\left|P^1(\tau)-P^2(\tau)\right|\mathrm{d}\tau\\&\le q_0L_{\mu_2}\cdot\int_0^t\left|c_{20}^1(\tau)-c_{20}^2(\tau)\right|\mathrm{d}\tau+q_0\lambda_2^0\int_0^t\int_0^m\left|p^1(s,\tau)-p^2(s,\tau)\right|\mathrm{d}s\mathrm{d}\tau\\&\le M_2\left(\int_0^t\int_0^m\left|p^1(s,\tau)-p^2(s,\tau)\right|\mathrm{d}s\mathrm{d}\tau+\int_0^t\left|c_{20}^1(\tau)-c_{20}^2(\tau)\right|\mathrm{d}\tau\right)\end{aligned}\quad(4)$$

其中，$M_2=\max\left\{q_0L_{\mu_2},q_0\lambda_2^0\right\}$。

$$\begin{aligned}&\left|c_{i0}^1-c_{i0}^2\right|\\&=\left|c_{i0}(0)\exp\{-(l+n)t\}+k\int_0^t c_e^1(\tau)\exp\{(\tau-t)(l+n)\}\mathrm{d}\tau\right.\\&\quad\left.-c_{i0}(0)\exp\{-(l+n)t\}+k\int_0^t c_e^2(\tau)\exp\{(\tau-t)(l+n)\}\mathrm{d}\tau\right|\\&=k\int_0^t\left|c_e^1(\tau)-c_e^2(\tau)\right|\exp\{(\tau-t)(l+n)\}\mathrm{d}\tau\\&\le M_3\int_0^t\left|c_e^1(\tau)-c_e^2(\tau)\right|\mathrm{d}\tau\quad(M_3=k)\end{aligned}\quad(5)$$

$$\begin{aligned}&\left|c_e^1-c_e^2\right|\\&=\left|\int_0^t\left[l_1c_{10}^1(\tau)P^1(\tau)+l_1c_{20}^1(\tau)q^1(\tau)+v(\tau)\right]\exp\left\{\int_t^\tau\left[k_1P^1(r)+k_1q^1(r)+h\right]\mathrm{d}r\right\}\mathrm{d}\tau\right.\\&\quad+c_e(0)\exp\left\{-\int_0^t\left(k_1P^1(r)+k_1q^1(r)+h\right)\mathrm{d}r\right\}\\&\quad-\int_0^t\left[l_1c_{10}^2(\tau)P^2(\tau)+l_1c_{20}^2(\tau)q^2(\tau)+v(\tau)\right]\exp\left\{\int_t^\tau\left[k_1P^2(r)+k_1q^2(r)+h\right]\mathrm{d}r\right\}\mathrm{d}\tau\\&\quad\left.+c_e(0)\exp\left\{-\int_0^t\left(k_1P^2(r)+k_1q^2(r)+h\right)\mathrm{d}r\right\}\right|\\&\le c_e(0)\cdot\int_0^t\left|k_1\left(P^1(r)-P^2(r)\right)+k_1\left(q^1(r)-q^2(r)\right)\right|\mathrm{d}r\\&\quad+\int_0^t\left|l_1\left(c_{10}^1(\tau)P^1(\tau)-c_{10}^2(\tau)P^2(\tau)\right)+l_1\left(c_{20}^1(\tau)q^1(\tau)-c_{20}^2(\tau)q^2(\tau)\right)\right|\mathrm{d}\tau\\&\quad+\int_0^t\left[l_1c_{10}^2(\tau)P^2(\tau)+l_1c_{20}^2(\tau)q^2(\tau)+v(\tau)\right]\cdot\left|\exp\left\{\int_t^\tau\left[k_1P^1(r)+k_1q^1(r)+h\right]\mathrm{d}r\right\}\right.\\&\quad\left.-\exp\left\{\int_t^\tau\left[k_1P^2(r)+k_1q^2(r)+h\right]\mathrm{d}r\right\}\right|\mathrm{d}\tau\end{aligned}$$

$$\begin{aligned}
&\le \left(k_1 + l_1 + TMk_1l_1 + Tq^*k_1l_1 + Tk_1v^0\right)\int_0^t\int_0^m \left|p^1(s,\tau) - p^2(s,\tau)\right|\mathrm{d}s\mathrm{d}\tau \\
&\quad + \left(k_1 + l_1 + TMk_1l_1 + Tq^*k_1l_1 + Tk_1v^0\right)\int_0^t \left|q^1(\tau) - q^2(\tau)\right|\mathrm{d}\tau \\
&\quad + Ml_1\cdot\int_0^t \left|c_{10}^1(\tau) - c_{10}^2(\tau)\right|\mathrm{d}\tau + q^*l_1\cdot\int_0^t \left|c_{20}^1(\tau) - c_{20}^2(\tau)\right|\mathrm{d}\tau \\
&\le M_4\left(\int_0^t\int_0^m \left|p^1(s,\tau) - p^2(s,\tau)\right|\mathrm{d}s\mathrm{d}\tau + \int_0^t \left|q^1(\tau) - q^2(\tau)\right|\mathrm{d}\tau + \sum_{i=1}^{2}\int_0^t \left|c_{20}^1(\tau) - c_{20}^2(\tau)\right|\mathrm{d}\tau\right)
\end{aligned} \tag{6}$$

且 $M_4 = \max\left\{k_1 + l_1 + TMk_1l_1 + Tq^*k_1l_1 + Tk_1v^0, Ml_1, q^*l_1\right\}$。

最后，定义解空间 X 上的范数为

$$\left\|(p,q,c_{10},c_{20},c_e)\right\|_X = Ess\sup_{t\in(0,T)} \mathrm{e}^{-\lambda t}\left\{\int_0^m |p(s,t)|\mathrm{d}s + |q(t)| + \sum_{i=1}^{2}|c_{i0}(t)| + |c_e(t)|\right\},$$

$\lambda > 0$ 足够大。

则由不等式(3)-(6)可得

$$\begin{aligned}
&\left\|F(x^1) - F(x^2)\right\|_X = \left\|F_1(x^1) - F_1(x^2), F_2(x^1) - F_2(x^2), \cdots, F_5(x^1) - F_5(x^2)\right\|_X \\
&\le M_5 Ess\sup_{t\in(0,T)} \mathrm{e}^{-\lambda t}\int_0^t\left\{\int_0^m \left|p^1(s,\tau) - p^2(s,\tau)\right|\mathrm{d}s + \left|q^1(\tau) - q^2(\tau)\right| + \sum_{i=1}^{2}\left|c_{i0}^1(\tau) - c_{i0}^2(\tau)\right| + \left|c_e^1(\tau) - c_e^2(\tau)\right|\right\} \\
&= M_5 Ess\sup_{t\in(0,T)} \mathrm{e}^{-\lambda t}\int_0^t \mathrm{e}^{\lambda\tau}\left\{\mathrm{e}^{-\lambda\tau}\left[\int_0^m \left|p^1(s,\tau) - p^2(s,\tau)\right|\mathrm{d}s + \left|q^1(\tau) - q^2(\tau)\right|\right.\right. \\
&\quad \left.\left. + \sum_{i=1}^{2}\left|c_{i0}^1(\tau) - c_{i0}^2(\tau)\right| + \left|c_e^1(\tau) - c_e^2(\tau)\right|\right]\right\}\mathrm{d}\tau \\
&\le M_5\left\|x^1 - x^2\right\|_X Ess\sup_{t\in(0,T)}\left\{\mathrm{e}^{-\lambda t}\int_0^t \mathrm{e}^{\lambda\tau}\mathrm{d}\tau\right\} \le \frac{M_5}{\lambda}\left\|x^1 - x^2\right\|_X
\end{aligned}$$

成立，其中 $M_5 = \max\{M_1, M_2, M_3, M_4\}$ 是常数，$\lambda > M_5$ 时 F 是压缩的，由 Banach 不动点定理可得不动点 (p,q,c_{10},c_{20},c_e) 是 F 的唯一解同时也是模型(1)的解。证明完毕。

3. 结论

受毒素种群模型和具有尺度结构的捕食种群模型的启发，本文在第一节中建立了一个污染环境下具有尺度结构的捕食种群模型，并对模型中相应的参数进行了解释，给出基本假设。第二节中通过运用特征线法得到了模型(1)的形式解，然后通过不等式估计和 Banach 不动点定理证明了系统解的存在唯一性。

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