Advances in Applied Mathematics 应用数学进展, **2018, 7(9), 1141-1146**
Published Online September 2018 in Hans. http://www.hanspub.org/journal/aam
https://doi.org/10.12677/aam.2018.79132

Hans汉斯

Note on the Global Asymptotic Stability of a Strengthening Type Predator-Prey Model with Stage Structure

Xiaoyan Huang, Hang Deng, Fengde Chen

College of Mathematics and Computer Science, Fuzhou University, Fuzhou Fujian
Email: fdchen@263.net

Received: Aug. 16th, 2018; accepted: Aug. 31st, 2018; published: Sep. 6th, 2018

Abstract

A strengthening type predator-prey model with stage structure is revisited in this paper. We first show that the main results of the previous paper are incorrect. After that, by constructing some suitable Lyapunov functions, a set of sufficient conditions which ensure the globally asymptotically stable of the positive equilibrium is obtained. We show that the conditions which ensure the existence of the positive equilibrium are enough to ensure the globally asymptotically stable, and consequently, the system is permanent. Our results supplement and complement some known results.

Keywords

Stage Structure, Predator, Prey, Global Asymptotic Stability

一类强身型食饵-捕食者模型正平衡点稳定性注记

黄小燕，邓　行，陈凤德

福州大学数学与计算机科学学院，福建　福州
Email: fdchen@263.net

收稿日期：2018年8月16日；录用日期：2018年8月31日；发布日期：2018年9月6日

摘　要

本文对一类强身型食饵-捕食者模型进行再探讨，首先指出前人的有关正平衡点全局稳定性的相关结果是

文章引用：黄小燕，邓行，陈凤德. 一类强身型食饵-捕食者模型正平衡点稳定性注记[J]. 应用数学进展, 2018, 7(9): 1141-1146. DOI: 10.12677/aam.2018.79132

错误的，之后通过构造适当的Lyapunov函数，我们证得了如果系统的正平衡点存在，则必是全局渐近稳定的，进而系统是一致持久的。本文所得结果推广和改进了前人的相关工作。

关键词

阶段结构，捕食，食饵，全局渐近稳定

1. 引言

近年来，阶段结构种群模型的动力学行为研究引起了学者们的极大关注，见文[1]-[18]以及所引文献。著名学者王稳地和他的学生于宇梅，张勇[16]在构造阶段结构捕食者-食饵种群模型时，原创性的提出如下假设：捕食者种群通过捕食活动不增加生育能力，只是增加体质减少死亡，且幼年捕食者没有捕食能力，在这样的假设下，作者们提出了如下强身型的捕食者-食饵模型：

$$\begin{aligned}&\dot{x}(t)=x(t)\left(r-bx(t)-ay_2(t)\right),\\&\dot{y}_1(t)=ey_2-(d_1+c)y_1(t),\\&\dot{y}_2(t)=ey_1(t)-d_2y_2(t)+kax(t)y_2(t),\end{aligned}\tag{1.1}$$

其中 $x(t)$ 为时刻 t 的食饵种群密度；$y_1(t)$ 为幼年捕食者在单位时间、单位面积内的生物量；$y_2(t)$ 为成年捕食者在单位时间、单位面积内的生物量，系统的各个参数的生态学含义见[16]。

作变换 $\overline{x}=\dfrac{bx}{r}$，$\overline{y}_1=\dfrac{ay_1}{e}$，$\overline{y}_2=\dfrac{ay_2}{r}$，$\overline{t}=rt$，并重新用 x,y_1,y_2,t 表示 $\overline{x},\overline{y}_1,\overline{y}_2,\overline{t}$，则系统(1.1)可以变换为

$$\begin{aligned}&\dot{x}(t)=x(t)\left(1-x(t)-y_2(t)\right),\\&\dot{y}_1(t)=y_2(t)-c_1y_1(t),\\&\dot{y}_2(t)=c_2y_1(t)-c_3y_2(t)+c_4x(t)y_2(t),\end{aligned}\tag{1.2}$$

计算易知系统可能有三个非负平衡点，$O(0,0,0)$，$E_0(1,0,0)$，此外，进一步假设

$$0<c_3-\frac{c_2}{c_1}<c_4\tag{1.3}$$

成立下，系统存在正平衡点 $\left(x^*,y_1^*,y_2^*\right)$，其中

$$x^*=\frac{c_3-\dfrac{c_2}{c_1}}{c_4},\quad y_1^*=\frac{y_2^*}{c_1},\quad y_2^*=1-x^*.\tag{1.4}$$

有关系统(1.2)正平衡点的稳定性和系统的持久性等，作者得到了如下四个结果(见原文定理 1，定理 2，定理 4 和定理 5)。

定理 A

当(1.3)成立时，正平衡点 $E\left(x^*,y_1^*,y_2^*\right)$ 是局部渐近稳定的。

定理 B

若条件

$$0 < c_3 - \frac{c_1}{c_2} \tag{1.5}$$

成立，则系统(1.2)是一致持续生存的。

定理 C

若条件

$$c_3 - \frac{c_1}{c_2} > c_4 \tag{1.6}$$

成立，则捕食者种群绝灭。

定理 D

若条件

$$c_3 > c_4 + \frac{c_1}{c_2} + c_4 y_1^* \tag{1.7}$$

成立，则是 $E\left(x^*, y_1^*, y_2^*\right)$ 全局渐近稳定的。

这里有两个问题。1)条件(1.6)成立时，必有条件(1.5)成立，由定理 B 此时系统(1.2)是一致持续生存的，而由定理 C 此时捕食者种群是绝灭的，这是两个相互矛盾的结论，所以，定理 B 和 C 必然有一个是错误的；2)条件(1.7)隐含 $c_3 - \frac{c_1}{c_2} > c_4$，也就是说，在(1.7)成立下，(1.3)不可能成立，从而系统(1.2)不可能有正平衡点，更不要说其稳定性了。既然在条件(1.3)成立下，系统(1.2)存在正平衡点，而文[16]所给条件是错误的，那么，到底什么条件才能保证系统(1.2)正平衡点的全局稳定性呢?此外，保证系统一致持久的条件是什么呢?定理 B 和定理 C 到底哪个会成立?哪个不成立呢?

本文的目的在于给出这些问题的肯定回答。我们首先给出保证系统的正平衡点全局渐近稳定性的充分性条件并给予严格证明，其后探讨了系统的持久性问题。本文所得结果补充和完善了文[16]的主要结果之一。

2. 主要结果

下面叙述本文的主要结果。

定理 2.1. 当(1.3)成立时，系统(1.2)的正平衡点 $E\left(x^*, y_1^*, y_2^*\right)$ 是全局渐近稳定的。

证明：我们将通过构造适当的Lyapunov函数来证明这一结论。

今构造Lyapunov函数

$$V(t) = K_1\left(x - x^* - x^* \ln\frac{x}{x^*}\right) + K_2\left(y_1 - y_1^* - y_1^* \ln\frac{y_1}{y_1^*}\right) + K_3\left(y_2 - y_2^* - y_2^* \ln\frac{y_2}{y_2^*}\right) \tag{2.1}$$

其中 $K_i, i = 1,2,3$ 是待定的正常数。注意到 x^*, y_1^*, y_2^* 满足方程

$$\begin{aligned} &1 - x^* - y_2^* = 0, \\ &y_2^* - c_1 y_1^* = 0, \\ &c_2 y_1^* - c_3 y_2^* + c_4 x^* y_2^* = 0. \end{aligned} \tag{2.2}$$

沿着系统(1.2)的正解计算导数，借助(2.2)，有

$$
\begin{aligned}
D^{+}V(t) &= K_1\frac{x-x^*}{x}x(1-x-y_2)+K_2\frac{y_1-y_1^*}{y_1}(y_2-c_1y_1) \\
&\quad +K_3\frac{y_2-y_2^*}{y_2}(c_2y_1-c_3y_2+c_4xy_2) \\
&= K_1(x-x^*)(x^*+y_2^*-x-y_2) \\
&\quad +K_2\frac{y_1-y_1^*}{y_1}\left(\frac{1}{y_1^*}(y_2y_1^*-y_1y_2^*)+\frac{y_1y_2^*}{y_1^*}-c_1y_1\right) \\
&\quad +K_3\frac{y_2-y_2^*}{y_2}\left(\frac{c_2}{y_2^*}(y_1y_2^*-y_1^*y_2)+\frac{c_2y_1^*y_2}{y_2^*}-c_3y_2+c_4xy_2\right) \\
&= -K_1(x-x^*)^2+K_1(x-x^*)(y_2^*-y_2) \\
&\quad +K_2\frac{y_1-y_1^*}{y_1}\left(\frac{1}{y_1^*}(y_2y_1^*-y_2y_1+y_2y_1-y_1y_2^*)\right) \\
&\quad +K_3\frac{y_2-y_2^*}{y_2}\left(\frac{c_2}{y_2^*}(y_1y_2^*-y_1y_2+y_1y_2-y_1^*y_2)-c_4x^*y_2+c_4xy_2\right) \\
&= -K_1(x-x^*)^2+K_1(x-x^*)(y_2^*-y_2) \\
&\quad +K_2\frac{y_1-y_1^*}{y_1}\left(\frac{1}{y_1^*}(y_2y_1^*-y_2y_1+y_2y_1-y_1y_2^*)\right) \\
&\quad +K_3\frac{y_2-y_2^*}{y_2}\left(\frac{c_2}{y_2^*}(y_1y_2^*-y_1y_2+y_1y_2-y_1^*y_2)-c_4x^*y_2+c_4xy_2\right) \\
&= -K_1(x-x^*)^2+K_1(x-x^*)(y_2^*-y_2)+K_2\frac{y_1-y_1^*}{y_1}\frac{y_2}{y_1^*}(y_1^*-y_1) \\
&\quad +K_2\frac{y_1-y_1^*}{y_1}\frac{y_1}{y_1^*}(y_2-y_2^*)+K_3\frac{y_2-y_2^*}{y_2}\frac{c_2}{y_2^*}y_1(y_2^*-y_2) \\
&\quad +K_3\frac{y_2-y_2^*}{y_2}\frac{c_2}{y_2^*}y_2(y_1-y_1^*)+K_3\frac{y_2-y_2^*}{y_2}y_2(-c_4x^*+c_4x) \\
&= -K_1(x-x^*)^2-K_1(x-x^*)(y_2-y_2^*)-\frac{K_2}{y_1^*}\frac{y_2}{y_1}(y_1-y_1^*)^2 \\
&\quad +\frac{K_2}{y_1^*}(y_1-y_1^*)(y_2-y_2^*)-\frac{K_3c_2}{y_2^*}\frac{y_1}{y_2}(y_2-y_2^*)^2 \\
&\quad +\frac{K_3c_2}{y_2^*}(y_2-y_2^*)(y_1-y_1^*)+K_3c_4(y_2-y_2^*)(x-x^*) \qquad (2.3) \\
&= -K_1(x-x^*)^2+(K_3c_4-K_1)(x-x^*)(y_2-y_2^*)-\frac{K_2}{y_1^*}\frac{y_2}{y_1}(y_1-y_1^*)^2 \\
&\quad +\left(\frac{K_2}{y_1^*}+\frac{K_3c_2}{y_2^*}\right)(y_1-y_1^*)(y_2-y_2^*)-\frac{K_3c_2}{y_2^*}\frac{y_1}{y_2}(y_2-y_2^*)^2
\end{aligned}
$$

今取

$$
K_1=c_4,K_2=\frac{c_2y_1^*}{y_2^*},K_3=1, \tag{2.4}
$$

则有

$$D^+V(t) = -c_4(x-x^*)^2 - \frac{c_2}{y_2^*}\left[\sqrt{\frac{y_2}{y_1}}(y_1-y_1^*) - \sqrt{\frac{y_1}{y_2}}(y_2-y_2^*)\right]^2. \tag{2.5}$$

因此，对所有的 $x, y_1, y_2 > 0$，且 $(x, y_1, y_2) \neq (x^*, y_1^*, y_2^*)$，均有 $D^+V(t) < 0$。且仅在 $E(x^*, y_1^*, y_2^*)$ 处 $D^+V(t) = 0$。由此可知 $E(x^*, y_1^*, y_2^*)$ 是全局渐近稳定的。

定理 2.1 证毕。

作为定理 2.1 的直接推论，我们立即得到如下有关系统(1.2)持久性的结论。

推论 2.1. 当(1.3)成立时，系统(1.2)是一致持久的。

3. 结论

于宇梅等学者[16]提出了强身型的捕食者-食饵模型(模型(1.1))，在作适当变换后，变成了系统(1.2)，作者探讨了系统的各个平衡点的存在性，正平衡点的局部稳定性和全局稳定性，系统的持久性和捕食者种群的绝灭性。我们注意到作者的主要结果：定理 B 和 C 是互相矛盾的；定理 D 的条件跟正平衡点存在的条件也是矛盾的，也就是说，文[16]的结果是否成立都需要进一步验证。本文中，我们证明了正平衡点如果存在，则必是全局吸引的，从而必有系统是一致持久的，我们的这一结果非常简洁完美，我们的结果补充和完善了文[16]的主要结果。

参考文献

[1] Chen, F.D., Chen, W.L., *et al.* (2013) Permanece of a Stage-Structured Predator-Prey System. *Applied Mathematics and Computation*, **219**, 8856-8862.

[2] Chen, F.D., Xie, X.D., *et al.* (2012) Partial Survival and Extinction of a Delayed Predator-Prey Model with Stage Structure. *Applied Mathematics and Computation*, **219**, 4157-4162.

[3] Chen, F.D., Wang, H.N., *et al.* (2013) Global Stability of a Stage-Structured Predator-Prey System. *Applied Mathematics and Computation*, **22**, 345-353.

[4] Li, T.T., Chen, F.D., *et al.* (2017) Stability of a Mutualism Model in Plant-Pollinator System with Stage-Structure and the Beddington-De Angelis Functional Response. *Journal of Nonlinear Functional Analysis*, **2017**, Article ID: 50.

[5] Li, Z., Han, M.A., *et al.* (2012) Global Stability of Stage-Structured Predator-Prey Model with Modified Leslie-Gower and Holling-Type II Schemes. *International Journal of Biomathematics*, **6**, Article ID: 1250057.

[6] Li, Z., Han, M., *et al.* (2014) Global Stability of a Predator-Prey System with Stage Structure and Mutual Interference. *Discrete and Continuous Dynamical Systems-Series B* (*DCDS-B*), **19**, 173-187.

[7] Lin, X., Xie, X., *et al.* (2016) Convergences of a Stage-Structured Predator-Prey Model with Modified Leslie-Gower and Holling-Type II Schemes. *Advances in Difference Equations*, **2016**, 181.

[8] Pu, L.Q., Miao, Z.S., *et al.* (2015) Global Stability of a Stage-Structured Predator-Prey Model. *Communications in Mathematical Biology and Neuroscience*, **2015**, Article ID: 5.

[9] Han, R.Y., Yang, L.Y., *et al.* (2015) Global Attractivity of a Single Species Stage-Structured Model with Feedback Control and Infinite Delay. *Communications in Mathematical Biology and Neuroscience*, **2015**, Article ID: 6.

[10] Wu, H.L. and Chen, F.D. (2009) Harvesting of a Single-Species System Incorporating Stage Structure and Toxicity. *Discrete Dynamics in Nature and Society*, **2009**, Article ID: 290123.

[11] Khajanchi, S. and Banerjee, S. (2017) Role of Constant Prey Refuge on Stage Structure Predator-Prey Model with Ratio Dependent Functional Response. *Applied Mathematics and Computation*, **314**, 193-198.

[12] Wei, F. and Fu, Q. (2016) Globally Asymptotic Stability of a Predator-Prey Model with Stage Structure Incorporating Prey Refuge. *International Journal of Biomathematics*, **9**, Article ID: 1650058.

[13] Xue, Y., Pu, L., *et al.* (2015) Global Stability of a Predator-Prey System with Stage Structure of Distributed-Delay Type. *Communications in Mathematical Biology and Neuroscience*, **2015**, Article ID: 12.

[14] Naji, R.K. and Majeed, S.J. (2016) The Dynamical Analysis of a Prey-Predator Model with a Refuge-Stage Structure Prey Population. *International Journal of Differential Equations*, **2016**, Article ID: 2010464.

[15] Lu, Y., Pawelek, K.A., *et al.* (2017) A Stage-Structured Predator-Prey Model with Predation over Juvenile Prey. *Applied Mathematics and Computation*, **29**, 7115-7130.

[16] Yu, Y.M., Zhang, Y. and Wang, W.D. (2001) The Asymptotic Behavior of a Strengthening Type Predator-Prey Model with Stage Structure. *Journal of Southwest China Normal University* (*Natural Science*), **26**, 363-367. (In Chinese)

[17] Hao, P.M., Wang, X.C. and Wei, J.J. (2018) Hopf Bifurcation Analysis of a Diffusive Single Species Model with Stage Structure and Strong Allee Effect. *Mathematics and Computers in Simulation*, **153**, 1-14.

[18] Li, Z. and Chen, F.D. (2009) Extinction in Periodic Competitive Stage-Structured Lotka-Volterra Model with the Effects of Toxic Substances. *Journal of Computational and Applied Mathematics*, **23**, 1143-1153.