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带有混合时滞和非线性扰动的中立系统指数稳定性改进的判别准则

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摘 要

本文主要研究带有混合时变时滞和非线性扰动的不确定中立型系统的指数稳定性问题。构造新的李雅谱诺夫泛函时考虑了区间时变时滞的上界和下界，结合改进的自由权矩阵方法，利用线性矩阵不等式给出了一些新的时滞相关的稳定性准则。提出的新算法适用于离散时滞下界不为零的情况。数值例子进一步表明本文算法的有效性。

关键词

中性系统，指数稳定性，时变时滞，非线性扰动

Improved Exponential Stability Criteria for Neutral System with Mixed Time-Varying Delays and Nonlinear Perturbations

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Abstract

This paper is concerned with the exponential stability problem for uncertain neutral systems with mixed time-varying delays and nonlinear perturbations. Some new stability criteria dependent on time delay are derived in terms of linear matrix inequalities (LMIs). The proposed method is based on the new Lyapunov-Krasovskii functionals involving lower and upper bounds of interval time-varying delays and Improved Free-Weighting Matrices approach (IFWM), and is applicable to the case that the lower bound of discrete delay is not zero. Two numerical examples are provided to show the effectiveness of the proposed method.

Keywords

Neutral System, Exponential Stability, Time-Varying Delay, Nonlinear Perturbation

1. 介绍

中立系统已在人口生态学、分布网络和热交换过程等许多实际工程中得到了广泛应用。目前，中立系统的渐近稳定性问题得到了广泛研究，并取得了丰硕成果 [1] [2] [3] [4] [5]。可以看出，时滞和非线性扰动往往会导致中立系统性能变差或不稳定。因此，具有混合时变时滞和非线性扰动的中立型系统稳定性问题成为近年来的研究热点。其中，矩阵分解方法和自由加权矩阵方法是研究渐近稳定性的主要方法。

同中立系统的渐近稳定性结果相比，其指数稳定性判据较少。例如，基于李雅谱诺夫泛函方法和线性矩阵不等式，文献 [6]研究了线性时滞系统的时滞相关指数稳定性，但文中没有考虑系统的不确定性。文献 [7]针对时不变时滞的情形，利用矩阵不等式研究了不确定时滞系统的指数稳定性。文献 [8]利用积分不等式技巧研究了具有非线性扰动的中立型系统的鲁棒指数稳定性问题。然而，文献 [9]在求解李雅谱诺夫泛函导数上界时，通过引入自由权矩阵方法改进了文献 [8]的结果，降低了结果的保守性。例如，多数文献通常将$h(t)\zeta^T(t)XS_1^{-1}X^T\zeta(t)$ 和$(h-h(t))\zeta^T(t)YS_1^{-1}Y^T\zeta(t)$

这两项扩大为$h\zeta^T(t)XS_1^{-1}X^T\zeta(t)$ 和$h\zeta^T(t)YS_1^{-1}Y^T\zeta(t)$, 并没有考虑它们之间的关系(事实上，$h=(h-h(t))+h(t)$)，这样的处理方法将会对稳定性结果产生一定的保守性。文献 [10]提出了时变时滞系统稳定性的自由权矩阵方法，得到了时滞系统的时滞相关条件。该方法考虑了李雅谱诺夫函数差分中的任何有用项，并证明了其对最大时滞界的改进。因此，基于改进的自由权矩阵法的非线性扰动中立型系统的指数稳定性问题仍值得进一步研究。

2. 预备引理

考虑一类具有混合时变时滞和非线性扰动的中立型系统：

$$\begin{cases} \dot{x}(t)=Ax(t)+Bx(t-h(t))+C\dot{x}(t-\tau(t))+f_1(x(t),t)+f_2(x(t-h(t)),t)+f_3(\dot{x}(t-\tau(t)),t), \\ x(\theta)=\phi(\theta), \quad \dot{x}(\theta)=\varphi(\theta), \qquad\qquad \forall\theta\in[-\max(h_2,\tau),0], \end{cases} \tag{1}$$

其中，$x(t)\in R^n$ 是状态变量, $A,B,C\in R^{n\times n}$ 是具有合适维数的常数矩阵。$h(t),\tau(t)$ 分别是时变离散时滞和中立时滞，并且满足下列条件：

$$0\leq\ h_1\leq h(t)\leq h_2, \quad \dot{h}(t)\leq h_d, \tag{2}$$

$$0\leq\tau(t)\leq\tau, \quad \dot{\tau}(t)\leq\tau_d<1, \tag{3}$$

h_1,h_2,τ,h_d,τ_d 是常数. $\phi(\theta),\varphi(\theta)$ 是在$[-\max(h_2,\tau),0]$上连续可微的初始条件函数。$f_1(x(t),t),f_2(x(t-h(t),t),f_3(\dot{x}(t-\tau(t)),t)$是未知的非线性扰动，且满足：$f_1(0,t)=0,f_2(0,t)=0,f_3(0,t)=0$和

$$\begin{cases} \| f_1(x(t),t)\|\leq\alpha\| x(t)\|, \\ \| f_2(x(t-h(t)),t)\|\leq\beta\| x(t-h(t))\|, \\ \| f_3(\dot{x}(t-\tau(t)),t)\|\leq\gamma\| \dot{x}(t-\tau(t))\|\ . \end{cases} \tag{4}$$

进一步，条件(4)可写为：

$$\begin{cases} f_1^T(x(t),t)f_1(x(t),t)\leq\alpha^2x^T(t)x(t), \\ f_2^T(x(t-h(t)),t)f_2(x(t-h(t)),t)\leq\beta^2x^T(t-h(t))x(t-h(t)), \\ f_3^T(\dot{x}(t-\tau(t)),t)f_3(\dot{x}(t-\tau(t)),t)\leq\gamma^2\dot{x}^T(t-\tau(t))\dot{x}(t-\tau(t)). \end{cases} \tag{5}$$

其中$\alpha\geq 0,\beta\geq 0$和$\gamma\geq 0$ 是已知常数。为了表述方便，我们定义$f_1:=f_1(x(t),t),f_2:=f_2(x(t-h(t)),t)$和$f_3=f_3(\dot{x}(t-\tau(t)),t)$.

算子$D:C([-\tau,0],\mathbf{R}^n)\to\mathbf{R}^n$ 定义为$Dx_t=x(t)-Cx(t-\tau)$.

定义1 [11]. 若齐次差分方程$Dx_t=0,t\geq 0,x_0=\psi\in\{\phi\in C([-\tau,0]:D\phi=0)\}$ 的零解是一致渐近稳定的，则称算子D 是稳定的。

定义2 [9]. 若标量$\varepsilon>0$ 和$\gamma>1$ 使得$\| x(t)\|\leq\gamma e^{-\varepsilon t}\mid\mu\mid_h$, 其中，$\mid\mu\mid_h=$ $\sup_{-h\leq\ s\leq\ 0}\sqrt{\|\phi(s)\|^2+\|\varphi(s)\|^2}$，则称系统(1)以衰减率$\varepsilon$ 指数稳定。

3. 主要结论

定理1. 已知$h_2 \geq h_1 \geq 0, \tau \geq 0, h_d$ 和τ_d, 如果存在正定矩阵$P > 0, R_1 > 0, R_2 > 0, R_3 > 0, Q > 0, S_1 > 0, S_2 > 0, Z_1 > 0, Z_2 > 0, M > 0, U \geq 0, V \geq 0, W \geq 0$ 和合适维数矩阵X_1, X_2, Y_1, Y_2, L_1 ，以及$\varepsilon_j \geq 0 (j = 1, 2, 3)$ 使得下列对称线性矩阵不等式成立：

$$\begin{pmatrix} U & X_1 \\ * & Z_1 \end{pmatrix} \geq 0, \tag{6}$$

$$\begin{pmatrix} V & L_1 \\ * & Z_2 \end{pmatrix} \geq 0, \tag{7}$$

$$\begin{pmatrix} W & Y_2 \\ * & M \end{pmatrix} \geq 0, \tag{8}$$

$$\begin{pmatrix} W & X_2 \\ * & M \end{pmatrix} \geq 0, \tag{9}$$

$$\begin{pmatrix} U+V & Y_1 \\ * & Z_1+Z_2 \end{pmatrix} \geq 0, \tag{10}$$

$$\begin{pmatrix} \Xi & \Gamma^T Q & h_2 \Gamma^T Z_1 & (h_2-h_1)\Gamma^T Z_2 & \tau \Gamma^T M \\ & -Q & 0 & 0 & 0 \\ & * & -h_2 Z_1 & 0 & 0 \\ & * & * & -(h_2-h_1)Z_2 & 0 \\ & * & * & * & -\tau M \end{pmatrix} < 0, \tag{11}$$

则系统(1)指数稳定，其中，

$$\Xi = \Xi_1 + \Xi_2 + \Xi_2^T + h_2 U + (h_2 - h_1)V + \tau W,$$

$$\Xi_1 = \begin{pmatrix} \Xi_{11} & PB & 0 & 0 & PC & 0 & 0 & P & P & P \\ & \Xi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & -e^{-2\varepsilon h_1}R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & -e^{-2\varepsilon h_2}R_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & \Xi_{55} & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & * & \Xi_{66} & 0 & 0 & 0 & 0 \\ & * & * & * & * & * & -S_2 e^{-2\varepsilon\tau} & 0 & 0 & 0 \\ & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ & * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix},$$

$\Xi_{11} = 2\varepsilon P + PA + A^T P + R_1 + R_2 + R_3 + S_1 + S_2 + \varepsilon_1 \alpha^2 I,$

$\Xi_{22} = -(1-h_d)R_1 e^{-2\varepsilon h_2} + \varepsilon_2 \beta^2 I,$

$\Xi_{55} = -(1-\tau_d)Q e^{-2\varepsilon\tau} + \varepsilon_3 \gamma^2 I,$

$\Xi_{66} = -(1-\tau_d)S_1 e^{-2\varepsilon\tau},$

$\Xi_2 = (X_1 + X_2, -X_1 + Y_1 - L_1, L_1, -Y_1, 0, -X_2 + Y_2, -Y_2, 0, 0, 0),$

$\Gamma = (A, B, 0, 0, C, 0, 0, I, I, I).$

证明. 对系统(1)，构造如下李雅普诺夫泛函：

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t), \tag{12}$$

其中，

$$V_1(t)=e^{2\varepsilon t}x^T(t)Px(t), \tag{13}$$

$$V_2(t)=\int_{t-h(t)}^{t}e^{2\varepsilon s}x^T(s)R_1x(s)ds+\int_{t-h_1}^{t}e^{2\varepsilon s}x^T(s)R_2x(s)ds$$
$$+\int_{t-h_2}^{t}e^{2\varepsilon s}x^T(s)R_3x(s)ds, \tag{14}$$

$$V_3(t)=\int_{t-\tau(t)}^{t}e^{2\varepsilon s}x^T(s)S_1x(s)ds+\int_{t-\tau}^{t}e^{2\varepsilon s}x^T(s)S_2x(s)ds, \tag{15}$$

$$V_4(t)=\int_{t-\tau(t)}^{t}e^{2\varepsilon s}\dot{x}^T(s)Q\dot{x}(s)ds, \tag{16}$$

$$V_5(t)=\int_{-h_2}^{0}\int_{t+\theta}^{t}e^{2\varepsilon s}\dot{x}^T(s)Z_1\dot{x}(s)dsd\theta+\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}e^{2\varepsilon s}\dot{x}^T(s)Z_2\dot{x}(s)dsd\theta$$
$$+\int_{-\tau}^{0}\int_{t+\theta}^{t}e^{2\varepsilon s}\dot{x}^T(s)M\dot{x}(s)dsd\theta, \tag{17}$$

且$P=P^T>0, R_1=R_1^T>0, R_2=R_2^T>0, R_3=R_3^T>0, S_1=S_1^T>0, S_2=S_2^T>0, Q=Q^T>0, Z_1=Z_1^T>0, Z_2=Z_2^T>0$ 以及$M=M^T>0$ 是待定矩阵。

计算$V(t)$ 沿系统(1)的导数为：

$$\begin{aligned}
\dot{V}_1(t) \;=\;& e^{2\varepsilon t}\Big[x^T(t)(2\varepsilon P+PA+A^TP)x(t)+x^T(t)2PBx(t-h(t))+x^T(t)2PC\dot{x}(t-\tau(t))\\
&+x^T(t)2Pf_1+x^T(t)2Pf_2+x^T(t)2Pf_3\Big],\\
\dot{V}_2(t) \;\le\;& e^{2\varepsilon t}\Big[x^T(t)(R_1+R_2+R_3)x(t)-(1-h_d)e^{-2\varepsilon h_2}x^T(t-h(t))R_1x(t-h(t))\\
&-e^{-2\varepsilon h_1}x^T(t-h_1)R_2x(t-h_1)-e^{-2\varepsilon h_2}x^T(t-h_2)R_3x(t-h_2)\Big],\\
\dot{V}_3(t) \;\le\;& e^{2\varepsilon t}\Big[x^T(t)(S_1+S_2)x(t)-(1-\tau_d)e^{-2\varepsilon\tau}x^T(t-\tau(t))S_1x(t-\tau(t))\\
&-e^{-2\varepsilon\tau}x^T(t-\tau)S_2x(t-\tau)\Big],\\
\dot{V}_4(t) \;\le\;& e^{2\varepsilon t}\Big[\dot{x}^T(t)Q\dot{x}(t)-(1-\tau_d)e^{-2\varepsilon\tau}\dot{x}^T(t-\tau(t))Q\dot{x}(t-\tau(t))\Big],\\
\dot{V}_5(t) \;=\;& e^{2\varepsilon t}\Big[\dot{x}^T(t)\big(h_2Z_1+(h_2-h_1)Z_2+\tau M\big)\dot{x}(t)-\int_{t-h_2}^{t-h(t)}e^{2\varepsilon(s-t)}\dot{x}^T(s)Z_1\dot{x}(s)ds\\
&-\int_{t-h(t)}^{t}e^{2\varepsilon(s-t)}\dot{x}^T(s)Z_1\dot{x}(s)ds-\int_{t-h_2}^{t-h(t)}e^{2\varepsilon(s-t)}\dot{x}^T(s)Z_2\dot{x}(s)ds-\\
&\int_{t-h(t)}^{t-h_1}e^{2\varepsilon(s-t)}\dot{x}^T(s)Z_2\dot{x}(s)ds-\int_{t-\tau}^{t-\tau(t)}e^{2\varepsilon(s-t)}\dot{x}^T(s)M\dot{x}(s)ds\\
&-\textstyle\int_{t-\tau(t)}^{t}e^{2\varepsilon(s-t)}\dot{x}^T(s)M\dot{x}(s)ds\Big].
\end{aligned}$$

对任何标量$s \in [t-h_2, t], [t-h_2, t-h_1]$ 和$[t-\tau, t]$，我们得到：

$$\begin{array}{llllll} e^{-2\varepsilon h_2} & \leq & e^{2\varepsilon(s-t)} & \leq & 1. & \\ e^{-2\varepsilon h_2} & \leq & e^{2\varepsilon(s-t)} & \leq & e^{-2\varepsilon h_1} & \leq \ 1. \\ e^{-2\varepsilon \tau} & \leq & e^{2\varepsilon(s-t)} & \leq & 1. & \end{array} \tag{18}$$

所以

$$\begin{aligned} \dot{V}_5(t) \ \leq \ & e^{2\varepsilon t}\Big[\dot{x}^T(t)\big(h_2 Z_1 + (h_2-h_1)Z_2 + \tau M\big)\dot{x}(t) - e^{-2\varepsilon h_2}\int_{t-h_2}^{t-h(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds \\ & -e^{-2\varepsilon h_2}\int_{t-h(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - e^{-2\varepsilon h_2}\int_{t-h_2}^{t-h(t)} \dot{x}^T(s)Z_2\dot{x}(s)ds \\ & -e^{-2\varepsilon h_2}\int_{t-h(t)}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds - e^{-2\varepsilon \tau}\int_{t-\tau}^{t-\tau(t)} \dot{x}^T(s)M\dot{x}(s)ds \\ & -e^{-2\varepsilon \tau}\textstyle\int_{t-\tau(t)}^{t} \dot{x}^T(s)M\dot{x}(s)ds\Big]. \end{aligned} \tag{19}$$

对任意实矩阵$X_i, Y_i, (i=1,2), L_1, U \geq 0, V \geq 0, W \geq 0$，由牛顿莱布尼茨公式知下列方程成立:

$$\rho_1(t) = 2e^{2\varepsilon t}\zeta^T(t)X_1\Big[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)ds\Big] = 0, \tag{20}$$

$$\rho_2(t) = 2e^{2\varepsilon t}\zeta^T(t)Y_1\Big[x(t-h(t)) - x(t-h_2) - \int_{t-h_2}^{t-h(t)} \dot{x}(s)ds\Big] = 0, \tag{21}$$

$$\rho_3(t) = 2e^{2\varepsilon t}\zeta^T(t)L_1\Big[x(t-h_1) - x(t-h(t)) - \int_{t-h(t)}^{t-h_1} \dot{x}(s)ds\Big] = 0, \tag{22}$$

$$\rho_4(t) = 2e^{2\varepsilon t}\zeta^T(t)X_2\Big[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds\Big] = 0, \tag{23}$$

$$\rho_5(t) = 2e^{2\varepsilon t}\zeta^T(t)Y_2\Big[x(t-\tau(t)) - x(t-\tau) - \int_{t-\tau}^{t-\tau(t)} \dot{x}(s)ds\Big] = 0, \tag{24}$$

$$\rho_6(t) = e^{2\varepsilon t}\Big[h_2\zeta^T(t)U\zeta(t) - \int_{t-h_2}^{t-h(t)} \zeta^T(t)U\zeta(t)ds - \int_{t-h(t)}^{t} \zeta^T(t)U\zeta(t)ds\Big] = 0, \tag{25}$$

$$\rho_7(t) = e^{2\varepsilon t}\Big[(h_2-h_1)\zeta^T(t)V\zeta(t) - \int_{t-h_2}^{t-h(t)} \zeta^T(t)V\zeta(t)ds - \int_{t-h(t)}^{t-h_1} \zeta^T(t)V\zeta(t)ds\Big] = 0, \tag{26}$$

$$\rho_8(t) = e^{2\varepsilon t}\Big[\tau\zeta^T(t)W\zeta(t) - \int_{t-\tau}^{t-\tau(t)} \zeta^T(t)W\zeta(t)ds - \int_{t-\tau(t)}^{t} \zeta^T(t)W\zeta(t)ds\Big] = 0, \tag{27}$$

其中，

$$\zeta^T(t) = [x^T(t), x^T(t-h(t)), x^T(t-h_1), x^T(t-h_2), \dot{x}^T(t-\tau(t)), x^T(t-\tau(t)), x^T(t-\tau), f_1^T, f_2^T, f_3^T].$$

对任意标量$\varepsilon_1>0,\varepsilon_2>0$ 和$\varepsilon_3>0$，我们进一步得到：

$$\begin{cases}\varepsilon_1\Big(\alpha^2x^T(t)x(t)-f_1^T(x(t),t)f_1(x(t),t)\Big)\geq 0,\\ \varepsilon_2\Big(\beta^2x^T(t-h(t))x(t-h(t))-f_2^T(x(t-h(t)),t)f_2(x(t-h(t)),t)\Big)\geq 0,\\ \varepsilon_3\Big(\gamma^2\dot{x}^T(t-\tau(t))\dot{x}(t-\tau(t))-f_3^T(\dot{x}(t-\tau(t)),t)f_3(\dot{x}(t-\tau(t)),t)\Big)\geq 0.\end{cases}\tag{28}$$

联合(18)-(28)，可得：

$$\begin{aligned}
&\dot{V}(t)\leq\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+\dot{V}_4(t)+\dot{V}_5(t)+e^{2\varepsilon t}\Big(\varepsilon_1\alpha^2x^T(t)x(t)-\varepsilon_1f_1^Tf_1\\
&+\varepsilon_2\beta^2x^T(t-h(t))x(t-h(t))-\varepsilon_2f_2^Tf_2+\varepsilon_3\gamma^2\dot{x}^T(t-\tau(t))\dot{x}(t-\tau(t))-\varepsilon_3f_3^Tf_3\Big)\\
&+\rho_1(t)+\rho_2(t)+\rho_3(t)+\rho_4(t)+\rho_5(t)+\rho_6(t)+\rho_7(t)+\rho_8(t)\\
&\leq\ e^{2\varepsilon t}\Bigg((x^T(t)(2\varepsilon p+PA+A^TP+R_1+R_2+R_3+S_1+S_2+\varepsilon_1\alpha^2I)x(t)\\
&+x^T(t)(PB+B^TP)x(t-h(t))+x^T(t)(PC+C^TP)\dot{x}(t-\tau(t))\\
&+x^T(t-h(t))\Big(-(1-h_d)R_1e^{-2\varepsilon h_2}+\varepsilon_2\beta^2I\Big)x(t-h(t))-x^T(t-h_1)R_2e^{-2\varepsilon h_1}x(t-h_1)\\
&-x^T(t-h_2)R_3e^{-2\varepsilon h_2}x(t-h_2)-x^T(t-\tau(t))(1-\tau_d)S_1e^{-2\varepsilon\tau}x(t-\tau(t))\\
&-x^T(t-\tau)S_2e^{-2\varepsilon\tau}x(t-\tau)+\dot{x}^T(t-\tau(t))\Big(-(1-\tau_d)Qe^{-2\varepsilon\tau}+\varepsilon_3\gamma^2I\Big)\dot{x}(t-\tau(t))\\
&+x^T(t)(2P)f_1+x^T(t)(2P)f_2+x^T(t)(2P)f_3-\varepsilon_1f_1^Tf_1-\varepsilon_2f_2^Tf_2-\varepsilon_3f_3^Tf_3\\
&+\dot{x}^T(t)\Big(Q+h_2Z_1+(h_2-h_1)Z_2+\tau M\Big)\dot{x}(t)\\
&-e^{-2\varepsilon h_2}\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)Z_1\dot{x}(s)ds\\
&-e^{-2\varepsilon h_2}\int_{t-h(t)}^{t}\dot{x}^T(s)Z_1\dot{x}(s)ds-e^{-2\varepsilon h_2}\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
&-e^{-2\varepsilon h_2}\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)Z_2\dot{x}(s)ds-e^{-2\varepsilon\tau}\int_{t-\tau}^{t-\tau(t)}\dot{x}^T(s)M\dot{x}(s)ds\\
&-e^{-2\varepsilon\tau}\int_{t-\tau(t)}^{t}\dot{x}^T(s)M\dot{x}(s)ds+2\zeta^T(t)X_1\Big[x(t)-x(t-h(t))-\int_{t-h(t)}^{t}\dot{x}(s)ds\Big]\\
&+2\zeta^T(t)Y_1\Big[x(t-h(t))-x(t-h_2)-\int_{t-h_2}^{t-h(t)}\dot{x}(s)ds\Big]\\
&+2\zeta^T(t)L_1\Big[x(t-h_1)-x(t-h(t))-\int_{t-h(t)}^{t-h_1}\dot{x}(s)ds\Big]\\
&+2\zeta^T(t)X_2\Big[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds\Big]\\
&+2\zeta^T(t)Y_2\Big[x(t-\tau(t))-x(t-\tau)-\int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds\Big]\\
&+h_2\zeta^T(t)U\zeta(t)-\int_{t-h_2}^{t-h(t)}\zeta^T(t)U\zeta(t)ds-\int_{t-h(t)}^{t}\zeta^T(t)U\zeta(t)ds\\
&+(h_2-h_1)\zeta^T(t)V\zeta(t)-\int_{t-h_2}^{t-h(t)}\zeta^T(t)V\zeta(t)ds-\int_{t-h(t)}^{t-h_1}\zeta^T(t)V\zeta(t)ds\\
&+\tau\zeta^T(t)W\zeta(t)-\int_{t-\tau}^{t-\tau(t)}\zeta^T(t)W\zeta(t)ds-\int_{t-\tau(t)}^{t}\zeta^T(t)W\zeta(t)ds\Bigg)
\end{aligned}\tag{29}$$

$$\begin{aligned}
&\leq\ e^{2\varepsilon t}\zeta^T(t)\Big(\Xi_1+\Xi_2+\Xi_2^T+h_2U+(h_2-h_1)V+\tau W+\Gamma^TQ\Gamma++h_2\Gamma^TZ_1\Gamma\\
&+(h_2-h_1)\Gamma^TZ_2\Gamma+\tau\Gamma^TM\Gamma\Big)\zeta(t).\\
&-e^{2\varepsilon(t-h_2)}\int_{t-h_2}^{t-h(t)}\begin{pmatrix}\zeta^T(t) & \dot{x}^T(s)\end{pmatrix}\begin{pmatrix}U+V & Y_1\\ & Z_1+Z_2\end{pmatrix}\begin{pmatrix}\zeta(t)\\ \dot{x}(s)\end{pmatrix}ds\\
&-e^{2\varepsilon(t-h_2)}\int_{t-h(t)}^{t}\begin{pmatrix}\zeta^T(t) & \dot{x}^T(s)\end{pmatrix}\begin{pmatrix}U & X_1\\ & Z_1\end{pmatrix}\begin{pmatrix}\zeta(t)\\ \dot{x}(s)\end{pmatrix}ds\\
&-e^{2\varepsilon(t-h_2)}\int_{t-h(t)}^{t-h_1}\begin{pmatrix}\zeta^T(t) & \dot{x}^T(s)\end{pmatrix}\begin{pmatrix}V & L_1\\ & Z_2\end{pmatrix}\begin{pmatrix}\zeta(t)\\ \dot{x}(s)\end{pmatrix}ds\\
&-e^{2\varepsilon(t-\tau)}\int_{t-\tau}^{t-\tau(t)}\begin{pmatrix}\zeta^T(t) & \dot{x}^T(s)\end{pmatrix}\begin{pmatrix}W & Y_2\\ & M\end{pmatrix}\begin{pmatrix}\zeta(t)\\ \dot{x}(s)\end{pmatrix}ds\\
&-e^{2\varepsilon(t-\tau)}\int_{t-\tau(t)}^{t}\begin{pmatrix}\zeta^T(t) & \dot{x}^T(s)\end{pmatrix}\begin{pmatrix}W & X_2\\ & M\end{pmatrix}\begin{pmatrix}\zeta(t)\\ \dot{x}(s)\end{pmatrix}ds
\end{aligned}\tag{30}$$

如果$\Pi<0$, (6)-(11) 成立，那么$\dot{V}(t)\leq e^{2\varepsilon t}\zeta^T(t)\Pi\zeta(t)<0$. 其中，

$$\Pi\ =\ \Xi_1+\Xi_2+\Xi_2^T+h_2U+(h_2-h_1)V+\tau W+\Gamma^TQ\Gamma++h_2\Gamma^TZ_1\Gamma+(h_2-h_1)\Gamma^TZ_2\Gamma+\tau\Gamma^TM\Gamma.$$

如果算子D是稳定的，且存在对称正定矩阵$P>0,R_1>0,R_2>0,R_3>0,Q>0,S_1>0,S_2>0,Z_1>0,Z_2>0,M>0,U\geq 0,V\geq 0,W\geq 0$ 和标量$\varepsilon_1\geq 0,\varepsilon_2\geq 0,\varepsilon_3\geq 0$ 使得线性矩阵不等式(6)-(11)成立，那么系统(1)是渐近稳定的。

因为$\dot{V}(t)<0$, 即李雅普诺夫泛函(12) 是单调下降的，那么对任意$t\in[h_1,h_2]$，不难得到：

$$e^{2\varepsilon t}\lambda_{min}(P)\parallel x(t)\parallel^2\ \leq\ V(t)\leq\ V(h_1)\leq\ V(0)\leq\ \lambda\mid\mu\mid_h^2\tag{31}$$

其中，$\lambda=\max\{\lambda_{max}(P),h_2\lambda_{max}(R_1),h_1\lambda_{max}(R_2),h_2\lambda_{max}(R_3),\tau\lambda_{max}(S_1),\tau\lambda_{max}(S_2),\tau\lambda_{max}(Q),h^2\lambda_{max}(Z_1),(h_2-h_1)^2\lambda_{max}(Z_2),\tau^2\lambda_{max}(M)\}$ 和定义2中给出的$\mid\mu\mid_h$。所以，

$$\parallel x(t)\parallel\ \leq\ \gamma e^{-\varepsilon t}\mid\mu\mid_h\tag{32}$$

其中$\gamma=\sqrt{\lambda/\lambda_{min}(P)}\geq\ 1$” 因此，系统(1)是以衰减率$\varepsilon$ 鲁棒指数稳定的。

注1. 如果$\varepsilon=0$，那么定理1在离散时滞下界不为零的条件下退化为渐近稳定性准则。如果$C=0$和$f_3(\dot{x}(t-\tau(t)),t)=0$，本文中的定理1将退化为文献 [10]中的相关定理，进一步说明定理1的稳定性判别方法具有一定的优越性。

注2. 在文献 [9]中，不等式$\int_{t-h(t)}^{t}\zeta^T(t)X_1V_1^{-1}X_1^T\zeta(t)ds\leq h\zeta^T(t)X_1V_1^{-1}X_1^T\zeta(t)$ 和 $\int_{t-h}^{t-h(t)}\zeta^T(t)Y_1Z_1^{-1}Y_1^T\zeta(t)ds\leq h\zeta^T(t)Y_1Z_1^{-1}Y_1^T\zeta(t)$ 用于获得时滞依赖的渐近稳定性准则。然而，此文献中没有进一步考虑$h(t)$ 和$h-h(t)$ 之间的关系。为了减少结果的保守性，我们在定理1中引入了$\rho_6(t),\rho_7(t)$ 和$\rho_8(t)$，并且在$\rho_6(t)$中将矩阵h_2U 分解为$h(t)U$ 和$(h_2-h(t))U$。

4. 数值仿真

例1. 考虑文献 [3]中的中立系统：

$$A=\begin{pmatrix}-1.2 & 0.1\\ -0.1 & -1\end{pmatrix},\quad B=\begin{pmatrix}-0.6 & 0.7\\ -1 & -0.8\end{pmatrix},\quad C=\begin{pmatrix}c & 0\\ 0 & c\end{pmatrix},$$

$$\begin{aligned}&f_1^T(x(t),t)f_1(x(t),t)\le \alpha^2x^T(t)x(t),\\ &f_2^T(x(t-h(t)),t)f_2(x(t-h(t)),t)\le \beta^2x^T(t-h(t))x(t-h(t)),\\ &f_3^T(\dot{x}(t-\tau(t)),t)f_3(\dot{x}(t-\tau(t)),t)\le \gamma^2\dot{x}^T(t-\tau(t))\dot{x}(t-\tau(t)),\end{aligned}$$

且$c=0.1,\alpha\ge 0,\beta\ge 0,\gamma\ge 0$. 对于$\beta=0.1,\tau=1,\tau_d=0,h_d=0.5,h_1=0,\varepsilon=0$, $\alpha=0$ 和$\alpha=0.1$, 表 1给出了不同的γ值下最大时滞h_2。这些数据表明，基于本文的稳定性判别方法得到的结果要优于文献 [2] [3] [4] [5]中的结果，并且最大时滞h_2会随着α或γ的增加而减小。

Table 1. Maximum upper bound of h_2 with different values of α and γ

表 1. 在不同的α 和γ下h_2 的最大上界

	$\alpha=0$					$\alpha=0.1$			
γ	0	0.1	0.2	0.3	γ	0	0.1	0.2	0.3
[2]	0.9488	0.7695	0.6087	0.4667	[2]	0.8408	0.6841	0.5420	0.4144
[3]	0.9839	0.8024	0.6392	0.4941	[3]	0.8752	0.7166	0.5727	0.4438
[4]	1.4886	1.2437	0.9921	0.7367	[4]	1.3244	1.0901	0.8475	0.6300
[5]	1.6325	1.3386	1.0816	0.8563	[5]	1.4440	1.1950	0.9734	0.7760
定理1	1.9744	1.6317	1.3073	1.0001	定理1	1.7591	1.4451	1.1414	0.8616

5. 结论

针对带有混合时变时滞和非线性扰动的不确定中立型系统，本文主要研究其指数稳定性判定方法。引入自由权矩阵方法求解李雅普诺夫泛函导数的上界。基于线性矩阵不等式，得到了一些新的指数稳定的充分条件。本文推广了一些已有结果，并且降低了结果的保守性。最后，通过数值例子说明本文结果的有效性。

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