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带乘性噪声的离子声波和朗缪尔波随机方程组的新行波解

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摘 要

本文利用多项式的完全判别法获得了带乘性噪声的离子声波和朗缪尔波随机方程组的行波解，这些解包括双曲函数解、三角函数解、有理函数解和雅克比椭圆函数解。本文所获得的解可以进一步解释该类随机微分方程的波的传播情况。

关键词

例子波，朗缪尔波，行波解，乘性噪声，完全判别系统

New Traveling Wave Solution of Stochastic Equations of the Ion Sound and Langmuir Waves with Multiplicative Noise

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Abstract

In this paper, by using the complete discriminant method of polynomials, new traveling wave solution of stochastic equations of the ion sound and Langmuir waves with multiplicative noise is obtained. These solutions include hyperbolic function solutions, trigonometric function solutions,

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rational function solutions and Jacobi elliptic function solutions. The solutions obtained in this paper can further explain the wave propagation of this kind of stochastic differential equations.

Keywords

Ion Sound Wave, Langmuir Wave, Traveling Wave Solution, Multiplicative Noise, Complete Discriminant System

1. 引言

随机非线性偏微分方程[1]-[7]通常被用于模拟等离子体物理学、量子力学、非线性光学和工程等领域中的非线性问题。通常，科研工作者更关注于这类方程的精确解。近年来，构造随机非线性偏微分方程的精确解的研究已经成为了一个非常重要的研究领域。一些非常重要的研究方法已经被提出。

本文考虑如下的带乘性噪声的离子声波和朗缪尔波随机方程组[8] [9]

$$\begin{cases} u_{tt}+u_{xx}-2\left(|v|^2\right)_x=0, \\ iv_t+\frac{1}{2}v_{xx}-vu=i\sigma v\beta_t, \end{cases} \tag{1.1}$$

其中 $u=u(t,x)$ 表示归一化的密度扰动。$\beta(t)$ 是标准的维纳过程，$\beta_t=\frac{\mathrm{d}\beta}{\mathrm{d}t}$ 为实参数。σ 代表噪声强度。特别地，当 $\sigma=0$ 时，方程(1.1)简化为确定性方程。

2. 方程(1.1)的行波解

对方程(1.1)作行波变换

$$v(t,x)=V(\xi)\mathrm{e}^{i\theta+\sigma\beta(t)-\sigma^2 t},\quad u(t,x)=U(\xi),\quad \xi=wx+\rho t,\quad \theta=kx+\lambda t, \tag{2.1}$$

其中 w,ρ,k 和 λ 为非零实常数，满足 $k\neq\pm1$ 和 $\rho^2\neq w^2$。将方程(2.1)代入方程(1.1)，可得：

$$-\frac{1}{2}k^2V(\xi)+\frac{1}{2}w^2V''(\xi)-\lambda V(\xi)-V(\xi)U(\xi)=0, \tag{2.2}$$

$$(\rho-w)(\rho+w)U''(\xi)-4w^2\mathrm{e}^{2\sigma[\beta(t)-\sigma t]}\left[V(\xi)V''(\xi)+\left(V'(\xi)\right)^2\right]=0. \tag{2.3}$$

由于 $E\left(\mathrm{e}^{2\sigma\beta(t)}\right)=\mathrm{e}^{2\sigma^2 t}$，则对方程(2.3)两边同时取数学期望，可得

$$(\rho-w)(\rho+w)U''(\xi)-4w^2\left[V(\xi)V''(\xi)+\left(V'(\xi)\right)^2\right]=0. \tag{2.4}$$

求解方程(2.4)可得

$$U(\xi)=\frac{2w^2V^2(\xi)}{\rho^2-w^2}. \tag{2.5}$$

结合方程(2.5)，则方程(2.2)被简化为

$$V''-\frac{4}{\rho^2-w^2}V^3-\frac{k^2+2\lambda}{w^2}V=0. \tag{2.6}$$

对方程(2.6)两边同时乘以V'，并积分可得

$$\left(V'\right)^2=\frac{2}{\rho^2-w^2}V^4+\frac{k^2+2\lambda}{w^2}V^2+2C, \tag{2.7}$$

其中 C 为积分常数。

对方程(2.7)作变换$V=\pm\sqrt{\left(\frac{8}{\rho^2-w^2}\right)^{-\frac{1}{3}}}\varphi$ 和 $\xi_1=\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}\xi$，可得

$$\varphi_{\xi_1}^2=\varphi\left(\varphi^2+p\varphi+q\right), \tag{2.8}$$

其中 $p=\frac{\left(k^2+2\lambda\right)\left(\rho^2-w^2\right)^{\frac{2}{3}}}{w^2}$ 和 $q=4C\left(\rho^2-w^2\right)^{\frac{1}{3}}$。则方程(2.8)可用如下积分变换表示

$$\pm\left(\xi_1-\xi_0\right)=\int\frac{\mathrm{d}\varphi}{\sqrt{\varphi\left(\varphi^2+p\varphi+q\right)}}, \tag{2.9}$$

假设$\Delta=p^2-4q$，用$F(\varphi)$表示二阶多项式的判别式，其中$F(\varphi)=\varphi^2+p\varphi+q$。由积分(2.10)的解，可以得到方程(1.1)的所有行波解的分类。

情形 1：当$\Delta=0$时，对于$\varphi>0$。

1) 如果 $p<0$，则方程(1.1)的解为：

$$v_1(t,x)=\pm\left[-\frac{\left(k^2+2\lambda\right)\left(\rho^2-w^2\right)^{\frac{2}{3}}}{2w^2}\tanh^2\left(\frac{1}{2}\left(-\frac{\left(k^2+2\lambda\right)\left(\rho^2-w^2\right)^{\frac{2}{3}}}{w^2}\right)^{\frac{1}{2}}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right)\right)\right]^{\frac{1}{2}} \\ \times e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \tag{2.10}$$

$$v_2(t,x)=\pm\left[-\frac{\left(k^2+2\lambda\right)\left(\rho^2-w^2\right)^{\frac{2}{3}}}{2w^2}\coth^2\left(\frac{1}{2}\left(-\frac{\left(k^2+2\lambda\right)\left(\rho^2-w^2\right)^{\frac{2}{3}}}{w^2}\right)^{\frac{1}{2}}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right)\right)\right]^{\frac{1}{2}} \\ \times e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \tag{2.11}$$

2) 如果 $p>0$，则方程(1.1)的解为：

$$v_3(t,x)=\pm\left[\frac{\left(k^2+2\lambda\right)\left(\rho^2-w^2\right)^{\frac{2}{3}}}{2w^2}\tan^2\left(\frac{1}{2}\left(\frac{\left(k^2+2\lambda\right)\left(\rho^2-w^2\right)^{\frac{2}{3}}}{w^2}\right)^{\frac{1}{2}}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right)\right)\right]^{\frac{1}{2}} \\ \times e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \tag{2.12}$$

3) 如果 $p=0$，则方程(1.1)的解为：

$$v_4(t,x)=\pm\left[\left(\frac{8}{\rho^2-w^2}\right)^{-\frac{1}{3}}\frac{4}{\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right)^2}\right]^{\frac{1}{2}}\mathrm{e}^{i(wx+\rho t)+\sigma\beta(t)-\sigma^2 t}. \quad (2.13)$$

情形 2：当 $\Delta>0$， $q=0$ 时，对于 $\varphi>-p$。

1) 如果 $p<0$，则方程(1.1)的解为：

$$v_5(t,x)=\pm\left[-\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{2w^2}+\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{2w^2}\right.$$
$$\left.\times\tanh^2\left(\frac{1}{2}\left(\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{w^2}\right)^{\frac{1}{2}}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right)\right)\right]^{\frac{1}{2}}\mathrm{e}^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \quad (2.14)$$

$$v_6(t,x)=\pm\left[-\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{2w^2}+\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{2w^2}\right.$$
$$\left.\times\coth^2\left(\frac{1}{2}\left(\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{w^2}\right)^{\frac{1}{2}}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right)\right)\right]^{\frac{1}{2}}\mathrm{e}^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \quad (2.15)$$

2) 如果 $p>0$，则方程(1.1)的解为：

$$v_7(t,x)=\pm\left[-\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{2w^2}-\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{2w^2}\right.$$
$$\left.\times\coth^2\left(\frac{1}{2}\left(-\frac{(k^2+2\lambda)(\rho^2-w^2)^{\frac{2}{3}}}{w^2}\right)^{\frac{1}{2}}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right)\right)\right]^{\frac{1}{2}}\mathrm{e}^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \quad (2.16)$$

情形 3：当 $\Delta>0$，$q\neq 0$ 时。若存在 α_1,α_2 和 α_3 满足 $\alpha_1<\alpha_2<\alpha_3$ 且其中一个为零，另外两个是 $F(\varphi)=0$ 的根。

1) 当 $\alpha_1<\varphi<\alpha_2$ 时，方程(1.1)的解为：

$$v_8(t,x)=\pm\left\{\left(\frac{8}{\rho^2-w^2}\right)^{-\frac{1}{3}}\left[\alpha_1+(\alpha_2-\alpha_1)sn^2\left(\frac{\sqrt{\alpha_3-\alpha_1}}{2}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right),m\right)\right]\right\}^{\frac{1}{2}} \tag{2.17}$$

$$\times e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t},$$

其中 $m^2=\dfrac{\alpha_2-\alpha_1}{\alpha_3-\alpha_1}$。

2) 当 $\varphi>\alpha_3$ 时，方程(1.1)的解为：

$$v_9(t,x)=\pm\left[\left(\frac{8}{\rho^2-w^2}\right)^{-\frac{1}{3}}\frac{\alpha_3-\alpha_2 sn^2\left(\frac{\sqrt{\alpha_3-\alpha_1}}{2}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right),m\right)}{cn^2\left(\frac{\sqrt{\alpha_3-\alpha_1}}{2}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right),m\right)}\right]^{\frac{1}{2}} e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t}. \tag{2.18}$$

情形 4：当 $\Delta<0$ 时。若 $\varphi>0$ 时，也可得方程(1.1)的解：

$$v_{10}(t,x)=\pm\left(\frac{8}{\rho^2-w^2}\right)^{-\frac{1}{6}}\left[-(2C)^{\frac{1}{2}}+\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\frac{2(2C)^{\frac{1}{2}}}{1+cn\left((2C)^{\frac{1}{4}}\left(\left(\frac{8}{\rho^2-w^2}\right)^{\frac{1}{3}}(wx+\rho t)-\xi_0\right),m\right)}\right]^{\frac{1}{2}} \tag{2.19}$$

$$\times e^{i(kx+\lambda t)+\sigma\beta(t)-\sigma^2 t},$$

其中 $m^2=\dfrac{4w^2C^{\frac{1}{2}}-(k^2+2\lambda)(\rho^2-w^2)^{\frac{1}{2}}}{8w^2C^{\frac{1}{2}}}$。

注：通过以上的讨论，获得方程(1.1)的解 $v(t,x)$。通过关系式(2.1)和(2.5)，很容易获得方程(1.1)的解 $u(t,x)$。

3. 结论

本文研究了带乘性噪声的离子声波和朗缪尔波随机方程组的行波解，获得了该类方程的 Jacobi 椭圆函数解、双曲函数解和、三角函数解和有理函数解。与已存在的文献相比，本文所获得的解是有意的。特别是所获得的雅克比椭圆函数解更一般。而且，本文所获得的解可以进一步解释该类随机微分方程的波的传播情况，为科学研究提供更多的依据。

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