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带乘性噪声的Biswas-Arshed方程的新行波解

孟　飞

成都大学计算机学院，四川 成都

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摘　要

利用多项式的完全判别法，构建了带乘性噪声的Biswas-Arshed方程的行波解。基于所提出的方法，获得了许多新的精确解，这些解包括双曲函数解、三角函数解、有理函数解、隐式解和雅克比椭圆函数解。

关键词

Biswas-Arshed方程，行波解，乘性噪声，完全判别系统

New Traveling Wave Solution of Biswas-Arshed Equation with Multiplicative Noise

Fei Meng

College of Computer Science, Chengdu University, Chengdu Sichuan

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Abstract

In this paper, by using the complete discriminant method of polynomials, new traveling wave solution of Biswas-Arshed equation with multiplicative noise is constructed. Based on the proposed method, many new exact solutions are obtained, these solutions include hyperbolic function solutions, trigonometric function solutions, rational function solutions, implicit solution and Jacobi elliptic function solutions.

Keywords

Biswas-Arshed Equation, Traveling Wave Solution, Multiplicative Noise, Complete Discriminant System

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1. 引言

随机微分方程被广泛应用于物理学、控制学、生物学、经济学等领域，主要研究的问题集中在随机控制、鞅表示论、变分不等式和稳定性等内容[1] [2] [3]。特别地，寻找随机微分方程的行波解是备受关注的问题之一[4] [5] [6] [7]。近年来，已经有许多方法被提出来获得随机偏微分方程的行波解，如指数函数展开法[8]、Kudryashov 方法[9]、平面动力系统方法[10]和扩展的(G'/G)-展开法[11]等。

本文考虑如下的带乘性噪声的 Biswas-Arshed 方程[12]

$$
\begin{aligned}
&iq_t + a_1 q_{xx} + a_2 q_{xt} + i\left(b_1 q_{xxx} + b_2 q_{xxt}\right) + \sigma\left(q - ia_2 q_x + b_2 q_{xx}\right)\frac{\mathrm{d}W(t)}{\mathrm{d}t} \\
&= i\left[\lambda\left(|q|^2 q\right)_x + \mu\left(|q|^2\right)_x q + \theta |q|^2 q_x\right],
\end{aligned} \tag{1.1}
$$

其中 $q = q(t,x)$ 表示波剖面。a_1，b_1，b_2，σ，θ，μ 和 λ 为实参数。$W(t)$ 是标准的维纳过程，$\frac{\mathrm{d}W(t)}{\mathrm{d}t}$ 代表白噪声。特别地，当 $\sigma = 0$ 时，方程(1.1)简化为著名的 Biswas-Arshed 方程。Zayed 等人利用一种特殊的变换和三种不同方法获得了带乘性噪声的 Biswas-Arshed 方程孤子解。

2. 方程(1.1)的行波解

对方程(1.1)作行波变换

$$
q(t,x) = Q(\xi)\mathrm{e}^{i\left[\phi(t,x)+\sigma W(t)-\sigma^2 t\right]},\ \phi(t,x) = -kx + wt,\ \xi = x - vt, \tag{2.1}
$$

其中 k，w 和 v 为非零实常数。将方程(2.1)代入方程(1.1)，则分离方程(1.1)实部和虚部可得：

实部：
$$
\begin{aligned}
&\left[a_1 + 3b_1 k - b_2\left(2kv + w - \sigma^2\right) - a_2 v\right]Q'' - k(\lambda+\theta)Q^3 \\
&-\left[a_1 k^2 + b_1 k^3 + \left(1 - a_2 k - b_2 k^2\right)\left(w - \sigma^2\right)\right]Q = 0,
\end{aligned} \tag{2.2}
$$

虚部：
$$
\begin{aligned}
&(b_1 - b_2 v)Q''' - (3\lambda + 2\mu + \theta)Q^2 Q' - \left[2a_1 k + 3b_1 k^2 - (a_2 - 2kb_2)\left(w - \sigma^2\right)\right. \\
&\left.+\left(1 - a_2 k - b_2 k^2\right)v\right]Q' = 0.
\end{aligned} \tag{2.3}
$$

对方程(2.3)关于 ξ 积分一次，可得

$$
\begin{aligned}
&(b_1 - b_2 v)Q'' - \frac{1}{3}(3\lambda + 2\mu + \theta)Q^3 - \left[2a_1 k + 3b_1 k^2 - (a_2 - 2kb_2)\left(w - \sigma^2\right)\right. \\
&\left.+\left(1 - a_2 k - b_2 k^2\right)v\right]Q = 0.
\end{aligned} \tag{2.4}
$$

方程(2.2)和方程(2.4)中 Q''，Q^3 和 Q 的系数相等，于是可得

$$
\begin{aligned}
&\frac{a_1 + 3b_1 k - b_2\left(2kv + w - \sigma^2\right) - a_2 v}{b_1 - b_2 v} = \frac{3k(\lambda+\theta)}{3\lambda + 2\mu + \theta} \\
&= \frac{a_1 k^2 + b_1 k^3 + \left(1 - a_2 k - b_2 k^2\right)\left(w - \sigma^2\right)}{2a_1 k + 3b_1 k^2 - (a_2 - 2kb_2)\left(w - \sigma^2\right) + \left(1 - a_2 k - b_2 k^2\right)v}.
\end{aligned} \tag{2.5}
$$

由方程(2.5)可得

$$w=\frac{\left(a_2-2b_2^2k^3-2a_2b_2k^2\right)\sigma^2+2\left(a_1b_2+2a_2b_1\right)k^3+4b_1b_2k^4+2\left(a_1a_2-b_1\right)k^2-a_1k}{a_2-2b_2^2k^3-2a_2b_2k^2},$$
$$v=\frac{\left(a_1+2b_1k\right)\left(1-4b_2k^2\right)}{a_2-2b_2^2k^3-2a_2b_2k^2},\quad \theta-\mu=0. \tag{2.6}$$

结合方程(2.2)，方程(2.4)和方程(2.5)，则方程(2.2)被简化为

$$AQ''-BQ-CQ^3=0, \tag{2.7}$$

其中 $A=a_1+3b_1k-b_2\left(2kv+w-\sigma^2\right)-a_2v\neq 0$， $B=a_1k^2+b_1k^3+\left(1-a_2k-b_2k^2\right)\left(w-\sigma^2\right)$， $C=k\left(\lambda+\theta\right)$。

对方程(2.7)两边同时乘以 Q'，并积分可得

$$\left(Q'\right)^2=\frac{C}{2A}Q^4+\frac{B}{A}Q^2+\frac{2D}{A}, \tag{2.8}$$

其中 D 为积分常数。

对方程(2.8)作变换 $Q=\pm\sqrt{\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\psi}$ 和 $\xi_1=\left(\frac{2C}{A}\right)^{\frac{1}{3}}\xi$，可得

$$\psi_{\xi_1}^2=\psi\left(\psi^2+d_1\psi+d_0\right), \tag{2.9}$$

其中 $d_1=\frac{4B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}$ 和 $d_0=\frac{8D}{A}\left(\frac{2C}{A}\right)^{-\frac{1}{3}}$。则方程(2.9)可用如下积分变换表示

$$\pm\left(\xi_1-\xi_0\right)=\int\frac{\mathrm{d}\psi}{\sqrt{\psi\left(\psi^2+d_1\psi+d_0\right)}}. \tag{2.10}$$

假设 $\Delta=d_1^2-4d_0$，用 $F(\psi)$ 表示二阶多项式的判别式，其中 $F(\psi)=\psi^2+d_1\psi+d_0$。由积分(2.10)的解，可以得到方程(1.1)的所有行波解的分类。

情形 1：当 $\Delta=0$ 时，对于 $\psi>0$。

1) 如果 $d_1<0$，则方程(1.1)的解为：

$$q_1(t,x)=\pm\left[-\frac{B}{C}\tanh^2\left(\frac{1}{2}\left(-\frac{2B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}\right)^{\frac{1}{2}}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right)\right)\right]^{\frac{1}{2}}\mathrm{e}^{i\left[-kx+wt+\sigma W(t)-\sigma^2t\right]}. \tag{2.11}$$

$$q_2(t,x)=\pm\left[-\frac{B}{C}\coth^2\left(\frac{1}{2}\left(-\frac{2B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}\right)^{\frac{1}{2}}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right)\right)\right]^{\frac{1}{2}}\mathrm{e}^{i\left[-kx+wt+\sigma W(t)-\sigma^2t\right]}. \tag{2.12}$$

2) 如果 $d_1>0$，则方程(1.1)的解为：

$$q_3(t,x)=\pm\left[\frac{B}{C}\tan^2\left(\frac{1}{2}\left(\frac{2B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}\right)^{\frac{1}{2}}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right)\right)\right]^{\frac{1}{2}}\mathrm{e}^{i\left[-kx+wt+\sigma W(t)-\sigma^2t\right]}. \tag{2.13}$$

3) 如果 $d_1=0$，则方程(1.1)的解为：

$$q_4(t,x)=\pm\left[\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\frac{4}{\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right)^2}\right]^{\frac{1}{2}}e^{i\left[-kx+wt+\sigma W(t)-\sigma^2 t\right]}. \tag{2.14}$$

情形 2： 当 $\Delta>0$，$d_0=0$ 时，对于 $\psi>-d_1$。

1) 如果 $d_1<0$，则方程(1.1)的解为：

$$q_5(t,x)=\pm\left[-\frac{4B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}+\frac{B}{C}\tanh^2\left(\frac{1}{2}\left(\frac{2B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}\right)^{\frac{1}{2}}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right)\right)\right]^{\frac{1}{2}}e^{i\left[-kx+wt+\sigma W(t)-\sigma^2 t\right]}. \tag{2.15}$$

$$q_6(t,x)=\pm\left[-\frac{4B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}+\frac{B}{C}\coth^2\left(\frac{1}{2}\left(\frac{2B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}\right)^{\frac{1}{2}}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right)\right)\right]^{\frac{1}{2}}e^{i\left[-kx+wt+\sigma W(t)-\sigma^2 t\right]}. \tag{2.16}$$

2) 如果 $d_1>0$，则方程(1.1)的解为：

$$q_7(t,x)=\pm\left[-\frac{4B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}-\frac{B}{C}\tan^2\left(\frac{1}{2}\left(-\frac{2B}{A}\left(\frac{2C}{A}\right)^{-\frac{2}{3}}\right)^{\frac{1}{2}}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right)\right)\right]^{\frac{1}{2}}e^{i\left[-kx+wt+\sigma W(t)-\sigma^2 t\right]}. \tag{2.17}$$

情形 3： 当 $\Delta>0$，$d_0\neq 0$ 时。若存在 α_1，α_2 和 α_3 满足 $\alpha_1<\alpha_2<\alpha_3$ 且其中一个为零，另外两个是 $F(\psi)=0$ 的根。

1) 当 $\alpha_1<\psi<\alpha_2$ 时，方程(1.1)的解为：

$$q_8(t,x)=\pm\left\{\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\left[\alpha_1+(\alpha_2-\alpha_1)sn^2\left(\frac{\sqrt{\alpha_3-\alpha_1}}{2}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right),m\right)\right]\right\}^{\frac{1}{2}}e^{i\left[-kx+wt+\sigma W(t)-\sigma^2 t\right]}, \tag{2.18}$$

其中 $m^2=\dfrac{\alpha_2-\alpha_1}{\alpha_3-\alpha_1}$。

2) 当 $\psi>\alpha_3$ 时，方程(1.1)的解为：

$$q_9(t,x)=\pm\left[\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\frac{\alpha_3-\alpha_2 sn^2\left(\frac{\sqrt{\alpha_3-\alpha_1}}{2}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right),m\right)}{cn^2\left(\frac{\sqrt{\alpha_3-\alpha_1}}{2}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right),m\right)}\right]^{\frac{1}{2}}e^{i\left[-kx+wt+\sigma W(t)-\sigma^2 t\right]}. \tag{2.19}$$

情形 4： 当 $\Delta<0$ 时。若 $\psi>0$ 时，也可得方程(1.1)的解：

$$q_{10}(t,x)=\pm\left[-\left(\frac{4D}{C}\right)^{\frac{1}{2}}+\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\frac{2\left(\frac{8D}{A}\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\right)^{\frac{1}{2}}}{1+cn\left(\left(\frac{8D}{A}\left(\frac{2C}{A}\right)^{-\frac{1}{3}}\right)^{\frac{1}{4}}\left(\left(\frac{2C}{A}\right)^{\frac{1}{3}}(x-vt)-\xi_0\right),m\right)}\right]^{\frac{1}{2}}e^{i\left[-kx+wt+\sigma W(t)-\sigma^2 t\right]}, \quad (2.20)$$

其中 $m^2=\frac{2\sqrt{CD}-B}{4\sqrt{CD}}$。

3. 结论

本文多项式完全判别法，研究了带乘性噪声的 Biswas-Arshed 方程的行波解，获得了该类方程的 Jacobi 椭圆函数解、双曲函数解和、三角函数解和有理函数解。所获得的解是有意的。特别是雅克比椭圆函数解更一般。

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