

Supersonic Aeroelastic Response of Circular Conical Shells

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Abstract

A study on the nonlinear flutter of circular conical shells in axial supersonic airflow was presented. The piston theory was used to calculate aerodynamic force. Nonlinear aeroelastic equations of circular truncated conical shells were used, when the truncated conical shell angle is not zero and the smaller radius is infinitely close to zero. Under this specific boundary conditions, the flutter critical aerodynamic pressures and nonlinear response of circular conical shells were investigated with one dimensional DQM (differential quadrature method). The results show that, the influence of different boundary conditions on the critical dynamic pressure and the limit cycle amplitude of the circular conical shell is not significant. The wave number corresponding to the minimum critical flutter aerodynamic pressure is relatively small. When the dynamic pressure parameters are large, the numerical simulation results show that there is a semi stable high order limit cycle, which maximum point is closer to the bottom of the cone shell.

Keywords

Flutter, Circular Conical Shells, Supersonic, Critical Aerodynamic Pressure, Limit Cycle

一类完全锥壳的超声速气动弹性响应问题

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摘要

为研究超声速轴向流中完全锥壳的气动弹性非线性响应, 本文采用活塞理论计算超声速气动力, 利用截锥壳的非线性气动弹性运动方程, 在截锥壳顶角不为零, 小半径无限趋近于零的特定边界条件下, 采用一维微分求积法(DQM)离散, 数值逼近求解了完全锥壳的颤振临界动压及气动弹性响应。结果表明, 不同边界条件对完全锥壳气动颤振临界动压和极限环幅值的影响不大, 最小颤振临界对应周向波数相对较小。当动压参数较大时, 数值模拟结果显示, 顶端自由, 底端固支的锥壳还存在一种半稳定的高阶响应极限环, 该极限环对应的振幅最大点更靠近锥壳底端。

关键词

颤振, 完全锥壳, 超声速, 临界动压, 极限环

1. 引言

对锥壳/截锥壳气动弹性力学的研究始于上个世纪五六十年代, Shulman [1]首先研究了截锥壳的自由振动和颤振问题; Dzygadlo [2]研究了锥壳的超声速颤振和发散问题; Sewall [3]研究了一类具有弹簧支承的完全锥壳超声速颤振的临界动压, 并进行了实验。Grigolyuk [4]研究了三层锥壳的气动颤振问题。从1970年起, Dixon 等人[5]做了大量更为细致的截锥壳振动和气动弹性颤振的理论和试验研究。首先, 用 Galerkin 方法研究了带有广义弹性边界约束的正交各向异性截锥壳的自由振动、气动颤振及屈曲, 采用了二维无粘准定常气动力, 通过计算多种边界、几何形状及外载荷验证了该做法的精度和正确性, 研究了顶角、长径比、径厚比等参数对各向同性简支截锥壳颤振性质的影响。随后, 用同样的方法研究了带有环支的钝截锥的自由振动、颤振及屈曲, 并作了相应的风洞试验, 对两个结果进行了对比, 发现尽管在数值上两者存在差异, 但是在性质上基本一致。指出导致这种差异的根源在于激波产生的振荡[6]。文献[7]研究了内压作用下截锥壳的颤振临界及模态, 并进行了试验, 两者结果较为一致。文[8]研究了带有环支截锥壳的非对称颤振和发散, 指出不同环支刚度会产生不同的失稳形式, 并对气动设计提出了相关建议。

但是, 目前尚缺乏对完全锥壳大挠度情况下的非线性响应研究。本文从完全截锥壳的大挠度气动弹性运动方程出发, 将完全锥壳的气动弹性力学问题认为是完全截锥壳在顶角不为零, 顶端半径无限趋近于零的条件下的特殊边界问题。从而采用分析完全截锥壳非线性气动弹性力学问题的方法, 研究了完全锥壳的颤振临界动压及极限环响应问题。

2. 气动弹性系统运动方程

考虑大挠度位移-应变关系, 含有母线、周向、法向三个方向位移参量 u , v , w 的大挠度截锥壳气动弹性运动方程为[9]:

$$\begin{aligned}
 & K \left\{ \frac{\sin \alpha_0}{r} \frac{\partial u}{\partial x} - \frac{\sin^2 \alpha_0}{r^2} u + \frac{\nu - 3}{2r^2} \sin \alpha_0 \frac{\partial v}{\partial \theta} + \frac{1 + \nu}{2r} \frac{\partial^2 v}{\partial x \partial \theta} - \frac{\cos \alpha_0 \sin \alpha_0}{r^2} w \right. \\
 & - \frac{1 + \nu}{2r^3} \sin \alpha_0 \left(\frac{\partial w}{\partial \theta} \right)^2 + \frac{1 + \nu}{2r^2} \left(\frac{\partial w}{\partial \theta} \right) \left(\frac{\partial^2 w}{\partial \theta \partial x} \right) + \frac{1 - \nu}{2r} \sin \alpha_0 \left(\frac{\partial w}{\partial x} \right)^2 + \frac{\partial^2 u}{\partial x^2} \\
 & \left. + \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} + \frac{1 - \nu}{2r^2} \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial \theta^2} \right\} + \frac{K \nu \cos \alpha_0}{r} \frac{\partial w}{\partial x} + \frac{1 - \nu}{2r^2} K \frac{\partial^2 u}{\partial \theta^2} - \rho h \frac{\partial^2 u}{\partial t^2} = 0
 \end{aligned} \tag{1.1a}$$

$$\begin{aligned}
& \frac{3-\nu}{2r^2} \sin \alpha_0 K \frac{\partial u}{\partial \theta} + \frac{1+\nu}{2r} K \frac{\partial^2 u}{\partial x \partial \theta} + \frac{1-\nu}{2} K \frac{\partial^2 v}{\partial x^2} + \frac{K}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{\nu-1}{2r^2} \sin^2 \alpha_0 K v \\
& + \frac{1-\nu}{2r} \sin \alpha_0 K \frac{\partial v}{\partial x} + \left\{ \frac{\cos \alpha_0}{r^2} K + \frac{2D(\nu-1)}{r^4} \sin^2 \alpha_0 \cos \alpha_0 \right\} \frac{\partial w}{\partial \theta} \\
& + \frac{1-2\nu}{r^3} D \sin \alpha_0 \cos \alpha_0 \frac{\partial^2 w}{\partial x \partial \theta} - \frac{D \cos \alpha_0}{r^2} \frac{\partial^3 w}{\partial x^2 \partial \theta} - \frac{D \cos \alpha_0}{r^4} \frac{\partial^3 w}{\partial \theta^3}
\end{aligned} \tag{1.1b}$$

$$\begin{aligned}
& + \frac{1-\nu}{2r^2} \sin \alpha_0 K \frac{\partial w}{\partial x} \frac{\partial w}{\partial \theta} + \frac{1-\nu}{2r} K \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial \theta} + \frac{1+\nu}{2r} K \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x \partial \theta} \\
& + \frac{K}{r^3} \frac{\partial w}{\partial \theta} \frac{\partial^2 w}{\partial \theta^2} - \rho h \frac{\partial^2 v}{\partial t^2} = 0 \\
& - D \frac{\partial^4 w}{\partial x^4} + D \frac{\sin^2 \alpha_0}{r^2} \frac{\partial^2 w}{\partial x^2} + D \frac{\nu-3}{r} \sin \alpha_0 \frac{\partial^3 w}{\partial x^3} - D \frac{\sin^3 \alpha_0}{r^3} \frac{\partial w}{\partial x} \\
& - D \frac{8-4\nu}{r^4} \sin^2 \alpha_0 \frac{\partial^2 w}{\partial \theta^2} - D \left(\frac{4\nu}{r^3} \sin \alpha_0 - \frac{6}{r^3} \sin \alpha_0 \right) \frac{\partial^3 w}{\partial x \partial \theta^2} \\
& + D \frac{\nu-3}{r^2} \frac{\partial^2 w}{\partial x^2 \partial \theta^2} - \frac{D}{r^4} \frac{\partial^4 w}{\partial \theta^4} - K \left\{ \frac{\sin \alpha_0 \cos \alpha_0}{r^2} u + \frac{\cos \alpha_0}{r^2} \frac{\partial v}{\partial \theta} \right. \\
& \left. + \frac{\cos^2 \alpha_0}{r^2} w + \frac{\cos \alpha_0}{2r^3} \left(\frac{\partial w}{\partial \theta} \right)^2 + \frac{\nu \cos \alpha_0}{r} \frac{\partial u}{\partial x} + \frac{\nu \cos \alpha_0}{2r} \left(\frac{\partial w}{\partial x} \right)^2 \right\} + \Delta p - \rho h \frac{\partial^2 w}{\partial t^2} = 0
\end{aligned} \tag{1.1c}$$

式中, $K = Eh/(1-\nu^2)$ 为薄膜刚度, $D = Eh^3/12(1-\nu^2)$ 为弯曲刚度, E 为杨氏模量, ν 为泊松比, ρ 为材料密度, t 为时间变量, 其它参数同图 1。若定义 $s = r/\sin \alpha_0$, 表示该点到顶点的母线长度, 并定义 $s = s_1$ 表示小半径边界, $s = s_2$ 表示大半径边界, 则当 $s_1 = 0$ 时, 方程(1)退化为完全锥壳方程。超声速气流沿壳体母线方向(图中为 x 方向), 当地马赫数 $M_{a_i} \gg 1$ 时, 采用近似的一阶线性活塞理论来计算气动力。其表达式如下[5]

$$\Delta p = - \frac{2q_i}{M_{a_i}} \left(\frac{\partial w}{\partial x} + \frac{1}{U_i} \frac{\partial w}{\partial t} \right) \tag{1.2}$$

其中, $q_i = \rho_0 U_i^2 / 2$, Ma_i 为当地马赫数, ρ_0 为气体密度, U_i 为当地气流速度。

3. 基于驻波假设的 DQM 离散

基于驻波颤振假设, 将位移向量写为下列形式[9]:

$$\begin{aligned}
X^T &= \{u(x, \theta, t) \quad v(x, \theta, t) \quad w(x, \theta, t)\} \\
&= \{U(x, t) \cos(n\theta) \quad V(x, t) \sin(n\theta) \quad W(x, t) \cos(n\theta)\}
\end{aligned} \tag{2}$$

此处, n 为周向波数。根据微分求积法的离散化思想, 可以将 $U(x, t)$, $V(x, t)$, $W(x, t)$ 的各阶导数表示成下列近似关系。

$$\begin{aligned}
U^{(m)}(x_i, t) &= \sum_{j=1}^N C_{ij}^{(m)} U(x_j, t), \quad V^{(m)}(x_i, t) = \sum_{j=1}^N C_{ij}^{(m)} V(x_j, t), \\
W^{(m)}(x_i, t) &= \sum_{j=1}^N C_{ij}^{(m)} W(x_j, t), \quad i = 1, 2, \dots, N.
\end{aligned} \tag{3}$$

此处, N 为 x 方向的离散点数目。 C 为权系数矩阵, $m = 1, 2, 3$ 和 4 , 表示求导阶数。

基于线性模态缩减方法, 将式(2)中含有时间变量的分量写成下列模态叠加的形式:

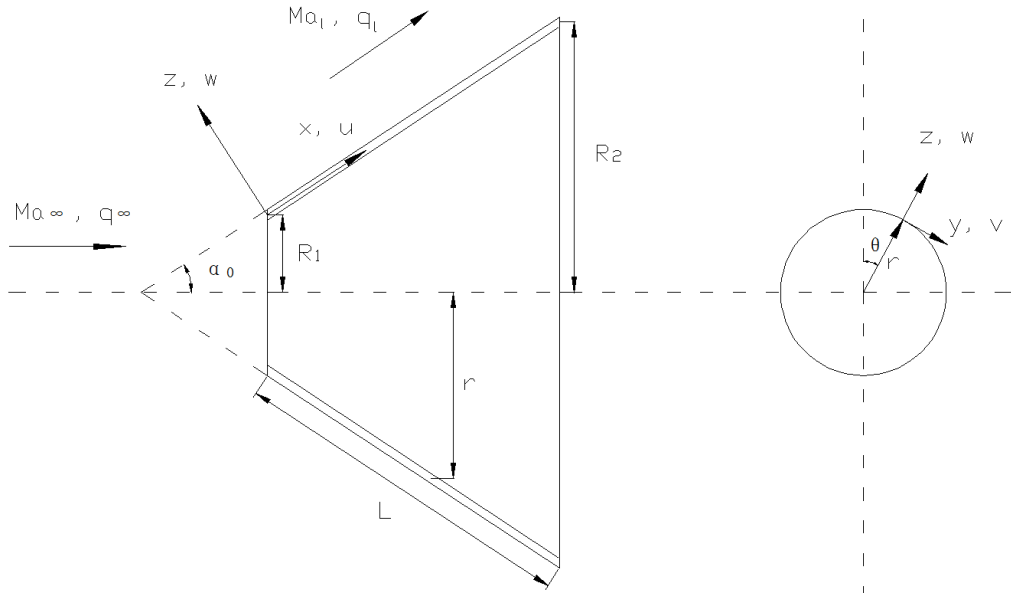


Figure 1. Truncate conical shell model
图 1. 截锥壳模型示意图

$$\begin{cases} U(x,t) \\ V(x,t) \\ W(x,t) \end{cases} = \sum_{i=1}^m \phi_i(x) q_i(t) \quad (4)$$

$\phi_i(x)$ 为基于线性系统特征值分析得到的特征向量, 在此作为非线性分析的基向量, q 是广义坐标, 可以建立如下形式的非线性运动方程

$$\{\phi\}^T [K] \{\phi\} \{q\} + \{\phi\}^T [G] \{\phi\} \{\dot{q}\} + \{\phi\}^T [M] \{\phi\} \{\ddot{q}\} + \{\phi\}^T \{N_q\} = 0 \quad (5)$$

其中 M, G, K, N_q 的形式参考文献[8]。对方程(5)在时域内直接积分, 求得响应。

4. 完全锥壳的线性颤振临界动压

完全锥壳就是当截锥壳顶角不为零, 顶端半径无限趋近于零时的特殊情况。取计算参数同文[7], 如下:

$$\begin{aligned} E &= 200 \text{ Gpa}, \rho = 7640 \text{ kg/m}^3, \mu = 0.28, L = 1.303 \text{ m}, \\ h &= 0.047 \text{ cm}, R_1 = 3.05 \text{ cm}, R_2 = 36.77 \text{ cm}, \alpha = 15^\circ, M_{a_1} = 3 \end{aligned} \quad (6)$$

图 2 给出了两端简支(文[10], p. 329, 第一类)时, 系统颤振临界动压随周向波数变化图, 系统最小颤振临界动压发生在 $n = 11$ 时, $\lambda_{cr}^{1/3} = 2.527$ 。文[7]采用 Galerkin 方法得到的结果为最小颤振临界动压发生在 $n = 12$ 时, $\lambda_{cr}^{1/3} = 2.530$ 。从图 1 可以看到, $n = 11$ 和 $n = 12$ 时的颤振临界动压非常接近, 本文 $n = 12$ 时的颤振临界动压 $\lambda_{cr}^{1/3} = 2.531$, 可见两种方法得到的结果较为一致。

由于完全锥壳方程在顶点的奇异性, 对其直接求解不容易得到正确结果。因此, 在参数(6)的基础上, 继续减小 R_1 , 逼近完全锥壳的真实情况。保持 R_2 不变, 逐渐增加壳体长度。将动压参数重新定义为

$$\lambda^* = \frac{2qR_2^3}{DM_{a_1}} \quad (7)$$

表 1 给出了两端第一类简支边界条件下, 完全锥壳线性颤振临界动压的近似解。引入顶端自由, 底端固支的边界条件:

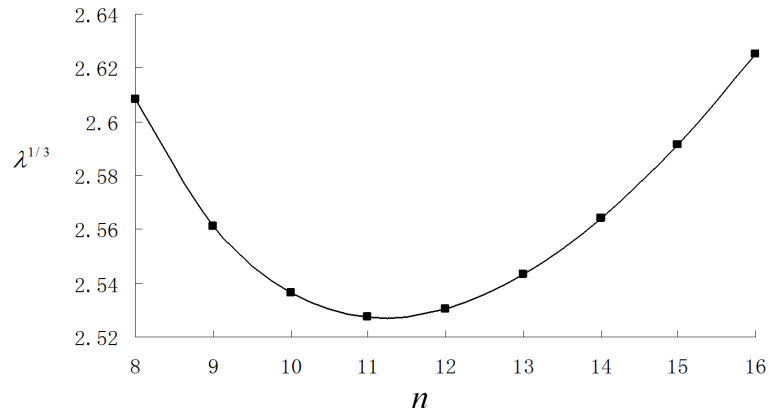


Figure 2. Flutter critical aerodynamic pressure $\lambda^{1/3} = \left(\frac{2qR_1^3}{DM_{a_1}} \right)^{1/3}$ vs n

图 2. 颤振临界动压参数 $\lambda^{1/3} = \left(\frac{2qR_1^3}{DM_{a_1}} \right)^{1/3}$ 随周向波数的变化图

Table 1. Flutter critical aerodynamic $\lambda^{1/3}$ with different radius ratios

表 1. 不同半径比时, 系统颤振临界动压 $\lambda^{1/3}$

$\frac{n}{R_2/R_1}$	8	9	10	11	12	13	14	15	16
12	31.447	30.881	30.582	30.475	30.513	30.667	30.914	31.246	31.651
80	31.446	30.879	30.579	30.471	30.509	30.661	30.907	31.238	31.641
∞	31.445	30.879	30.578	30.478	30.508	30.660	30.906	31.236	31.640

1) $x=0$ 时,

$$\begin{aligned} U(0)\sin\alpha_0 - W(0)\cos\alpha_0 = 0; \quad V(0) = 0; \\ \frac{\partial W(0)}{\partial x} = 0; \quad N_x(0)\cos\alpha_0 - Q_x(0)\sin\alpha_0 = 0; \end{aligned} \quad (8.a)$$

2) $x=L$ 时,

$$U(L) = V(L) = W(L) = \frac{\partial W(L)}{\partial x} = 0 \quad (8.b)$$

图 3 给出了两种不同边界条件下, 系统的颤振临界动压随周向波数变化情况。由图可见, 顶端自由、底端固定的完全锥壳颤振边界要低于两端简支时, 最小颤振临界动压对应的周向波数为 12。

5. 完全锥壳的驻波颤振极限环

计算参数同(6), 将 R_1 无限减小, 来逼近完全锥壳的真实情况。图 4 给出了周向波数 $n=11$ 时, 两端中面简支边界下, 振幅最大点的极限环响应幅值图。图 5 给出了周向波数 $n=12$ 时, 顶端自由, 底端固定边界条件下, 振幅最大点的极限环响应幅值图。由图可见, 两种边界条件下, 极限环幅值随动压参数的变化趋势基本一致, 数值上也较为接近。这说明不同边界条件对完全锥壳气动响应的影响不大。在图 5 中, 当动压参数 λ^* 较大时, 系统还存在一个半稳定极限环, 该极限环幅值略小, 表示更高阶的响应形式。

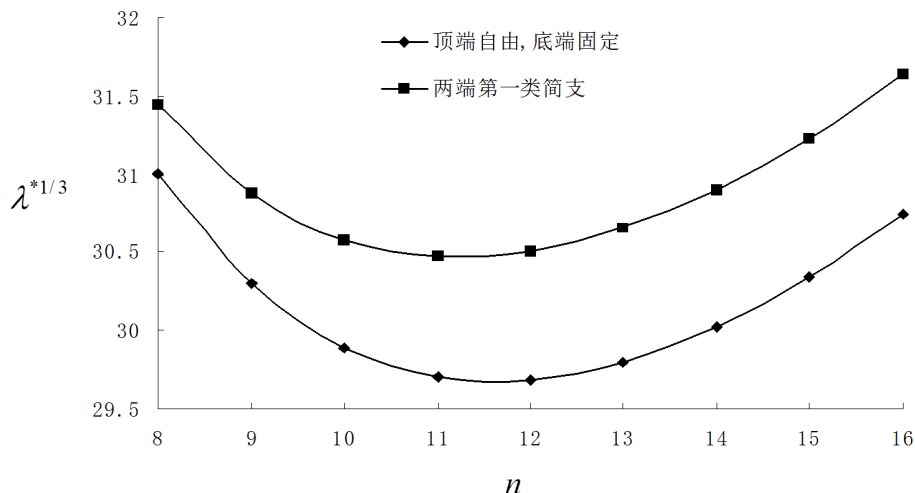


Figure 3. Flutter critical aerodynamic pressure $\lambda^* = \frac{2qR_2^3}{DM_{a_1}}$ vs n , under different boundary conditions

图 3. 不同边界条件, 颤振临界动压 $\lambda^* = \frac{2qR_2^3}{DM_{a_1}}$ 随周向波数变化图

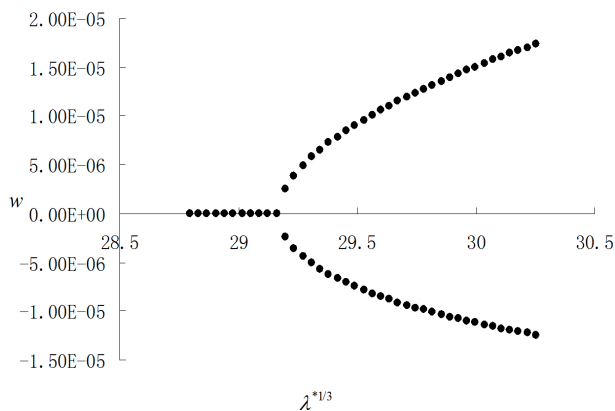


Figure 4. Both medium surface simple supported boundaries, limit cycle amplitude vs aerodynamic pressure (NMs = 4, $\theta = 15^\circ$, $x/L = 0.81$)

图 4. NMs = 4, $\theta = 15^\circ$, $x/L = 0.81$, 两端中面约束简支边界, 极限环幅值随动压变化图

6. 结论

本文给出了完全锥壳在超声速轴向流作用下的气动弹性颤振的临界动压及极限环响应的求解近似方法, 研究了完全锥壳的数值逼近问题, 得到了不同边界条件下的颤振临界动压, 并研究了极限环响应幅值随动压的变化情况。结论如下:

- 1) 顶端自由、底端固定的完全锥壳颤振边界要低于两端简支时, 最小颤振临界动压对应的周向波数较小。
- 2) 不同边界条件下, 完全锥壳颤振临界动压和极限环幅值差别不大。
- 3) 当动压参数较大时, 数值模拟结果显示, 顶端自由, 底端固支的锥壳还存在一种半稳定的高阶响

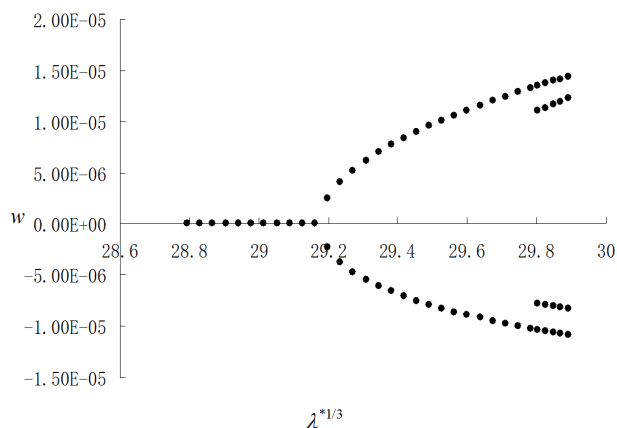


Figure 5. Top free, bottom fixed boundary, limit cycle amplitude vs aerodynamic pressure (NMs = 4, $\theta = 15^\circ$, $x/L = 0.81$)

图 5. NMs = 4, $\theta = 15^\circ$, $x/L = 0.81$, 顶端自由, 底端固定边界, 极限环幅值随动压变化图

应极限环, 该极限环对应的振幅最大点更靠近锥壳底端。

本文采用数值逼近的方法研究了完全锥壳超声速颤振临界动压和极限环响应的幅值问题, 研究结果可以为相关锥壳蒙皮设计和颤振抑制提供一定的理论依据和参考。

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