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Blowup of Solutions for a Class of Doubly Nonlinear Parabolic Equations

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Abstract

This paper is concerned with a class of doubly nonlinear parabolic systems. Under the homogeneous Dirichlet conditions and suitable conditions on the nonlinearity and certain initial datum, a sufficient condition for finite time blowup of its solution in a bounded domain is gave by using a modification of Levine's concavity method.

Keywords

Blowup of Solution, Doubly Nonlinear Parabolic Equations, Levine's Concavity Method

多重非线性抛物方程组解的爆破

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摘　要

本文研究了一类多重非线性抛物方程组解的爆破，利用修正的Levine凸性方法，对齐次Dirichlet边界和非线性项和初始条件的适当条件下，给出了解爆破时间的充分条件。

关键词

爆破，多重非线性抛物方程组，Levine凸性方法

1. 引言

本文研究如下非线性抛物方程组解的爆破性

$$\frac{\partial}{\partial t}\left(u+|u|^{m_1-2}u\right)-\Delta u=f_1(u,v) \tag{1.1}$$

$$\frac{\partial}{\partial t}\left(v+|v|^{m_2-2}v\right)-\Delta v=f_2(u,v) \tag{1.2}$$

$$u(x,t)=v(x,t)=0,\quad x\in\partial\Omega,\quad t\ge 0 \tag{1.3}$$

$$u(x,0)=u_0(x),\quad v(x,0)=v_0(x),\quad x\in\Omega \tag{1.4}$$

其中 Ω 是 $R^N\,(N\ge 1)$ 上的有界区域且有光滑边界 $\partial\Omega$，Δ 是 Ω 上的 Laplace 算子，$f_i(u,v)$，$i=1,2$ 为以后给定的函数。

型如(1.1)的单个多重非线性抛物方程

$$\frac{\partial}{\partial t}b(u)-\Delta u=f(u) \tag{1.5}$$

就是经典的所谓双非线性抛物方程,这类方程可以描述诸多化学反应、热传导过程和种群动力学过程(详细见文献[1])。方程(1.5)的初值问题或初边值问题已经有许多文献研究其局部和整体可解性[2]-[6]，文献[7]-[12]则研究了其整体吸引子的存在性和正则性。最近几十年，该类非线性抛物方程的爆破问题吸引了许多人的注意，基于 Levine [13] [14]凸性方法这一开创性的证明爆破的结果，Iami 和 Mochizuki [15]则给出了方程(1.5)带 Neumann 初边值问题解爆破的充分条件，该凸性方法还被 Levine [16] [17]用于如下渗流方程

$$v_t-\Delta\varphi(v)=f(v) \tag{1.6}$$

Sacks [18]研究了如下包含方程解的爆破问题

$$b(u)_t\ni\Delta u+q\cdot\nabla r(u)+f(u) \tag{1.7}$$

Zhang [19]和 Ding 和 Guo [20]-[22]通过构造适当的辅助函数，利用一阶微分不等式考虑了下面带梯度项和 Neumann (或 Robin)初边值问题解的爆破条件

$$b(u)_t=\nabla\left(a(u)\nabla u\right)+f\left(x,t,|\nabla u|^2,u\right) \tag{1.8}$$

Korpusou 和 Sveshnikov [23] [24]给出了如下方程初边值问题弱解爆破的充分条件

$$\left(u+\sum_{k=1}^{m}a_k(x)|u|^{p_k-2}u\right)_t-\mathrm{div}h\left(x,|\nabla u|\nabla u\right)+g(x,u)=f(x,u) \tag{1.9}$$

最后，还应提及 Ouardi 和 Hachimi [25] [26]研究了如下多重非线性抛物方程组

$$\beta_i(u_i)_t-\Delta u_i=f_i(x,t,u_1,u_2),\quad i=1,2$$

得到了其整体吸引子的存在性和正则性以及 Hausdorff 维数估计。

本文用修正的 Levine 凸性方法证明问题(1.1)~(1.4)的解在有限时刻爆破，该方法比原始的 Levine 凸

性方法更简洁，其基本技巧是 Korpousov [24]给出的一个微分不等式，本文把[24]的方法用于多重非线性抛物方程组。据作者所知，关于多重非线性抛物方程组的爆破问题的研究还比较少。本文的安排如下：第二节将给出一些假设和基本引理，第三节给出主要结果和证明。

2. 假设和基本引理

本文用 $H_0^1(\Omega)$ 和 $L^p(\Omega)$ 表示通常的 Soblev 空间,其范数分别记为 $\|.\|_{H_0^1}=\|\nabla.\|_2$ 和 $\|.\|_p$，特别是当 $p=2$ 时，记 $\|.\|_p=\|.\|$，这些符号的含义和记法同文献[2]。

本文始终假设 $m_1, m_2>2$。关于非线性项 $f_i(u,v)$，$i=1,2$ 的假设如下：

(A1) $f_i(u,v)\in C^1(R^2), i=1,2$，存在函数 $F(u,v)\in C^2(R^2)$ 使得

$$f_1(u,v)=\frac{\partial F(u,v)}{\partial u},\quad f_2(u,v)=\frac{\partial F(u,v)}{\partial v},$$

且存在常数 $\beta_0>2, \beta_1>0$ 使得

$$\beta_1\left(|u|^{p+1}+|v|^{p+1}\right)\le F(u,v)\le\frac{1}{\beta_0}\left(uf_1(u,v)+vf_2(u,v)\right),$$

其中， $p>0$ 当 $n=1,2$ 时， $0<p<\dfrac{N+2}{N-2}$ 当 $n>2$ 时。

注：满足条件(A1)的函数是存在的。事实上，一个典型的例子是取

$$F(u,v)=a|u+v|^{p+1}+2b|uv|^{\frac{p+1}{2}}$$

且 $f_1(u,v)=\dfrac{\partial F(u,v)}{\partial u}, f_2(u,v)=\dfrac{\partial F(u,v)}{\partial v}$，即

$$f_1(u,v)=(p+1)\left(a|u+v|^{p-1}(u+v)+b|u|^{\frac{p-3}{2}}u|v|^{\frac{p+1}{2}}\right),$$

$$f_2(u,v)=(p+1)\left(a|u+v|^{p-1}(u+v)+b|v|^{\frac{p-3}{2}}v|u|^{\frac{p+1}{2}}\right),$$

这时， $(p+1)F(u,v)=uf_1(u,v)+vf_2(u,v)$，其中 $a>0$， $b>0$， $p\ge1$， $\beta_0=p+1$。该例的详细情况可见文献[27]。

利用 Galerkin 方法，结合单调性理论和紧性方法[2]，类似文献[24]可得问题(1.1)~(1.4)解的局部存在性。

定理 2.1：假设条件(A1)成立， $u_0, v_0\in H_0^1$， $\max(m_1,m_2)\le p+1$，则问题(1.1)~(1.4)存在弱解 (u,v)，即，存在 $T>0$ 使得

$$u,\ v\in L^2(0,T;H_0^1),\quad u_t,\ v_t\in L^2(0,T;L^2(\Omega)),\quad \left(|u|^{\frac{m_1}{2}}\right)_t,\ \left(|v|^{\frac{m_2}{2}}\right)_t\in L^2((0,T)\times\Omega).$$

且对任意 $\phi(x)\in H_0^1, \varphi(t)\in D(0,T)$ 成立：

$$\int_0^T\left[\int_\Omega\left[\left(u_t+\frac{2(m_1-1)}{m_1}|u|^{m_1/2-2}u\left(|u|^{m_1/2}\right)_t\right)\phi+\nabla u\nabla\phi-f_1(u,v)\phi\right]\mathrm{d}x\varphi(t)\right]\mathrm{d}t=0,$$

$$\int_0^T\left[\int_\Omega\left[\left(v_t+\frac{2(m_1-1)}{m_1}|v|^{m_1/2-2}v\left(|v|^{m_1/2}\right)_t\right)\phi+\nabla v\nabla\phi-f_2(u,v)\phi\right]\mathrm{d}x\varphi(t)\right]\mathrm{d}t=0,$$

以及 $u(x,0)=u_0\in H_0^1,\ v(x,0)=v_0\in H_0^1$。

下面给出本文的基本引理。

引理 2.2 [13] [24] [28]：设 $H(t)$ 是 R 上非负二次连续可导函数且满足不等式

$$\Phi''(t)\Phi(t)-\alpha\left[\Phi'(t)\right]^2+\beta\Phi(t)\geq 0$$

其中 $\alpha>1,\ \beta>0$ 为常数。若 $\Phi(0)>0$，$\Phi'(0)>0,\ \left[\Phi'(0)\right]^2>\frac{2\beta}{2\alpha-1}\Phi(0)$，则必存在时刻 $T<T_1=A^{-1}\Phi^{1-\alpha}(0)$，使当 $t\to T^-$ 时有 $\Phi(t)\to+\infty$，其中

$$A^2=(\alpha-1)^2\Phi^{-2\alpha}(0)\left[\left(\Phi'(0)\right)^2-\frac{2\beta}{2\alpha-1}\Phi(0)\right]>0\text{。}$$

3. 主要结果及证明

首先引入泛函

$$\Phi(t)=\int_0^t\left[\frac{1}{2}\|u\|^2+\frac{1}{2}\|v\|^2+\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}\right]\mathrm{d}s+\frac{1}{2}\left(\|u_0\|^2+\|v_0\|^2\right)+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}\quad(3.1)$$

$$J(t)=\int_0^t\int_\Omega\left[u_t^2+v_t^2+(m_1-1)|u|^{m_1-2}u_t^2+(m_2-1)|v|^{m_2-2}v_t^2\right]\mathrm{d}s+\frac{1}{2}\|u_0\|^2+\frac{1}{2}\|v_0\|^2+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}\quad(3.2)$$

$$E(t)=\frac{1}{2}\|u\|^2+\frac{1}{2}\|v\|^2+\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}+\|\nabla u\|^2+\|\nabla v\|^2-\int_\Omega F(u,v)\mathrm{d}x\quad(3.3)$$

$$E(0)=\frac{1}{2}\|u_0\|^2+\frac{1}{2}\|v_0\|^2+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}+\|\nabla u_0\|^2+\|\nabla v_0\|^2-\int_\Omega F(u_0,v_0)\mathrm{d}x\quad(3.4)$$

现给出主要引理。

引理 3.1：对任意 $t\in[0,T)$，下面不等式成立

$$\left[\Phi'(t)\right]^2\leq(m_1+m_2)\Phi(t)J(t)\quad(3.5)$$

证明 注意到

$$\begin{aligned}\Phi'(t)&=\frac{1}{2}\left(\|u\|^2+\|v\|^2\right)+\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}\\&=\int_0^t\left[\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}s}\left(\|u\|^2+\|v\|^2\right)\mathrm{d}s+\int_0^t\frac{\mathrm{d}}{\mathrm{d}s}\left[\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}\right]\mathrm{d}s+\frac{1}{2}\left(\|u_0\|^2+\|v_0\|^2\right)\right.\\&\quad+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2},\end{aligned}\quad(3.6)$$

而由 Holder 不等式得

$$\left|\int_0^t\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}s}\|u\|^2\mathrm{d}s\right|\leq\int_0^t\|u\|\|u_t\|\mathrm{d}s\leq\left(\int_0^t\|u\|^2\mathrm{d}s\right)^{\frac{1}{2}}\left(\int_0^t\|u_t\|^2\mathrm{d}s\right)^{\frac{1}{2}}\quad(3.7)$$

$$\left|\int_0^t\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}s}\|v\|^2\mathrm{d}s\right|\leq\left(\int_0^t\|v\|^2\mathrm{d}s\right)^{\frac{1}{2}}\left(\int_0^t\|v_t\|^2\mathrm{d}s\right)^{\frac{1}{2}}\quad(3.8)$$

$$\left|\int_0^t\frac{m_1-1}{m_1}\frac{\mathrm{d}}{\mathrm{d}s}\|u\|_{m_1}^{m_1}\mathrm{d}s\right|\leq(m_1-1)\int_0^t\int_\Omega|u|^{\frac{m_1}{2}}\left(|u|^{\frac{m_1-2}{2}}|u_t|\right)\mathrm{d}s\leq(m_1-1)\left(\int_0^t\|u\|_{m_1}^{m_1}\mathrm{d}s\right)^{\frac{1}{2}}\left(\int_0^t\int_\Omega|u|^{m_1-2}|u_t|^2\mathrm{d}s\right)^{\frac{1}{2}}\quad(3.9)$$

$$\left|\int_0^t\frac{m_2-1}{m_2}\frac{\mathrm{d}}{\mathrm{d}s}\|v\|_{m_2}^{m_2}\mathrm{d}s\right|\leq(m_2-1)\left(\int_0^t\|v\|_{m_2}^{m_2}\mathrm{d}s\right)^{\frac{1}{2}}\left(\int_0^t\int_\Omega|v|^{m_2-2}|v_t|^2\mathrm{d}s\right)^{\frac{1}{2}}\quad(3.10)$$

考虑到(3.7)~(3.10)，则由(3.6)得

$$\Phi'(t)\le\left(\int_0^t\|u\|^2\mathrm{d}s\right)^{\frac12}\left(\int_0^t\|u_t\|^2\mathrm{d}s\right)^{\frac12}+\left(\int_0^t\|v\|^2\mathrm{d}s\right)^{\frac12}\left(\int_0^t\|v_t\|^2\mathrm{d}s\right)^{\frac12}$$
$$+(m_1-1)\left(\int_0^t\|u\|_{m_1}^{m_1}\mathrm{d}s\right)^{\frac12}\left(\int_0^t\int_\Omega|u|^{m_1-2}|u_t|^2\mathrm{d}s\right)^{\frac12}+(m_2-1)\left(\int_0^t\|v\|_{m_2}^{m_2}\mathrm{d}s\right)^{\frac12}\left(\int_0^t\int_\Omega|v|^{m_2-2}|v_t|^2\mathrm{d}s\right)^{\frac12}$$
$$+\frac12\left(\|u_0\|^2+\|v_0\|^2\right)+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2},$$

再利用不等式

$$\left(\sum_{i=1}^k a_ib_i\right)^2\le\left(\sum_{i=1}^k a_i^2\right)\left(\sum_{i=1}^k b_i^2\right),$$

得

$$\left(\Phi'(t)\right)^2\le(m_1+m_2)\left[\int_0^t\frac{1}{m_1+m_2}\left(\|u\|^2+\|v\|^2\right)\mathrm{d}s+\int_0^t\left(\frac{m_1-1}{m_1+m_2}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_1+m_2}\|v\|_{m_2}^{m_2}\right)\mathrm{d}s\right.$$
$$\left.+\frac{1}{2(m_1+m_2)}\left(\|u_0\|^2+\|v_0\|^2\right)+\frac{m_1-1}{m_1(m_1+m_2)}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2(m_1+m_2)}\|v_0\|_{m_2}^{m_2}\right]J$$
$$\le(m_1+m_2)\Phi(t)J(t),$$

于是，引理得证。

下面，给出主要定理。

定理 3.2：设定理 2.1 的条件成立，且

$$\left[4(m_1+m_2)-5\right]\left[\frac12\left(\|u_0\|^2+\|v_0\|^2\right)+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}\right]\ge4\left[\|\nabla u_0\|^2+\|\nabla v_0\|^2-\int_\Omega F(u_0,v_0)\mathrm{d}x\right]\quad(3.11)$$

则问题(1.1)~(1.4)的弱解 (u,v) 必在某有限时刻 $T<T_1=A^{-1}\Phi^{1-\alpha}(0)$ 爆破，即

$$\lim\sup_{t\to T}\left(\|\nabla u\|+\|\nabla v\|\right)=+\infty$$

证明：方程(1.1)，(1.2)两边分别同乘 u 和 v，然后关于 x 积分并相加，得

$$\Phi''(t)+\|\nabla u\|^2+\|\nabla v\|^2=\int_\Omega\left(f_1(u,v)u+f_2(u,v)v\right)\mathrm{d}x\quad(3.12)$$

方程(1.1)，(1.2)两边分别同乘 u_t 和 v_t，然后关于 x 积分并相加，得

$$J'(t)+\frac12\frac{\mathrm{d}}{\mathrm{d}t}\left(\|\nabla u\|^2+\|\nabla v\|^2\right)=\int_\Omega\left(f_1(u,v)u_t+f_2(u,v)v_t\right)\mathrm{d}x=\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega F(u,v)\mathrm{d}x\quad(3.13)$$

(3.13)关于 t 积分得

$$J(t)+\frac12\left(\|\nabla u\|^2+\|\nabla v\|^2\right)-E(0)=\int_\Omega F(u,v)\mathrm{d}x\quad(3.14)$$

再利用条件(A1)得

$$\beta_0\left[J(t)+\frac12\left(\|\nabla u\|^2+\|\nabla v\|^2\right)-E(0)\right]\le\int_\Omega\left(f_1(u,v)u+f_2(u,v)v\right)\mathrm{d}x\quad(3.15)$$

(3.12)结合(3.15)，并用到 $\beta_0>2$，得

$$\Phi''(t)+\|\nabla u\|^2+\|\nabla v\|^2\ge2\left[J(t)+\frac12\left(\|\nabla u\|^2+\|\nabla v\|^2\right)-E(0)\right]\ge2\left(J(t)+\|\nabla u\|^2+\|\nabla v\|^2-2E(0)\right)$$

即

$$\Phi''(t)-2J(t)+2E(0)\ge 0 \tag{3.16}$$

注意到$\Phi(t)\ge 0$得

$$\Phi''(t)\Phi(t)-2J(t)\Phi(t)+2E(0)\Phi(t)\ge 0 \tag{3.17}$$

利用引理3.1，得

$$\Phi''(t)\Phi(t)-\alpha\left[\Phi'(t)\right]^2+\beta\Phi(t)\ge 0 \tag{3.18}$$

其中$\alpha=2(m_1+m_2)$，$\beta=2E(0)$。

如果$E(0)>0$，由(3.1)，(3.6)和(3.11)知

$$\Phi(0)>0,\ \Phi'(0)>0,\ \left[\Phi'(0)\right]^2>\frac{2\beta}{2\alpha-1}\Phi(0)$$

于是，由引理2.2得结论。如果$E(0)\le 0$，取$\beta=0$，则(3.18)变为

$$\Phi''(t)\Phi(t)-\alpha\left[\Phi'(t)\right]^2\ge 0$$

于是，由标准的凸性引理得结论。

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