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非线性混合气体方程的可积性求解

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摘 要

文章主要对约化摄动法理论进行了深入研究，给出了(1+1) 玻色-费米混合超流气体的孤波模型，给出其非线性波方程的解析解，并讨论了孤子的相互作用行为。玻色-费米混合气体中的二维物质波脉冲，包括线性的和非线性的，以及在幺正性体系的限制条件中运用约化摄动法进行计算，得到一个耦合KdV方程，进一步研究和讨论其方程的可积性。

关键词

玻色-费米混合气体，孤子解，约化摄动法

On the Integrability of Nonlinear Mixed Gas Equations

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Abstract

In this paper, the theory of reduced perturbation method is well investigated. The

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solitary wave model of (1+1) Bose-Fermi mixed superfluid gas is given, the analytical solution of its nonlinear wave equation is given, and the interaction behavior of solitons is discussed. A coupled KdV equation is obtained by calculating the two-dimensional matter wave pulses in Bose-Fermi mixture gas, including linear and nonlinear, and by using the reduced perturbation method under the constraint conditions of unitary system. The integrability of the equation is further studied and discussed.

Keywords

Bose-Fermi Gas Mixture, Soliton Solution, Reductive Perturbation Technique

1. 引言

近几年，激光冷却技术的发展使得玻色-爱因斯坦凝聚 (BEC) 得以实现。此后人们在实验和理论上对它进行了很多有意义的研究 [1, 2]。利用玻色-爱因斯坦凝聚的Gross-Pitaevskii (GP)方程对 BEC 的研究 [3–5]。以及玻色-费米混合气体调查了 BEC 和弱关联的原子 BCS 凝聚，在对 BCS-BEC 的区域里做了大量的研究工作，并定义了一些参量。早在1975 年，Wahlquist 和 Estabrook 基于外微分形式系统及李代数表示理论建立了 1+1 维非线性演化方程的延拓结构理论，给出了一个系统求解非线性演化方程线性谱问题的有效方法。该理论主要是将要研究的(1+1)维可积非线性微分方程表达为一组外微分形式 2-形式，使得这些外微分形式构成闭理想，然后引进势或伪势和与之相联系的外微分 1-形式，并要求引入的外微分 1-形式与原来的外微分形式 2-形式构成新的闭理想，从而成功给出可积方程的 Lax 表示以及贝克隆变换。随后该理论被广泛应用于研究(1+1)可积非线性演化方程 [6, 7]。最近人们利用该理论对海森堡铁磁链方程 [8]，高阶非线性薛定谔方程 [9], 反应扩散方程 [10]进行了深入分析和研究。

近年来对于各种耦合 KdV 方程 [11]的研究得到人们的普遍关注，本文主要研究了玻色费米混合气体中二维物质波脉冲。包括线性和非线性在 BEC 和 BCS 及幺正性体系的限制条件中获得了一个孤立波方程，然后我们将运用约化摄动法进行计算得到一个耦合 KdV方程，进一步研究和讨论其方程的可积性。

2. 动力学方程

含有大量粒子的玻色-费米混合气体系统，在温度很低的时候，满足如下二维无量纲化动力学

方程方 [12]：

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\Psi_b = [-\frac{\hbar^2}{2m_b}\nabla^2 + U_b + \mu_b(n_b, a_b)]\Psi_b, \\ i\hbar\frac{\partial}{\partial t}\Psi_p = [-\frac{\hbar^2}{2m_p}\nabla^2 + U_p + \mu_p(n_p, a_p)]\Psi_p, \end{cases} \tag{1}$$

其中 $\mu_b(n_b, a_b) = c_{b_0} n_b^{r_{b_0}}(1 + c_{b_1} n_b^{r_{b_1}})$ 表示的是玻色子化学势，$\mu_p(n_p, a_p) = c_{p_0} n_p^{r_{p_0}}(1 + c_{p_1} n_p^{r_{p_1}})$ 是费米子化学势，$U_b = U_p = \frac{1}{2}m[\omega_\perp^2(x^2+y^2) + \omega_z^2 z^2]$表示外势，$\Psi_b(r,t) = \sqrt{n_b(r,t)}e^{i\phi_b(r,t)}$ 是玻色子的序参量，$\Psi_p(r,t) = \sqrt{n_p(r,t)}e^{i\phi_p(r,t)}$是费米子的序参量，$n_j(j=b,p)$表示的是离子密度。

将 $\Psi_b(r,t) = \sqrt{n_b(r,t)}e^{i\phi_b(r,t)}$ 代入式（1）第一个方程，$\Psi_p(r,t) = \sqrt{n_p(r,t)}e^{i\phi_p(r,t)}$ 代入式(2)第二个方程，化简整理得

$$\begin{cases} \frac{\partial n_b}{\partial t} + \frac{\hbar}{m_b}\nabla\cdot(n_b, \bigtriangledown\phi_b) = 0, \\ \frac{\partial \phi_b}{\partial t} + \frac{\hbar}{2m_b}[(\nabla\phi_b)^2 - \frac{1}{\sqrt{n_b}}\nabla^2\sqrt{n_b}] + \frac{1}{\hbar}[U_b + \mu_b(n_b, a_b)] = 0, \\ \frac{\partial n_p}{\partial t} + \frac{\hbar}{m_p}\nabla\cdot(n_p, \bigtriangledown\phi_p) = 0, \\ \frac{\partial \phi_p}{\partial t} + \frac{\hbar}{2m_p}[(\nabla\phi_p)^2 - \frac{1}{\sqrt{n_p}}\nabla^2\sqrt{n_p}] + \frac{1}{\hbar}[U_p + \mu_p(n_p, a_p)] = 0, \end{cases} \tag{2}$$

假设粒子被限制在具有盘状势 $U_b = U_p = \frac{1}{2}m[\omega_\perp^2(x^2+y^2) + \omega_z^2 z^2]$ 的阱中, $\omega_\perp$ 代表 x 轴或 y 轴的横向频率, ω_z 为 z 方向的势阱频率。为了方便变量无量纲化, 规定将方程组(2)做如下变换

$$\begin{cases} x, y, z = a_z(x', y', z') \\ t = \omega_\perp^{-1} t' \\ n_j = n_{j0} n', \quad (j = b, p) \\ a_\perp = [\frac{\hbar}{m\omega_\perp}]^{\frac{1}{2}} \\ n_{j0} = N/a_\perp^3 \\ \mu_j = \hbar\omega_\perp\mu_j', \quad (j = b, p) \end{cases} \tag{3}$$

将这些变量带入方程 (2) 中, 由于盘状势 $\omega_z/\omega_\perp$ 非常小, 因此 $(\omega_z/\omega_\perp)^2$ 可以被忽略。然后有:

$$U_b = U_p = \frac{1}{2}m[\omega_\perp^2(x^2+y^2) + \omega_z^2 z^2](\omega_\perp \ll \omega_z)$$

所以方程组(2)中的(1)式和(3)式变为：

$$\begin{cases} \frac{\partial' n_b'}{\partial' t'} + [\nabla'\cdot(n_b'\nabla'\phi_b')] = 0 \Longleftrightarrow \frac{\partial n_b}{\partial t} + [\nabla\cdot(n_b\nabla\phi_b)] = 0 \\ \frac{\partial' n_p'}{\partial' t'} + [\nabla'\cdot(n_p'\nabla'\phi_p')] = 0 \Longleftrightarrow \frac{\partial n_p}{\partial t} + [\nabla\cdot(n_p\nabla\theta_p)] = 0 \end{cases} \tag{4}$$

(注：$\nabla' = \frac{1}{a_z}\nabla$), 方程组(2)中的 (2) 式和 (4) 式等价于：

$$\begin{cases} \frac{\partial'\phi_b'}{\partial' t'} + \frac{1}{2}[(\nabla'\phi_b')^2 - \frac{1}{\sqrt{n_b'}}\nabla'^2\sqrt{n_b'}] + \frac{1}{2}[(x'^2+y'^2) + (\frac{\omega_z}{\omega_\perp})^2 z^2] + \frac{m}{\hbar^2}a_z^2\mu_b = 0 \\ \frac{\partial'\theta_p'}{\partial' t'} + \frac{1}{2}[(\nabla'\phi_p')^2 - \frac{1}{\sqrt{n_p'}}\nabla'^2\sqrt{n_p'}] + \frac{1}{2}[(x'^2+y'^2) + (\frac{\omega_z}{\omega_\perp})^2 z^2] + \frac{m}{\hbar^2}a_z^2\mu_p = 0 \end{cases} \tag{5}$$

其中

$$(a_z = [\frac{\hbar}{m\omega_z}]^{\frac{1}{2}} \Longrightarrow a_z^2 = [\frac{\hbar}{m\omega_z}] \Longrightarrow \frac{m}{\hbar^2}a_z^2\mu_j = \frac{1}{\hbar\omega_z}\mu_j = \mu_j') \tag{6}$$

(5) 式变为

$$\begin{cases} \frac{\partial\phi_b'}{\partial' t'} + \frac{1}{2}[(\nabla'\phi_b')^2 - \frac{1}{\sqrt{n_b'}}\nabla'^2\sqrt{n_b'}] + \frac{1}{2}[(x'^2+y'^2) + (\frac{\omega_z}{\omega_\perp})^2 z^2] + \mu_b' = 0 \\ \frac{\partial\theta_p'}{\partial' t'} + \frac{1}{2}[(\nabla'\phi_p')^2 - \frac{1}{\sqrt{n_p'}}\nabla'^2\sqrt{n_p'}] + \frac{1}{2}[(x'^2+y'^2) + (\frac{\omega_z}{\omega_\perp})^2 z^2] + \mu_p' = 0 \end{cases} \tag{7}$$

进一步得：

$$\begin{cases} \frac{\partial\theta_b}{\partial t} + \frac{1}{2}[(\nabla\phi_b)^2 - \frac{1}{\sqrt{n_b}}\nabla^2\sqrt{n_b}] + \frac{1}{2}[(x^2+y^2) + (\frac{\omega_z}{\omega_\perp})^2 z^2] + \mu_b = 0 \\ \frac{\partial\phi_p}{\partial t} + \frac{1}{2}[(\nabla\phi_p)^2 - \frac{1}{\sqrt{n_p}}\nabla^2\sqrt{n_p}] + \frac{1}{2}[\frac{\omega_\perp^2}{\omega_z^2}(x^2+y^2) + z^2] + \mu_p = 0 \end{cases} \tag{8}$$

所以方程组(2)转换为如下形式：

$$\begin{cases} \frac{\partial n_b}{\partial t} + [\nabla\cdot(n_b \bigtriangledown \phi_b)] = 0 \\ \frac{\partial\phi_b}{\partial t} + \frac{1}{2}[(\nabla\phi_b)^2 - \frac{1}{\sqrt{n_b}}\nabla^2\sqrt{n_b}] + \frac{1}{2}(x^2+y^2) + \frac{1}{2}(\frac{\omega_z}{\omega_\perp})^2 z^2 + \mu_b(n_b, a_b) = 0 \\ \frac{\partial n_p}{\partial t} + [\nabla\cdot(n_p \bigtriangledown \phi_p)] = 0 \\ \frac{\partial\phi_p}{\partial t} + \frac{1}{2}[(\nabla\phi_p)^2 - \frac{1}{\sqrt{n_p}}\nabla^2\sqrt{n_p}] + \frac{1}{2}(x^2+y^2) + \frac{1}{2}(\frac{\omega_z}{\omega_\perp})^2 z^2 + \mu_p(n_p, a_p) = 0 \end{cases} \tag{9}$$

令

$$\begin{cases} \sqrt{n_j} = A_j(x,y,z)G_j(z), \quad (j=b,p) \\ \phi_j = -\xi t + \varphi_j(x,y,z) \end{cases} \tag{10}$$

并假定：

$$-\frac{1}{2}\frac{d^2G_j}{dz^2} + \frac{1}{2}z^2G_j = \nu G_j \tag{11}$$

当 $\nu = \frac{1}{2}$ 时，$G_j = e^{-\frac{z^2}{2}}$ (本征值)

将(10)，(11)式代入方程组(9)中，即有

$$\begin{cases} \frac{\partial A_b}{\partial t} + \frac{\partial A_b}{\partial x}\frac{\partial\varphi_b}{\partial x} + \frac{\partial A_b}{\partial y}\frac{\partial\varphi_b}{\partial y} + \frac{1}{2}A_b(\frac{\partial^2\varphi_b}{\partial x^2} + \frac{\partial^2\varphi_b}{\partial y^2}) = 0 \\ -\frac{1}{2}(\frac{\partial^2 A_b}{\partial x^2} + \frac{\partial^2 A_b}{\partial y^2}) + (-\xi + \frac{1}{2})A_b + A_b\{\frac{\partial\varphi_b}{\partial t} + \frac{1}{2}[(\frac{\partial\varphi_b}{\partial x})^2 + (\frac{\partial\varphi_b}{\partial y})^2]\} + \mu_b A_b = 0 \\ \frac{\partial A_p}{\partial t} + \frac{\partial A_p}{\partial x}\frac{\partial\varphi_p}{\partial x} + \frac{\partial A_p}{\partial y}\frac{\partial\varphi_p}{\partial y} + \frac{1}{2}A_p(\frac{\partial^2\varphi_p}{\partial x^2} + \frac{\partial^2\varphi_p}{\partial y^2}) = 0 \\ -\frac{1}{2}(\frac{\partial^2 A_p}{\partial x^2} + \frac{\partial^2 A_p}{\partial y^2}) + (-\xi + \frac{1}{2})A_p + A_p\{\frac{\partial\varphi_p}{\partial t} + \frac{1}{2}[(\frac{\partial\varphi_p}{\partial x})^2 + (\frac{\partial\varphi_p}{\partial y})^2]\} + \mu_p A_p = 0 \end{cases} \tag{12}$$

(本式的推导中：$\omega_\perp \ll \omega_z$, 推出 $\frac{\omega_z^2}{\omega_\perp^2}(x^2+y^2) \to 0$) 根据已知条件：

$$\begin{cases} \mu_j(n_j, a_j) = c_{j0}n_j^{r_{j0}}(1 + c_{j1}n_j^{r_{j1}}) \\ n_j = A_j^2G_j^2, \quad (j=b,p) \end{cases} \tag{13}$$

将方程(13)代入方程组(12)中的第二个方程和第四个方程，得

$$\begin{cases} -\frac{1}{2}(\frac{\partial^2 A_b}{\partial x^2} + \frac{\partial^2 A_b}{\partial y^2}) + (-\xi + \frac{1}{2})A_b + A_b\{\frac{\partial\varphi_b}{\partial t} + \frac{1}{2}[(\frac{\partial\varphi_b}{\partial x})^2 + (\frac{\partial\varphi_b}{\partial y})^2]\} \\ +A_b c_{b0}n_b^{r_{b0}} + A_b c_{b0}n_b^{r_{b0}}c_{b1}n_b^{r_{b1}} = 0 \\ -\frac{1}{2}(\frac{\partial^2 A_p}{\partial x^2} + \frac{\partial^2 A_p}{\partial y^2}) + (-\xi + \frac{1}{2})A_p + A_p\{\frac{\partial\varphi_p}{\partial t} + \frac{1}{2}[(\frac{\partial\varphi_p}{\partial x})^2 + (\frac{\partial\varphi_p}{\partial y})^2]\} \\ +A_p c_{p0}n_p^{r_{p0}} + A_p c_{p0}n_p^{r_{p0}}c_{p1}n_p^{r_{p1}} = 0 \end{cases} \tag{14}$$

进一步得：

$$\begin{cases} -\frac{1}{2}(\frac{\partial^2 A_b}{\partial x^2}+\frac{\partial^2 A_b}{\partial y^2})+(-\xi+\frac{1}{2})A_b+A_b\{\frac{\partial \varphi_b}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_b}{\partial x})^2+(\frac{\partial \varphi_b}{\partial y})^2]\} \\ +c_{b0}A_b^{2r_{b0}+1}G_b^{2r_{b0}}+c_{b0}c_{b1}A_b^{2r_{b0}+2r_{b1}+1}G_b^{2r_{b0}+2r_{b1}}=0 \\ -\frac{1}{2}(\frac{\partial^2 A_p}{\partial x^2}+\frac{\partial^2 A_p}{\partial y^2})+(-\xi+\frac{1}{2})A_p+A_p\{\frac{\partial \varphi_p}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_p}{\partial x})^2+(\frac{\partial \varphi_p}{\partial y})^2]\} \\ +c_{p0}A_p^{2r_{p0}+1}G_p^{2r_{p0}}+c_{p0}c_{p1}A_p^{2r_{p0}+2r_{p1}+1}G_p^{2r_{p0}+2r_{p1}}=0 \end{cases} \tag{15}$$

又当 $\nu=\frac{1}{2}$ 时，$G_j=e^{-\frac{z^2}{2}}$ (本征值)，所以上式等价于

$$\begin{cases} -\frac{1}{2}(\frac{\partial^2 A_b}{\partial x^2}+\frac{\partial^2 A_b}{\partial y^2})+(-\xi+\frac{1}{2})A_b+A_b\{\frac{\partial \varphi_b}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_b}{\partial x})^2+(\frac{\partial \varphi_b}{\partial y})^2]\} \\ +c_{b0}A_b^{2r_{b0}+1}e^{-r_{b0}z^2}+c_{b0}c_{b1}A_b^{2r_{b0}+2r_{b1}+1}e^{-(r_{b0}+r_{b1})z^2}=0 \\ -\frac{1}{2}(\frac{\partial^2 A_p}{\partial x^2}+\frac{\partial^2 A_p}{\partial y^2})+(-\xi+\frac{1}{2})A_p+A_p\{\frac{\partial \varphi_p}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_p}{\partial x})^2+(\frac{\partial \varphi_p}{\partial y})^2]\} \\ +c_{p0}A_p^{2r_{p0}+1}e^{-r_{p0}z^2}+c_{p0}c_{p1}A_p^{2r_{p0}+2r_{p1}+1}e^{-(r_{p0}+r_{p1})z^2}=0 \end{cases} \tag{16}$$

对上式两边同乘以 e^{-z^2}, 并对 z 积分 $(\int_{-\infty}^{\infty}e^{-z^2}dz=\sqrt{\pi})$，有

$$\begin{cases} -\frac{1}{2}(\frac{\partial^2 A_b}{\partial x^2}+\frac{\partial^2 A_b}{\partial y^2})+(-\xi+\frac{1}{2})A_b+A_b\{\frac{\partial \varphi_b}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_b}{\partial x})^2+(\frac{\partial \varphi_b}{\partial y})^2]\} \\ +\frac{c_{b0}}{\sqrt{r_{b0}+1}}A_b^{2r_{b0}+1}+\frac{c_{b0}c_{b1}}{\sqrt{r_{b0}+r_{b1}+1}}A_b^{2r_{b0}+2r_{b1}+1}=0 \\ -\frac{1}{2}(\frac{\partial^2 A_p}{\partial x^2}+\frac{\partial^2 A_p}{\partial y^2})+(-\xi+\frac{1}{2})A_p+A_p\{\frac{\partial \varphi_p}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_p}{\partial x})^2+(\frac{\partial \varphi_p}{\partial y})^2]\} \\ +\frac{c_{p0}}{\sqrt{r_{p0}+1}}A_p^{2r_{p0}+1}+\frac{c_{p0}c_{p1}}{\sqrt{r_{p0}+r_{p1}+1}}A_p^{2r_{p0}+2r_{p1}+1}=0 \end{cases} \tag{17}$$

所以方程组（17）通过计算得;

$$\begin{cases} \frac{\partial A_b}{\partial t}+\frac{\partial A_b}{\partial x}\frac{\partial \varphi_b}{\partial x}+\frac{\partial A_b}{\partial y}\frac{\partial \varphi_b}{\partial y}+\frac{1}{2}A_b(\frac{\partial^2 \varphi_b}{\partial x^2}+\frac{\partial^2 \varphi_b}{\partial y^2})=0 \\ -\frac{1}{2}(\frac{\partial^2 A_b}{\partial x^2}+\frac{\partial^2 A_b}{\partial y^2})+(-\xi+\frac{1}{2})A_b+A_b\{\frac{\partial \varphi_b}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_b}{\partial x})^2+(\frac{\partial \varphi_b}{\partial y})^2]\} \\ +\frac{c_{b0}}{\sqrt{r_{b0}+1}}A_b^{2r_{b0}+1}+\frac{c_{b0}c_{b1}}{\sqrt{r_{b0}+r_{b1}+1}}A_b^{2r_{b0}+2r_{b1}+1}=0 \\ \frac{\partial A_p}{\partial t}+\frac{\partial A_p}{\partial x}\frac{\partial \varphi_p}{\partial x}+\frac{\partial A_p}{\partial y}\frac{\partial \varphi_p}{\partial y}+\frac{1}{2}A_p(\frac{\partial^2 \varphi_p}{\partial x^2}+\frac{\partial^2 \varphi_p}{\partial y^2})=0 \\ -\frac{1}{2}(\frac{\partial^2 A_p}{\partial x^2}+\frac{\partial^2 A_p}{\partial y^2})+(-\xi+\frac{1}{2})A_p+A_p\{\frac{\partial \varphi_p}{\partial t}+\frac{1}{2}[(\frac{\partial \varphi_p}{\partial x})^2+(\frac{\partial \varphi_p}{\partial y})^2]\} \\ +\frac{c_{p0}}{\sqrt{r_{p0}+1}}A_p^{2r_{p0}+1}+\frac{c_{p0}c_{p1}}{\sqrt{r_{p0}+r_{p1}+1}}A_p^{2r_{p0}+2r_{p1}+1}=0 \end{cases} \tag{18}$$

再令 $A_j=u_{j0}+\varepsilon^2 a_{j0}+\varepsilon^4 a_{j1}, \quad \varphi_j=\varepsilon\varphi_{j0}+\varepsilon^3\varphi_{j1},(j=b,p)$
$\xi=\varepsilon(z-c_jt)(j=b,p), \quad \tau=\varepsilon^3 t.$
把上式代入方程组（18) 中，通过计算整理得

$$\begin{cases} \frac{\partial a_{b_0}}{\partial \tau}-\frac{1}{8c_b}\frac{\partial^3 a_{b_0}}{\partial \xi^3}+\frac{3c_b}{2u_{b_0}}a_{b_0}\frac{\partial a_{b_0}}{\partial \xi}=0, \\ \frac{\partial a_{p_0}}{\partial \tau}-\frac{1}{8c_p}\frac{\partial^3 a_{p_0}}{\partial \xi^3}+\frac{3c_p}{2u_{p_0}}a_{p_0}\frac{\partial a_{p_0}}{\partial \xi}=0. \end{cases} \tag{19}$$

从而我们进一步得到耦合 KdV方程：

$$\begin{cases} \frac{\partial a_{b_0}}{\partial \tau}-\frac{1}{8c_b}\frac{\partial^3 a_{b_0}}{\partial \xi^3}+\frac{3c_b}{2u_{b_0}}a_{b_0}\frac{\partial a_{b_0}}{\partial \xi}=0, \\ \frac{\partial a_{p_0}}{\partial \tau}-\frac{1}{8c_p}\frac{\partial^3 a_{p_0}}{\partial \xi^3}+\frac{3c_p}{2u_{p_0}}a_{p_0}\frac{\partial a_{p_0}}{\partial \xi}=0. \end{cases} \tag{20}$$

3. 主要结论

本文利用非线性问题的约化摄动法基本理论，着重对非线性动力学方程进行分析，针对非线性问题的求解方法进行讨论并推广 [13–16]，获得新的耦合 KdV 方程，研究结果为实际应用给予理论依据。对于低维度非线性系统计算方法的讨论与研究，提出新的求解非线性系统精确解的方法，构建新孤立子方程。接下来构造一个简单有效的方法给出大气科学中特别是灾害性天气重要可积模型；孤子理论在台风的研究中已具有初步应用，通过研究非线性大气重力波模型的可积性质，并结合数值方法的解和解析方法的解以及现有经验为灾害性天气、气候的预报、预警提供可能的参考方案，是本论文的突出特色。

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