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The Stabilities for a Class of Nonlinear Differential Systems with Time-Delay

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Abstract

In this paper, we discuss the stability for a class of nonlinear differential systems by using integral inequalities.

Keywords

Integral Inequalities, Monotonous, Time-Delay Nonlinear Differential System, Stability

一类非线性时滞微分系统的稳定性

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摘　要

由一类积分不等式推导给出一类非线性时滞微分系统Lipschitz稳定性判断准则。

关键词

积分不等式，单调，非线性时滞系统，稳定

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1. 准备工作

定义 1 [1]：若时滞微分方程中未知函数的最高阶导数含有两个不同的变元值，称之为中立型时滞微分方程；

考虑时滞微分系统：$\frac{\mathrm{d}x}{\mathrm{d}t}=f\left(t,x(t),x(t-\tau),\dot{x}(t-\tau)\right)$ (1.1)

其中 $x\in R^n$，$y=col\left(x_1,x_2,\cdots,x_m\right)$，$z=col\left(x_{m+1},x_{m+2},\cdots,x_n\right)$，$x=col(y,z)$，$f(t,0,0,0)\equiv 0$，$\tau\geq 0$ 为常数，给定初始函数：

$$x(t)=\varphi(t),\quad \dot{x}(t)=\dot{\varphi}(t)\quad \forall t\in E_{t_0}=\left[t_0-\tau,t_0\right]\text{连续可微} \tag{1.2}$$

我们总假定(1.1)满足初始条件(1.2)的解存在唯一，并用 $x\left(t,t_0,\varphi\right)$ 表示(1.1)满足条件(1.2)的解。

定义 2 [2]：称(1.1)的平凡解关于部分变元 y 是 Lipschitz 稳定的，如果存在常数 $M\left(t_0\right)>0$ 和 $\delta\left(t_0\right)>0$，使当 $\|\varphi\|+\|\dot{\varphi}\|<\delta$ (对 $\forall t\in E_{t_0}$)时，有：$\left\|y\left(t;t_0,\varphi\right)\right\|+\left\|\dot{y}\left(t;t_0,\varphi\right)\right\|\leq M\left(t_0\right)\left(\|\varphi\|+\|\dot{\varphi}\|\right)$ 于 $t\geq t_0\geq 0$ 成立，简记为 LS；

定义 3 [3] [4]：称(1.1)的平凡解关于部分变元 y 是一致 Lipschitz 稳定的，若定义 2 中的 M 和 δ 均与 t_0 无关，简记为 ULS；

引理 1：设如下条件于 $t\geq 0$ 成立

(I)：常数 $c>0$；

(II)：

$$\begin{aligned}u(t)\leq\ & c+\int_0^t a(s)u(s)\mathrm{d}s+\sum_{i=1}^{I}\int_0^t b_i(s)u^{\alpha_i+1}(s)\mathrm{d}s\\&+\sum_{j=1}^{J}\int_0^t c_j(s)\int_0^s d_j(\sigma)u(\sigma)\mathrm{d}\sigma\mathrm{d}s\\&+\sum_{k=1}^{K}\int_0^t e_k(s)\int_0^s f_k(\sigma)u^{\beta_k+1}(\sigma)\mathrm{d}\sigma\mathrm{d}s\\&+\int_0^t\sum_{i=1}^{I}b_i(s)u^{\alpha_i}(s)\left[\sum_{l=1}^{L}\int_0^s g_l(\sigma)\int_0^\sigma h_l(\tau)u(\tau)\mathrm{d}\tau\mathrm{d}\sigma\right]\mathrm{d}s\\&+\int_0^t\sum_{i=1}^{I}b_i(s)u^{\alpha_i}(s)\left[\sum_{m=1}^{M}\int_0^s p_m(\sigma)\int_0^\sigma q_m(\tau)u^{\gamma_m+1}(\tau)\mathrm{d}\tau\mathrm{d}\sigma\right]\mathrm{d}s\end{aligned}$$

其中：$u(t)$，$a(t)$，$b_i(t)\left(i=1,2,\cdots,I\right)$，$c_j(t),d_j(t)\left(j=1,2,\cdots,J\right)$，

$e_k(t),f_k(t)\left(k=1,2,\cdots,K\right)$，$g_l(t),h_l(t)\left(l=1,2,\cdots,L\right)$，

$p_m(t),q_m(t)\left(m=1,2,\cdots,M\right)$ 均为 R_+ 上非负连续函数，

且：$\alpha_i\left(i=1,2,\cdots,I\right)$，$\beta_k\left(k=1,2,\cdots,K\right)$，$\gamma_m\left(m=1,2,\cdots,M\right)$ 均为大于 1 的常数，$1\leq\alpha_1\leq\alpha_2\leq\cdots\leq\alpha_I$，$1\leq\beta_1\leq\beta_2\leq\cdots\leq\beta_K$，$1\leq\gamma_1\leq\gamma_2\leq\cdots\leq\gamma_M$

令：$\bar{\alpha}=\max\left(\alpha_I,\beta_K,\gamma_M\right)$，$\underline{\alpha}=\min\left(\alpha_1,\beta_1,\gamma_1\right)$

(III) 设

$$A(t)=a(t)+\sum_{i=1}^{I}b_i(t)+\sum_{j=1}^{J}c_j(t)\int_0^t d_j(s)\mathrm{d}s+2\sum_{k=1}^{K}e_k(t)\int_0^t f_k(s)\mathrm{d}s$$
$$+\sum_{i=1}^{I}b_i(t)\int_0^t\sum_{l=1}^{L}g_l(s)\int_0^s h_l(\sigma)\mathrm{d}\sigma\mathrm{d}s+\sum_{i=1}^{I}b_i(t)\int_0^t\sum_{m=1}^{M}p_m(s)\int_0^s q_m(\sigma)\mathrm{d}\sigma\mathrm{d}s$$

$$B(t)=\sum_{i=1}^{I}b_i(t)+\sum_{k=1}^{K}e_k(t)\int_0^t f_k(s)\mathrm{d}s+\sum_{i=1}^{I}b_i(t)\int_0^t\sum_{l=1}^{L}g_l(s)\int_0^s h_l(\sigma)\mathrm{d}\sigma\mathrm{d}s$$
$$+2\sum_{i=1}^{I}b_i(t)\int_0^t\sum_{m=1}^{M}p_m(s)\int_0^s q_m(\sigma)\mathrm{d}\sigma\mathrm{d}s$$

$$C(t)=\int_0^t\sum_{m=1}^{M}p_m(s)\int_0^s q_m(\sigma)\mathrm{d}\sigma\mathrm{d}s$$

$$E_1(t)=1-\overline{\alpha}c^{\overline{\alpha}}\int_0^t\left[B(\tau)+C(\tau)\right]\exp\left(\overline{\alpha}\int_0^\tau A(\sigma)\mathrm{d}\sigma\right)\mathrm{d}\tau$$

$$F(t)=a(t)+\sum_{j=1}^{J}c_j(t)\int_0^t d_j(s)\mathrm{d}s$$

$$G(t)=\sum_{i=1}^{I}b_i(t)\text{Щ}^{\alpha_i-\underline{\alpha}}+\sum_{k=1}^{K}e_k(t)\int_0^t f_k(s)\text{Щ}^{\beta_k-\underline{\alpha}}\mathrm{d}s$$
$$+\sum_{i=1}^{I}b_i(t)N^{\alpha_i-\underline{\alpha}}\int_0^t\sum_{l=1}^{L}g_l(s)\int_0^s h_l(\sigma)\mathrm{d}\sigma$$

$$H(t)=\sum_{m=1}^{M}p_m(t)\int_0^t q_m(s)\text{Щ}^{\gamma_m-\underline{\alpha}}\mathrm{d}s$$

$$E_2(t)=1-\underline{\alpha}c^{\underline{\alpha}}\int_0^t\left[G(\tau)+H(\tau)\right]\exp\left(\underline{\alpha}\int_0^\tau F(\sigma)\mathrm{d}\sigma\right)\mathrm{d}\tau$$

且 $\int_0^{+\infty}A(s)\mathrm{d}s<+\infty$，

$$E_1(t)>0,\quad\left[1-(\overline{\alpha}-1)c^{\overline{\alpha}}\int_0^t B(s)E_1^{-\frac{1}{\overline{\alpha}}}(s)\exp\left(\overline{\alpha}\int_0^s A(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]>0$$

$$E_2(t)>0,\quad\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E_2^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]>0$$

则有：$u(t)\le c\exp\left(\int_0^t F(s)\mathrm{d}s\right)\cdot\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E_2^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]^{-\frac{1}{\underline{\alpha}-1}}$

注：该引理已在文献[5]中证明。

2. 一类非线性时滞微分系统的稳定性

考虑如下微分系统：

$$\frac{\mathrm{d}x}{\mathrm{d}t}=A(t)x(t)+f\left(t,x(t-\delta(t)),\int_0^t h\left\{s,x(s-\delta(s)),\int_0^s g\left[\sigma,x(\sigma-\delta(\sigma))\right]\mathrm{d}\sigma\right\}\mathrm{d}s\right)\tag{2.1}$$

其中：$A(t)=\left(a_{ij}(t)\right)_{n\times n}$，$f\in C\left[I\times R^n\times R^n\times R^n,R^n\right]$

$h\in C\left[I\times R^n\times R^n\times R^n,R^n\right]$，$g\in C\left[I\times R^n\times R^n,R^n\right]$

$0<\delta_0\le\delta(t)\le\delta$，$A(t)$ 有界，$f(t,0,0,0)=0$，$x(t)=\varphi(t)$，于 $-\delta\le t\le 0$

由引理 1，可得与系统(2.1)的解等价的积分系统的解：

$$x(t)=Y(t,0)\varphi(0)+\int_0^t Y(t,s)\,f\left(s,x(s-\delta(s)),\int_0^s h\left\{\tau,x(\tau-\delta(\tau)),\int_0^\tau g\left[\sigma,x(\sigma-\delta(\sigma))\right]\mathrm{d}\sigma\right\}\mathrm{d}\tau\right)\mathrm{d}s \tag{2.2}$$

主要结论：对于系统(2.1)而言，假设下列条件于 $t\ge 0$ 时成立：

1) $\|Y(t,s)\|\le\exp(r(t-s))$，$(t\ge s\ge 0)$，$r\le 0$

2)
$$\begin{aligned}&\left\|f\left(t,x(t-\delta(t)),\int_0^t h\left\{s,x(s-\delta(s)),\int_0^s g\left[\sigma,x(\sigma-\delta(\sigma))\right]\mathrm{d}\sigma\right\}\mathrm{d}s\right)\right\|\\
&\le a(t)\|x(t-\delta(t))\|+\sum_{i=1}^I b_i(t)\|x(t-\delta(t))\|^{\alpha_i+1}\\
&\quad+\sum_{j=1}^J c_j(t)\int_0^t\exp(r(t-s))d_j(s)\|x(s-\delta(s))\|\mathrm{d}s\\
&\quad+\sum_{k=1}^K e_k(t)\int_0^t\exp(r(t-s))f_k(s)\|x(s-\delta(s))\|^{\beta_k+1}\mathrm{d}s\\
&\quad+\sum_{i=1}^I b_i(t)\|x(t-\delta(t))\|^{\alpha_i}\int_0^t\exp(r(t-s))\sum_{l=1}^L g_l(s)\int_0^s h_l(\sigma)\|x(\sigma-\delta(\sigma))\|\mathrm{d}\sigma\mathrm{d}s\\
&\quad+\sum_{i=1}^I b_i(t)\|x(t-\delta(t))\|^{\alpha_i}\int_0^t\sum_{m=1}^M\exp(r(t-s))p_m(s)\int_0^s q_m(\sigma)\|x(\sigma-\delta(\sigma))\|^{\gamma_m+1}\mathrm{d}\sigma\mathrm{d}s\end{aligned}$$

3) 其中 $a(t)$，$b_i(t)(i=1,2,\cdots,I)$，$c_j(t),d_j(t)(j=1,2,\cdots,J)$
$e_k(t),f_k(t)(k=1,2,\cdots,K)$，$g_l(t),h_l(t)(l=1,2,\cdots,L)$，
$p_m(t),q_m(t)(m=1,2,\cdots,M)$ 如[6]中定理 3.1.1 所设，且单调不增。

4) $F(t)=M_2a(t)+M_2\sum_{j=1}^J c_j(t)\int_0^t d_j(t)\mathrm{d}t$ 且 $\int_0^{+\infty}F(s)\mathrm{d}s<+\infty$

$$\begin{aligned}G(t)=&\sum_{i=1}^I M_2b_i(t)\mathrm{e}^{\alpha_i rt}Ш^{\alpha_i-\underline{\alpha}}+\sum_{k=1}^K M_2e_k(t)\int_0^t f_k(s)\mathrm{e}^{\beta_k rs}Ш^{\beta_k-\underline{\alpha}}\mathrm{d}s\\
&+\sum_{i=1}^I M_2b_i(t)\mathrm{e}^{\alpha_i rt}N^{\alpha_i-\underline{\alpha}}\int_0^t\sum_{l=1}^L g_l(s)\int_0^s h_l(\sigma)\mathrm{d}\sigma\end{aligned}$$

$$H(t)=\sum_{m=1}^M M_2p_m(t)\int_0^t q_m(s)\mathrm{e}^{\gamma_m rs}Ш^{\gamma_m-\underline{\alpha}}\mathrm{d}s$$

$$E(t)=1-\underline{\alpha}c^{\underline{\alpha}}\int_0^t[G(\tau)+H(\tau)]\exp\left(\underline{\alpha}\int_0^\tau F(\sigma)\mathrm{d}\sigma\right)\mathrm{d}\tau$$

$E(t)>0$，且，$\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]>0$

$\int_0^\infty G(s)E^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s<\infty$ 其中 Ш 如[7] [8] [9]中引理所设

则系统(2.1)的零解在 C_1 中一致 Lipschitz 渐近稳定。

证明：由系统(2.1)的等价系统(2.2)：

$$x(t)=Y(t,0)\varphi(0)+\int_0^t Y(t,s)\,f\left(s,x(s-\delta(s)),\int_0^s h\left\{\tau,x(\tau-\delta(\tau)),\int_0^\tau g\left[\sigma,x(\sigma-\delta(\sigma))\right]\mathrm{d}\sigma\right\}\mathrm{d}\tau\right)\mathrm{d}s$$

由条件 1)，2)，得到：

$$
\begin{aligned}
\|x(t)\| \le & \|\varphi(0)\|\exp(rt)+\int_0^t \exp(r(t-s))a(s)\|x(s-\delta(s))\|\mathrm{d}s \\
& +\int_0^t \sum_{i=1}^{I} b_i(s)\exp(r(t-s))\|x(s-\delta(s))\|^{\alpha_i+1}\mathrm{d}s \\
& +\int_0^t \sum_{j=1}^{J} c_j(s)\exp(r(t-s))\int_0^s \exp(r(s-\theta))d_j(\theta)\|x(\theta-\delta(\theta))\|\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{k=1}^{K} e_k(s)\exp(r(t-s))\int_0^s \exp(r(s-\theta))f_k(\theta)\|x(\theta-\delta(\theta))\|^{\beta_k+1}\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{i=1}^{I} b_i(s)\|x(s-\delta(s))\|^{\alpha_i}\exp(r(t-s)) \\
& \times\int_0^s \exp(r(s-\theta))\sum_{l=1}^{L} g_l(\theta)\int_0^\theta h_l(\sigma)\|x(\sigma-\delta(\sigma))\|\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{i=1}^{I} b_i(s)\|x(s-\delta(s))\|^{\alpha_i}\exp(r(t-s)) \\
& \times\int_0^s \sum_{m=1}^{M} \exp(r(s-\theta))p_m(\theta)\int_0^\theta q_m(\sigma)\|x(\sigma-\delta(\sigma))\|^{\gamma_m+1}\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s
\end{aligned}
\tag{2.3}
$$

于是有：

$$
\begin{aligned}
\|x(t)\|\mathrm{e}^{-rt} \le & \|\varphi(0)\|\int_0^t a(s)\|x(s-\delta(s))\|\mathrm{e}^{-rs}\mathrm{d}s+\int_0^t \sum_{i=1}^{I} b_i(s)\|x(s-\delta(s))\|^{\alpha_i+1}\mathrm{e}^{-rs}\mathrm{d}s \\
& +\int_0^t \sum_{j=1}^{J} c_j(s)\mathrm{e}^{-rs}\int_0^s \mathrm{e}^{r(s-\theta)}d_j(\theta)\|x(\theta-\delta(\theta))\|\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{k=1}^{K} e_k(s)\mathrm{e}^{-rs}\int_0^s \mathrm{e}^{r(s-\theta)}f_k(\theta)\|x(\theta-\delta(\theta))\|^{\beta_k+1}\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{i=1}^{I} b_i(s)\|x(s-\delta(s))\|^{\alpha_i}\mathrm{e}^{-rs}\int_0^s \mathrm{e}^{r(s-\theta)}\sum_{l=1}^{L} g_l(\theta)\int_0^\theta h_l(\sigma)\|x(\sigma-\delta(\sigma))\|\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{i=1}^{I} b_i(s)\|x(s-\delta(s))\|^{\alpha_i}\mathrm{e}^{-rs}\int_0^s \mathrm{e}^{r(s-\theta)}\sum_{m=1}^{M} p_m(\theta)\int_0^\theta q_m(\sigma)\|x(\sigma-\delta(\sigma))\|^{\gamma_m+1}\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s
\end{aligned}
\tag{2.4}
$$

令 $u(t)=\|x(t)\|\mathrm{e}^{-rt}$，由 $\mathrm{e}^{-rt}=\mathrm{e}^{-r(t-\delta(t))}\cdot\mathrm{e}^{-r\delta(t)}$，注意到 $\delta(t)\le\delta$，故可设 $\sup_{0\le t<+\infty}\mathrm{e}^{-r\delta(t)}=M_1<+\infty$，由此可得：

$$
\begin{aligned}
u(t)\le & \|\varphi(0)\|+M_1\int_0^t a(s)u(s-\delta(s))\mathrm{d}s+\int_0^t \sum_{i=1}^{I} M_1^{\alpha_i+1}b_i(s)\mathrm{e}^{\alpha_i rs}u^{\alpha_i+1}(s-\delta(s))\mathrm{d}s \\
& +M_1\int_0^t \sum_{j=1}^{J} c_j(s)\int_0^s d_j(\theta)u(\theta-\delta(\theta))\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{k=1}^{K} M_1^{\beta_k+1}e_k(s)\int_0^s \mathrm{e}^{\beta_k r\theta}f_k(\theta)u^{\beta_k+1}(\theta-\delta(\theta))\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{i=1}^{I} M_1^{\alpha_i+1}b_i(s)u^{\alpha_i}(s-\delta(s))\mathrm{e}^{\alpha_i rs}\int_0^s \sum_{l=1}^{L} g_l(\theta)\int_0^\theta h_l(\sigma)u(\sigma-\delta(\sigma))\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s \\
& +\int_0^t \sum_{i=1}^{I} M_1^{\alpha_i+1}b_i(s)u^{\alpha_i}(s-\delta(s))\mathrm{e}^{\alpha_i rs}\int_0^s \sum_{m=1}^{M} p_m(\theta)\int_0^\theta M_1^{\gamma_m}\mathrm{e}^{\gamma_m r\sigma}q_m(\sigma)u^{\gamma_m+1}(\sigma-\delta(\sigma))\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s
\end{aligned}
\tag{2.5}
$$

令：$\varphi_1=\sup_{-\delta\le t\le 0}\|\varphi(t)\|$，$\varphi_2=\sup_{-\delta\le t\le 0}\|\varphi(t)\|^{\alpha_i+1}$

$\varphi_3 = \sup_{-\delta\le t\le 0}\|\varphi(t)\|^{\beta_k+1}$， $\varphi_4 = \sup_{-\delta\le t\le 0}\|\varphi(t)\|^{\gamma_m+1}$

$\varphi = \max\{\varphi_1,\varphi_2,\varphi_3,\varphi_4,\varphi_2\varphi_4\}$

令：$\|u(t)\| = \begin{cases}\max\{\varphi_1,\max(u(\xi))\}, & 0\le\xi\le t\\ \varphi_1 & -\delta\le t\le 0\end{cases}$

显然$\|u(t)\|$单调不减，且由$u(t)$的定义，有$u(t-\delta(t))\le\varphi_1$，因此：

$$\begin{aligned} u(t) \le{}& \varphi + M_2\int_0^t a(s)\|u(s)\|\mathrm{d}s + \int_0^t\sum_{i=1}^{I}M_2 b_i(s)\mathrm{e}^{\alpha_i rs}\|u(s)\|^{\alpha_i+1}\mathrm{d}s\\ &+M_2\int_0^t\sum_{j=1}^{J}c_j(s)\int_0^s d_j(\theta)\|u(\theta)\|\mathrm{d}\theta\mathrm{d}s\\ &+\int_0^t\sum_{k=1}^{K}M_2e_k(s)\int_0^s\mathrm{e}^{\beta_k r\theta}f_k(\theta)\|u(\theta)\|^{\beta_k+1}\mathrm{d}\theta\mathrm{d}s\\ &+\int_0^t\sum_{i=1}^{I}M_2b_i(s)\|u(s)\|^{\alpha_i}\mathrm{e}^{\alpha_i rs}\int_0^s\sum_{l=1}^{L}g_l(\theta)\int_0^\theta h_l(\sigma)\|u(\sigma)\|\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s\\ &+\int_0^t\sum_{i=1}^{I}M_2b_i(s)\|u(s)\|^{\alpha_i}\mathrm{e}^{\alpha_i rs}\int_0^s\sum_{m=1}^{M}p_m(\theta)\int_0^\theta\mathrm{e}^{\gamma_m r\sigma}q_m(\sigma)\|u(\sigma)\|^{\gamma_m+1}\mathrm{d}\sigma\mathrm{d}\theta\mathrm{d}s \end{aligned} \tag{2.6}$$

其中$M_2=\max\{M_1,M_1^{\alpha_i+1},M_1^{\beta_k+1},M_1^{\gamma_m+1},M_1^{\alpha_i+\gamma_m+1}\}$

注意到定理条件，由引理可得；

$$u(t)\le c\exp\left(\int_0^t F(s)\mathrm{d}s\right)\cdot\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]^{-\frac{1}{\underline{\alpha}-1}}$$

其中：$c=\varphi$，注意到$u(t)$的定义，可得：

$$\|x(t)\|\le\varphi\mathrm{e}^{rt}\exp\left(\int_0^t F(s)\mathrm{d}s\right)\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]^{-\frac{1}{\underline{\alpha}-1}}$$

注意到定理的条件，有如下事实：

$$r<0,\quad \int_0^{+\infty}F(s)\mathrm{d}s<+\infty,\quad E(t)>0,\quad \underline{\alpha}\ge 1$$

$\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]>0$，且有：

$\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]<1$，于是令：

$$\begin{aligned} M(t,0) &= \mathrm{e}^{rt}\exp\left(\int_0^t F(s)\mathrm{d}s\right)\left[1-(\underline{\alpha}-1)c^{\underline{\alpha}}\int_0^t G(s)E^{-\frac{1}{\underline{\alpha}}}(s)\exp\left(\underline{\alpha}\int_0^s F(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right]^{-\frac{1}{\underline{\alpha}-1}}\\ &\le \mathrm{e}^{rt}\exp\left(\int_0^t F(s)\mathrm{d}s\right) \end{aligned}$$

于是，对一切$t\ge0$时，有$\lim_{t\to\infty}M(t,t_0)=0$一致的成立。于是，可得结论成立。

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