Pure Mathematics 理论数学, 2022, 12(12), 2081-2105
Published Online December 2022 in Hans. http://www.hanspub.org/journal/pm
https://doi.org/10.12677/pm.2022.1212225

具有恐惧效应和 Beddington-DeAngelis 功能反应的时空共位群内捕食模型的稳定性和分叉分析

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收稿日期：2022年11月17日；录用日期：2022年12月15日；发布日期：2022年12月22日

摘 要

竞争和捕食是生态学中经常出现的一种现象，当两物种争夺相同的有限资源时，捕食者和食饵的共存是维持捕食者-食饵系统所必需的。研究共位群内捕食者和食饵在争夺同一资源时是否可能共存是非常有意义的。本文研究了具有恐惧效应的时空共位群内捕食模型的稳定性与 **Hopf** 分支，该模型包含了以 **Beddington-DeAngelis** 功能反应为特征的物种相互干扰，导出了共位群内捕食者和食饵共存的条件，这是通过讨论平衡点的存在性，局部和全局渐近稳定性以及一致持久性来实现，其中平衡点的稳定性的条件是通过李雅普诺夫方法和赫尔维兹判据获得的。最后，我们以恐惧因子为分支参数，得到了各平衡点处 **Hopf** 分支存在的条件。

关键词

共位群内捕食模型，**Beddington-DeAngelis** 型功能反应，恐惧效应，稳定性，**Hopf** 分支

Stability and Bifurcation Analysis of a Spatiotemporal Intraguild Predation Model with Fear Effect and Beddington-DeAngelis Functional Response

文章引用: 王小宁. 具有恐惧效应和Beddington-DeAngelis功能反应的时空共位群内捕食模型的稳定性和分叉分析[J]. 理论数学, 2022, 12(12): 2081-2105. DOI: 10.12677/pm.2022.1212225

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Received: Nov. 17^{th}, 2022; accepted: Dec. 15^{th}, 2022; published: Dec. 22^{nd}, 2022

Abstract

Competition and predation are a common phenomenon in ecology. When two species compete for the same limited resources, the coexistence of predator and prey is necessary to sustain the predator-prey system. It is of great significance to study whether predators and prey can coexist in a intraguild predation model when competing for the same resource. In this paper, we study the stability and Hopf bifurcation of a spatiotemporal intraguild predation model with a fear effect, the conditions for the coexistence of predator and prey in a intraguild predation model are derived by discussing the existence of equilibrium points, local and global asymptotic stability and uniform persistence, the condition of stability of equilibrium point is obtained by Lyapunov method and Helvetz criterion. Finally, we take the fear factor as the branching parameter and obtain the conditions for the existence of Hopf bifurcation at each equilibrium point.

Keywords

Intraguild Predation Model, Beddington-DeAngelis Functional Response, Fear Effect, Stability, Hopf Bifurcation

1. 引言

在自然环境中, 除竞争和捕食外, 共位群内捕食(Intraguild predation, 简称 IGP) 是另一种普遍存在于生态系统种群间相互作用的现象, 其影响不容忽视. 在 IGP 系统中, IG 捕食者以 IG 食饵

为食, 并与 IG 食饵相互竞争, 争夺共享资源, 从而 IG 捕食者, IG 食饵和共享资源构成一个简单的食物网. 对共位群内捕食系统的研究最早可以追溯到文 [1–3], 学者 Polis, McCormick, Myer和Holt首次提出了共位群内捕食系统的概念. 在 1997 年文 [4]中生物学家 Holt 和 Polis 建立了一个简单的共位群内捕食系统模型, 表明只有当 IG 食饵是共享资源的优势竞争者时, 所有种群才会共存,这说明在理论上共位群内捕食系统中的三种群共存是非常困难的, 而实际生物系统中与之正好相反, 这就产生了一个悖论. 因而, 现在许多学者从生物的各个角度对文 [4]中的模型进行修正和补充, 来解释理论和现实之间的矛盾. 因此, 共位群内捕食系统的研究具有重要的理论价值和现实意义.

在捕食-食饵系统中, 功能反应函数是极其重要的组成部分. 为了描述捕食者个体的消耗率随食饵密度梯度的变化, 已经提出了各种类型的功能反应函数. 其中包括经典的 Holling-I 型, II 型和 III 型功能反应 [5–7], Beddington-DeAngelis (BD) 型功能反应 [8,9]和其他一些类型 [10,11]. 而 Holling 型功能反应假设捕食者消耗率仅依赖食饵密度, 而 BD 型功能反应则假设捕食者消耗率也受到捕食者间相互干扰的影响, 即 BD 功能反应函数也依赖于捕食者密度. 研究表明, 非线性功能反应极大地影响着物种间相互作用的结果, 特别是食物网中包含三个甚至三个以上物种时 [12–17]. 对于 IGP 模型, Kang 和 Wedekin [18]假设了 Holling III 型功能反应, 这是基于 Schaber 等人的实验结果的显著拟合 [19]. 随着 Holling I 型和 Holling II 型功能反应的混合, 混沌在 IGP 模型中被证明是可能的 [20,21]. 此后, 具有 BD 功能反应函数的生态模型被大量研究 [22–26]. 然而, 由于复杂性, 对具有 BD 功能反应的 IGP 模型的研究相对有限 [27–32].

值得注意的是, 文 [27–30]中假定 IG 食饵对共享资源的捕食率与 IG 捕食者的密度无关。这可能不能反映 IG 捕食者对捕食率的影响. 由于三个物种(共享资源, IG食饵和 IG捕食者)共享同一环境, 当 IG 食饵和 IG 捕食者外出寻找食物时, 这三个物种可能会同时相遇, 因此这三个物种都会影响搜索效率 [6].一方面,我们都知道生态系统中影响捕食-食饵系统动力学的影响因素有很多,例如食饵庇护 [33,34], 气味干扰 [35], 狩猎合作 [36], 种群抵御 [37]等, 其中食饵对捕食者的恐惧就是其中之一 [38–40]. 因此, 基于以上考虑, 本文现将共享资源对 IG 食饵的恐惧以及各物种的随机扩散纳入到模型 [32]中进行进一步研究.文 [32]的模型为:

$$\begin{cases} \dfrac{\mathrm{d}R}{\mathrm{d}\tau} = r_1R(1-\dfrac{R}{K}) - \dfrac{c_1RN}{1+A_1R+A_2N+A_3P} - \dfrac{c_2RP}{1+A_1R+A_2N+A_3P}, \\ \dfrac{\mathrm{d}N}{\mathrm{d}\tau} = \dfrac{\varepsilon_1c_1RN}{1+A_1R+A_2N+A_3P} - \dfrac{c_2NP}{1+A_1R+A_2N+A_3P} - \eta_1N, \\ \dfrac{\mathrm{d}P}{\mathrm{d}\tau} = \dfrac{\varepsilon_2c_2RP}{1+A_1R+A_2N+A_3P} + \dfrac{\varepsilon_3c_3NP}{1+A_1R+A_2N+A_3P} - \eta_2P, \end{cases} \tag{1.1}$$

这里, $R(\tau)$, $N(\tau)$ 和 $P(\tau)$ 分别表示在 τ 时刻共享资源, IG 食饵和 IG 捕食者种群的密度. 参数 r_1, K 分别表示共享资源的内禀增长率和环境承载力. A_1, A_2, A_3 分别表示共享资源, IG 食饵和 IG 捕食者的干扰效应. c_1, c_2, c_3 分别表示 IG 食饵, IG 捕食者对共享资源的最大捕食率以及 IG 捕食者对 IG食饵的最大捕食率, ε_1, ε_2, ε_3 分别表示 IG 食饵, IG 捕食者对共享资源的转化率以及 IG 捕食者对 IG 食饵的转化率, η_1, η_2 分别代表 IG 食饵和 IG 捕食者的自然死亡率.

在该模型中, 我们假设共享资源由于害怕 IG 食饵而导致共享资源数量下降, 而 IG 食饵的生长仅受其 IG 捕食者的影响, 从而将恐惧因子 $h(f,N)=\frac{1}{1+fN}$乘到系统 (1.1) 中, 其中 f 表示共享资源

对 IG 食饵的恐惧程度, 得到修改后的模型如下:

$$\begin{cases} \dfrac{\mathrm{d}R}{\mathrm{d}\tau} = \dfrac{r_1 R}{1+fN}(1-\dfrac{R}{K}) - \dfrac{c_1 RN}{1+A_1R+A_2N+A_3P} - \dfrac{c_2 RP}{1+A_1R+A_2N+A_3P}, \\ \dfrac{\mathrm{d}N}{\mathrm{d}\tau} = \dfrac{\varepsilon_1 c_1 RN}{1+A_1R+A_2N+A_3P} - \dfrac{c_2 NP}{1+A_1R+A_2N+A_3P} - \eta_1 N, \\ \dfrac{\mathrm{d}P}{\mathrm{d}\tau} = \dfrac{\varepsilon_2 c_2 RP}{1+A_1R+A_2N+A_3P} + \dfrac{\varepsilon_3 c_3 NP}{1+A_1R+A_2N+A_3P} - \eta_2 P, \end{cases} \tag{1.2}$$

为方便起见, 引入以下无量纲变量和参数，以减少系统 (1.2)中参数的数量:

$$t = r_1\tau,\ u = \frac{R}{N},\ v = \frac{c_1}{r_1}N,\ w = \frac{c_2}{r_1}P,\ k = \frac{r_1 f}{c_1},\ e_1 = \frac{\varepsilon_1 c_1 K}{r_1},\ e_2 = \frac{\varepsilon_2 c_2 K}{r_1},$$

$$c = \frac{c_3}{c_2},\ \varepsilon = \frac{\varepsilon_3 c_2}{c_1},\ a = A_1 K,\ b_1 = \frac{A_2 r_1}{c_1},\ b_2 = \frac{A_3 r_1}{c_2},\ \delta_1 = \frac{\eta_1}{r_1},\ \delta_2 = \frac{\eta_2}{r_2},$$

可将系统 (1.2) 约化为

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = u(\dfrac{1-u}{1+kv} - \dfrac{v+w}{1+au+b_1v+b_2w}), \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = v(\dfrac{e_1u-cw}{1+au+b_1v+b_2w} - \delta_1), \\ \dfrac{\mathrm{d}w}{\mathrm{d}t} = w(\dfrac{e_2u+\varepsilon cv}{1+au+b_1v+b_2w} - \delta_2), \\ u(0)\geq 0,\ v(0)\geq 0,\ w(0))\geq 0, \end{cases} \tag{1.3}$$

此外, 在种群演化进程中, 环境等因素导致了空间分布的不均匀性.因此, 将扩散引入到具有 Beddington-DeAngelis 功能反应的 IGP 模型 (1.3)中. 而本文, 我们考虑流量等于零的 Neumann 边界条件下的自扩散模型:

$$\begin{cases} u_t - d_1\Delta u = u(\dfrac{1-u}{1+kv} - \dfrac{v+w}{1+au+b_1v+b_2w}), & x\in\Omega,\ t>0, \\ v_t - d_2\Delta v = v(\dfrac{e_1u-cw}{1+au+b_1v+b_2w} - \delta_1), & x\in\Omega,\ t>0, \\ w_t - d_3\Delta w = w(\dfrac{e_2u+\varepsilon cv}{1+au+b_1v+b_2w} - \delta_2), & x\in\Omega,\ t>0, \\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = \dfrac{\partial w}{\partial\nu} = 0, & x\in\partial\Omega,\ t>0, \\ u(x,0)=u_0(x)\geq 0,\ v(x,0)=v_0(x)\geq 0,\ w(x,0)=w_0(x)\geq 0, & x\in\Omega. \end{cases} \tag{1.4}$$

而 Ω 是 $\mathbb{R}^N$ $(N\geq 1)$ 中具有光滑边界 $\partial\Omega$ 的有界区域; ν是边界 $\partial\Omega$的单位外法向量; $d_1,\ d_2,\ d_3>0$ 表示物种的自扩散系数, 自扩散项反映了个体从高浓度向低浓度扩散, 初值 $u_0(x), v_0(x), w_0(x)$为 Ω 上的连续函数，假设模型中的所有参数均为正常数.

下面我们定义

$$\mathbf{F}(\mathbf{E})=(f_1(\mathbf{E}),f_2(\mathbf{E}),f_3(\mathbf{E}))^\top=\begin{pmatrix} u(\dfrac{1-u}{1+kv}-\dfrac{v+w}{1+au+b_1v+b_2w}) \\ v(\dfrac{e_1u-cw}{1+au+b_1v+b_2w}-\delta_1) \\ w(\dfrac{e_2u+\varepsilon cv}{1+au+b_1v+b_2w}-\delta_2) \end{pmatrix}.$$

本文的组织结构如下: 第 2 节讨论了时间系统解的存在性和有界性, 可行平衡点的局部和全局稳定性, 以及分析了系统 (1.3) 的 Hopf 分叉. 第 3 节致力于时空系统的分析: 讨论了系统 (1.4) 的解的持久性和平衡点的局部稳定性. 第 4 节给出了结论.

2. 系统 (1.3)的动力学分析

2.1. 正不变性和有界性

定理1 系统 (1.3) 的所有解在 $\mathbb{R}^3_+=\{(u,v,w):u(t)>0,v(t)>0,w(t)>0\}$ 内存在且保持正性.

证明 设 $(u(t),v(t),w(t))$ 为系统 (1.3) 的解, 很容易看出, f_1,f_2,f_3 是 $\mathbb{R}^3_+$ 上的连续函数且是局部 Lipschitizian连续的. 因此, 满足初值条件 $(u(0),v(0),w(0))\geq(0,0,0)$ 的系统 (1.3) 的所有解在 $\mathbb{R}^3_+=\{(u,v,w):u(t)>0,v(t)>0,w(t)>0\}$内存在且唯一, 进而表明了对 $\forall t>0$ 的所有解存在且保持正性.

定理2 在 $\mathbb{R}^3_+$ 内的系统 (1.3) 的所有解都是一致有界的.

证明 设 $(u(t),v(t),w(t))$ 为系统 (1.3)的任意一个解,因为 $\dfrac{\mathrm{d}u}{\mathrm{d}t}\leq u\frac{1-u}{1+kv}\leq u(1-u)$, 由比较原理可得, $\lim\limits_{t\to\infty}u(t)\leq 1$, 即对 $\forall\epsilon_1>0,\exists T_1$,当 $t>T_1$ 时,有 $u(t)\leq 1+\epsilon_1$.

从 (1.3)得第二个方程，我们有

$$\frac{\mathrm{d}v}{\mathrm{d}t}+\delta_1v=\frac{(e_1u-cw)v}{1+au+b_1v+b_2w}\leq\frac{e_1uv}{b_1v}\leq\frac{e_1(1+\epsilon_1)}{b_1}$$

根据 Gronwall's不等式 [41]，我们有

$$0<v(t)<\frac{e_1(1+\epsilon_1)(1-e^{-\delta_1t})}{b_1\delta_1}+v(0)e^{-\delta_1t}$$

. 由比较原理可得, $\lim\limits_{t\to\infty}v(t)\leq\frac{e_1(1+\epsilon_1)}{\delta_1b_1}$, 即对 $\forall\epsilon_2>0,\exists T_2$,当 $t>\max\{T_1,T_2\}$时,有 $v(t)\leq\frac{e_1(1+\epsilon_1)}{\delta_1b_1}+\epsilon_2$.

从 (1.3)的第三个方程, 我们有

$$\frac{\mathrm{d}w}{\mathrm{d}t}+\delta_2w=\frac{(e_2u+\varepsilon cv)w}{1+au+b_1v+b_2w}\leq\frac{(e_2u+\varepsilon cv)w}{b_2w}\leq\frac{e_2(1+\epsilon_1)+c\varepsilon[\frac{e_1(1+\epsilon_1)}{b_1\delta_1}+\epsilon_2]}{b_2}$$

根据 Gronwall's不等式 [41]，我们有

$$0 < w(t) < \frac{e_2(1+\epsilon_1) + c\varepsilon[\frac{e_1(1+\epsilon_1)}{b_1\delta_1} + \epsilon_2]}{b_2\delta_2}(1 - e^{-\delta_2 t}) + w(0)e^{-\delta_2 t}$$

由比较原理可得, $\lim\limits_{t\to\infty} w(t) \le \frac{e_2(1+\epsilon_1)+c\varepsilon[\frac{e_1(1+\epsilon_1)}{b_1\delta_1}+\epsilon_2]}{b_2\delta_2}$, 即对 $\forall\epsilon_3 > 0, \exists T_3$,当 $t > \max\{T_1, T_2, T_3\}$时,有 $w(t) \le \frac{e_2(1+\epsilon_1)+c\varepsilon[\frac{e_1(1+\epsilon_1)}{b_1\delta_1}+\epsilon_2]}{b_2\delta_2} + \epsilon_3 \le \frac{[(1+\epsilon_1)(e_2+\frac{\varepsilon c e_1}{b_1\delta_1})+\varepsilon c\epsilon_2]}{b_2\delta_2} + \epsilon_3$.

故系统 (1.3)的所有解一致有界并最终趋于以下区域:

$$\Theta = \left\{(u,v,w) \in \mathbb{R}^3_+ : u(t) \le 1+\epsilon_1, v(t) \le \frac{e_1(1+\epsilon_1)}{b_1}, w(t) \le \frac{(1+\epsilon_1)(e_2+\frac{e_1 c\varepsilon}{b_1\delta_1}) + c\varepsilon\epsilon_2}{b_2\delta_2} + \epsilon_3, \forall\epsilon_1,\epsilon_2,\epsilon_3 > 0\right\}$$

2.2. 平衡点的存在性

系统 (1.3)有以下具有生物意义的可行平衡点.

(i) 总存在平凡平衡点 $\mathbf{E}_0 = (0,0,0)$ 和半平凡平衡点 $\mathbf{E}_1 = (1,0,0)$;

(ii) 其他边界平衡点以 $\mathbf{E}_2 = (u_2, v_2, 0), u_2, v_2 > 0$, $\mathbf{E}_3 = (u_3, 0, w_3), u_3, w_3 > 0$ 的形式给出.令 $\Re^i = \frac{e_i}{\delta_i(1+a)}$, $i = 1,2$,那么 $\Re^1$ 是在无 IG 捕食者（物种w）的情况下, IG 食饵(物种v)的基本再生数, $\Re^2$ 是在无 IG 食饵的情况下, IG 捕食者的基本再生数.

当 IG 捕食者不存在时, 即 $w = 0$, 系统 (1.3)退化为以下捕食者- 食饵系统:

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = u(\dfrac{1-u}{1+kv} - \dfrac{v}{1+au+b_1v}), \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = v(\dfrac{e_1 u}{1+au+b_1v} - \delta_1), \end{cases}$$

若 $\Re^1 > 1$, 则系统 (1.3）有唯一 $\mathbf{E}_2 = (u_2, v_2, 0)$ 形式的边界平衡点, 而 v_2 满足以下方程:

$$A_1 v_2^2 + B_1 v_2 - C_1 = 0$$

其中

$$A_1 = e_1 b_1^2 \delta_1^2 + k\delta_1(e_1 - a\delta_1)^2 > 0,$$

$$B_1 = \delta_1(e_1 - a\delta_1)^2 + 2e_1 b_1 \delta_1^2 - e_1 b_1 \delta_1(e_1 - a\delta_1),$$

$$C_1 = e_1\delta_1(e_1 - \delta_1 - a\delta_1) > 0,$$

则 $v_2 = \frac{-B_1+\sqrt{B_1^2+4A_1C_1}}{2A_1} > 0, u_2 = \frac{\delta_1(1+b_1v_2)}{e_1-a\delta_1}$ $(0 < u_2 < 1)$.

类似地, 当 IG 食饵不存在时, 即 $v=0$, 若 $\Re^2>1$,则 $u_3=\frac{(a\delta_2+b_2e_2-e_2)+\sqrt{(a\delta_2+b_2e_2-e_2)^2+4b_2e_2\delta_2}}{2b_2e_2}$, $w_3=\frac{e_2u_3(1-u_3)}{\delta_2}(0<u_3<1)$.

(iii) 若 $\mathbf{E}^*=(u^*,v^*,w^*)$为系统 (1.3) 的正常数平衡点, 则有以下方程:

$$u(\frac{1-u}{1+kv}-\frac{v+w}{1+au+b_1v+b_2w})=0 \tag{1.3a}$$

$$v(\frac{e_1u-cw}{1+au+b_1v+b_2w}-\delta_1)=0 \tag{1.3b}$$

$$w(\frac{e_2u+\varepsilon cv}{1+au+b_1v+b_2w}-\delta_2)=0 \tag{1.3c}$$

由 (1.3b)和 (1.3c)得, $w=\frac{[(e_1\delta_2-e_2\delta_1)u-c\varepsilon\delta_1v]}{c\delta_2}$, 将其代入 (1.3b), (1.3c)中得到

$$v=\frac{[(e_2-a\delta_2)c+b_2(e_2\delta_1-e_1\delta_2)]u-c\delta_2}{c[b_1\delta_2-\varepsilon(c+b_2\delta_1)]}$$

$$w=\frac{\varepsilon c\delta_1-[(e_1-a\delta_1)c\varepsilon+b_1(e_2\delta_1-e_1\delta_2)]u}{c[b_1\delta_2-\varepsilon(c+b_2\delta_1)]}$$

定义

$$\Lambda_v:=e_2-a\delta_2+\frac{b_2(e_2\delta_1-e_1\delta_2)}{c}$$

,

$$\Lambda_w:=e_1-a\delta_1+\frac{b_1(e_2\delta_1-e_1\delta_2)}{c\varepsilon}$$

. 当 $u\in S_v$时, $v>0$, 而

$$S_v=\begin{cases}(\frac{\delta_2}{\Lambda_v},\infty), & L>0,\Lambda_v>0;\\ (0,\infty), & L<0,\Lambda_v\le 0;\\ (0,\frac{\delta_2}{\Lambda_v}), & L<0,\Lambda_v>0.\end{cases} \tag{1}$$

类似地, 当 $u\in S_w$时, $w>0$, 而

$$S_w=\begin{cases}(0,\infty), & L>0,\Lambda_w\le 0;\\ (0,\frac{\delta_1}{\Lambda_w}), & L>0,\Lambda_w>0;\\ (\frac{\delta_2}{\Lambda_w},\infty), & L<0,\Lambda_w>0.\end{cases} \tag{2}$$

令 $L=b_1\delta_2-\varepsilon(c+b_2\delta_1)$, $M=(e_2-a\delta_2)c+b_2(e_2\delta_1-e_1\delta_2)$, $N=(e_1-a\delta_1)c\varepsilon+b_1(e_2\delta_1-e_1\delta_2)$, 则

$$v=\frac{Mu-c\delta_2}{cL} \tag{1.3d}$$

$$w = \frac{\varepsilon c\delta_1 - Nu}{cL} \tag{1.3e}$$

将 (1.3d), (1.3e)代入 (1.3a)中得到关于 u的表达式如下:

$$f(u) = A_2u^2 + B_2u + C_2 = 0 \ \ (0 < u < 1)$$

$$A_2 = ac^2L^2 + kM^2 + cb_1ML - cb_2NL - kMN,$$

$$B_2 = c^2(1-a)L^2 + c^2(\varepsilon b_2\delta_1 - b_1\delta_2)L + ck(\varepsilon\delta_1 - 2\delta_2)M + ck\delta_2N + c(1-b_1)LM - c(1-b_2)LN,$$

$$C_2 = c^2\varepsilon\delta_1(1-b_2)L - c^2\delta_2(1-b_1)L - c^2L^2 - k\varepsilon c^2\delta_1\delta_2 - c^2k\delta_2^2,$$

当($\mathbf{H_1}$) $\Re_1 > 1 > \Re_2, L \neq 0$且 $S_E = S_v \cap S_w \neq \emptyset$ 成立时, 系统 (1.3)至多有两个正常数平衡点, 具体如下: $f(0) = C_2, f(1) = A_2 + B_2 + C_2$,

(a) 若 $A_2C_2 > 0$, $\Delta = B_2^2 - 4A_2C_2 > 0, u = -\frac{B_2}{2A_2}$,
当 $C_2(A_2 + B_2 + C_2) > 0$时, $u_1^* = \frac{-B_2-\sqrt{B_2^2-2A_2C_2}}{2A_2}$, $u_2^* = \frac{-B_2+\sqrt{B_2^2-2A_2C_2}}{2A_2}$. 则系统 (1.3)存在两个正常数平衡点, 分别为 $\mathbf{E}_1^* = (u_1^*, v_1^*, w_1^*)$, $\mathbf{E}_2^* = (u_2^*, v_2^*, w_2^*)$;
当 $C_2(A_2 + B_2 + C_2) < 0$时,有 $u_1^* = \frac{-B_2-\sqrt{B_2^2-2A_2C_2}}{2A_2}$,则系统 (1.3) 只有一个正常数平衡点 $\mathbf{E}_1^* = (u_1^*, v_1^*, w_1^*)$.

(b) 若 $A_2C_2 < 0, C_2(A_2 + B_2 + C_2) < 0$时, 有 $u_2^* = \frac{-B_2+\sqrt{B_2^2-2A_2C_2}}{2A_2}$, 则系统 (1.3) 只有一个正常数平衡点 $\mathbf{E}_2^* = (u_2^*, v_2^*, w_2^*)$.

(c) 若 $A_2C_2 > 0$, $\Delta = B_2^2 - 4A_2C_2 = 0$时, $u_3^* = -\frac{B_2}{2A_2}$, 则系统 (1.3)的正常数平衡点为 $\mathbf{E}_3^* = (u_3^*, v_3^*, w_3^*)$.

(d)若 $A_2C_2 = 0$,
当 $A_2 = 0$, $-B_2^2 < B_2C_2 < 0$时, $u_4^* = -\frac{C_2}{B_2}$, 则系统 (1.3)的正常数平衡点为 $\mathbf{E}_4^* = (u_4^*, v_4^*, w_4^*)$;
当 $C_2 = 0$, $-A_2^2 < A_2B_2 < 0$时, $u_5^* = -\frac{B_2}{A_2}$, 则系统 (1.3)的正常数平衡点为 $\mathbf{E}_5^* = (u_5^*, v_5^*, w_5^*)$.

2.3. 平衡点的局部稳定性

系统 (1.3) 在 (u, v, w) 处的 Jacobi 矩阵如下

$$\mathbf{J} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}, \tag{3}$$

其中
$J_{11} = \frac{1-2u}{1+kv} - \frac{(v+w)(1+b_1v+b_2w)}{(1+au+b_1v+b_2w)^2}$, $J_{12} = -[\frac{ku(1-u)}{(1+kv)^2} + \frac{u(1+au)+(b_2-b_1)uw}{(1+au+b_1v+b_2w)^2}]$, $J_{13} = \frac{(b_2-b_1)uv-u(1+au)}{(1+au+b_1v+b_2w)^2}$, $J_{21} = \frac{e_1v(1+b_1v)+vw(ac+e_1b_2)}{(1+au+b_1v+b_2w)^2}$, $J_{22} = \frac{(e_1u-cw)(1+au+b_2w)}{(1+au+b_1v+b_2w)^2} - \delta_1$, $J_{23} = -\frac{cv(1+b_1v)+uv(ac+e_1b_2)}{(1+au+b_1v+b_2w)^2}$, $J_{31} = \frac{e_2w(1+b_1v+b_2w)-ac\varepsilon vw}{(1+au+b_1v+b_2w)^2}$, $J_{32} = \frac{c\varepsilon w(1+b_2w)+uw(ac\varepsilon-b_1e_2)}{(1+au+b_1v+b_2w)^2}$, $J_{33} = \frac{(e_2u+\varepsilon cv)(1+au+b_1v)}{(1+au+b_1v+b_2w)^2} - \delta_2$.

下面通过计算系统 (1.3) 在每个平衡点处的 Jacobi 矩阵的特征值, 来确定这些平衡点的稳定性.

定理 3 (i) 平凡平衡点 $\mathbf{E}_0 = (0,0,0)$ 是无条件不稳定的.

(ii) 若 $max\{\Re_1, \Re_2\} < 1$, 则半平凡平衡点 $\mathbf{E}_1 = (1,0,0)$ 是局部渐近稳定的; 否则 $\mathbf{E}_1$ 是不稳定的.

证明 (i) 系统 (1.3) 在平衡点 $\mathbf{E}_0$ 处的 Jacobi 矩阵为

$$\mathbf{J}_{\mathbf{E}_0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\delta_1 & 0 \\ 0 & 0 & -\delta_2 \end{pmatrix}. \tag{4}$$

矩阵 (4) 的特征值为 $\lambda_1^{(0)} = 1 > 0$, $\lambda_2^{(0)} = -\delta_1$ 和 $\lambda_3^{(0)} = -\delta_2$. 因此, 平衡点 $\mathbf{E}_0$ 是不稳定的.

(ii) 系统 (1.3) 在平衡点 $\mathbf{E}_1$ 处的 Jacobi 矩阵为

$$\mathbf{J}_{\mathbf{E}_1} = \begin{pmatrix} -1 & -\dfrac{1}{1+a} & -\dfrac{1}{1+a} \\ 0 & \dfrac{e_1}{1+a} - \delta_1 & 0 \\ 0 & 0 & \dfrac{e_2}{1+a} - \delta_2 \end{pmatrix}. \tag{5}$$

矩阵 (5) 的特征方程为

$$(\lambda + 1)\left[\lambda - (\frac{e_1}{1+a} - \delta_1)\right]\left[\lambda - (\frac{e_2}{1+a} - \delta_2)\right] = 0.$$

所以, 矩阵 (5) 的特征值为 $\lambda_1^{(1)} = -1$, $\lambda_2^{(1)} = \dfrac{e_1}{1+a} - \delta_1 = \delta_1(\Re^1 - 1)$, $\lambda_3^{(1)} = \dfrac{e_2}{1+a} - \delta_2 = \delta_2(\Re^2 - 1)$. 当 $max\{\Re_1, \Re_2\} < 1$时, 平衡点 $\mathbf{E}_1$ 是局部渐近稳定的; 反之, $\mathbf{E}_1$ 是不稳定的.

定理 4 假设 $\Re_1 > 1, k > b_1$成立, 若 $\lambda_1^{(2)} < 0$, $T^{(2)} < 0$, 则无 IG 捕食者平衡点 $E_2(u_2, v_2, 0)$ 是局部渐近稳定的; 若 $\lambda_1^{(2)} > 0$或 $T^{(2)} > 0$, 则 $E_2(u_2, v_2, 0)$是不稳定的.

证明 系统 (1.3) 在平衡点 $\mathbf{E}_2$ 处的 Jacobi 矩阵为

$$\mathbf{J}_{\mathbf{E}_2} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}. \tag{6}$$

其中

$a_{11} = \frac{1}{1+kv_2}\left[\frac{a\delta_1(1-u_2)}{e_1} - u_2\right]$, $a_{12} = -\left[\frac{ku_2(1-u_2)}{(1+kv_2)^2} + \frac{\delta_1}{e_1} - \frac{b_1\delta_1(1-u_2)}{e_1(1+kv_2)}\right]$,
$a_{13} = \frac{(b_2-b_1)u_2v_2 - u_2(1+au_2)}{(1+au_2+b_1v_2)^2}$, $a_{21} = (e_1 - a\delta_1)\frac{1-u_2}{1+kv_2}$, $a_{22} = -\frac{b_1\delta_1(1-u_2)}{1+kv_2}$,
$a_{23} = -\frac{cv_2(1+b_1v_2)+u_2v_2(e_1b_2+ac)}{(1+au_2+b_1v_2)^2}$, $a_{33} = \frac{e_2\delta_1}{e_1} - \delta_2 + \frac{\varepsilon c(1-u_2)}{1+kv_2}$.

矩阵 (6) 的特征方程为

$$(\lambda - a_{33})\left[\lambda^2 - (a_{11}+a_{22})\lambda + a_{11}a_{22} - a_{12}a_{21}\right] = 0$$

令 $D^{(2)} = a_{11}a_{22} - a_{12}a_{21} = \frac{1-u_2}{1+kv_2}\left[\frac{\delta_1(k-b_1)(1-u_2)}{(1+kv_2)^2} + \frac{b_1\delta_1 u_2}{1+kv_2} + \frac{\delta_1(e_1-a\delta_1)}{e_1}\right]$,
$T^{(2)} = a_{11} + a_{22} = \frac{\delta_1(\frac{a}{e_1}-b_1)(1-u_2)-u_2}{1+kv_2}$, $\lambda_1^{(2)} = \frac{e_2\delta_1}{e_1} - \delta_2 + \frac{\varepsilon c(1-u_2)}{1+kv_2}$,
由韦达定理可得 $\lambda_2^{(2)} + \lambda_3^{(2)} = T^{(2)}$, $\lambda_2^{(2)}\lambda_3^{(2)} = D^{(2)} > 0$. 当 $\lambda_1^{(2)} < 0$ 且 $T^{(2)} < 0$ 时, 矩阵 (6) 的特征值 $\lambda_2^{(2)}$, $\lambda_3^{(2)}$ 具有负实部, 因此, 平衡点 $\mathbf{E}_2$ 是局部渐近稳定的; 当 $\lambda_1^{(2)} > 0$或 $T^{(2)} > 0$ 时, 平衡点 $\mathbf{E}_2$ 是不稳定的.

定理 5 假设 $\Re^2 > 1$, 若 $\Re^2 > \Re^1$且 $a\delta_2 < b_2e_2$, 则无 IG 食饵平衡点 $\mathbf{E}_3(u_3, 0, w_3)$ 是局部渐近稳定的; 若 $\Re^1 > \Re^2$, $\delta_1(\Re^1 - \Re^2) > c(1-u_3)\Re^2$或 $e_2u_3 < (a\delta_2 - b_2e_2)(1-u_3)$ 时, 平衡点 $\mathbf{E}_3(u_3, 0, w_3)$ 是不稳定的.

证明 系统 (1.3) 在平衡点 $\mathbf{E}_3$ 处的 Jacobi 矩阵为

$$\mathbf{J}_{\mathbf{E}_3} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix}. \tag{7}$$

其中
$b_{11} = \frac{a\delta_2}{e_2}(1-u_3) - u_3$, $b_{12} = (1-u_3)(\frac{b_1\delta_2}{e_2} - ku_3) - \frac{\delta_2}{e_2}$, $b_{13} = -\frac{\delta_2[1-b_2(1-u_3)]}{e_2}$,
$b_{22} = \frac{e_1\delta_2 - e_2\delta_1}{e_2} - c(1-u_3)$, $b_{31} = (e_2 - a\delta_2)(1-u_3)$, $b_{32} = (\varepsilon c - b_1\delta_2)(1-u_3)$, $b_{33} = -b_2\delta_2(1-u_3)$.
矩阵 (7) 的特征方程为

$$(\lambda - b_{22})\left[\lambda^2 - (b_{11}+b_{33})\lambda + b_{11}b_{33} - b_{13}b_{31}\right] = 0.$$

令 $D^{(3)} = b_{11}b_{33} - b_{13}b_{31} = \delta_2(1-u_3)\sqrt{(e_2 - b_2e_2 - a\delta_2)^2 + 4b_2e_2\delta_2} > 0$,
$T^{(3)} = b_{11} + b_{33} = (\frac{a\delta_2}{e_2} - b_2)(1-u_3) - u_3$, $\lambda_1^{(3)} = \frac{e_1\delta_2 - e_2\delta_1}{e_2} - c(1-u_3)$.
由韦达定理可得 $\lambda_2^{(3)} + \lambda_3^{(3)} = T^{(3)}$, $\lambda_2^{(3)}\lambda_3^{(3)} = D^{(3)} > 0$. 当 $\lambda_1^{(3)} < 0$ 且 $T^{(2)} < 0$, 即 $\Re^2 > \Re^1$且 $a\delta_2 < b_2e_2$时, 矩阵 (7) 的特征值 $\lambda_2^{(3)}$, $\lambda_3^{(3)}$ 具有负实部, 因此, 平衡点 $\mathbf{E}_3$ 是局部渐近稳定的; 当 $\lambda_1^{(3)} > 0$或 $T^{(3)} > 0$, 即$\Re^1 > \Re^2$, $\delta_1(\Re^1 - \Re^2) > c(1-u_3)\Re^2$或 $e_2u_3 < (a\delta_2 - b_2e_2)(1-u_3)$ 时, 平衡点 $\mathbf{E}_3$ 是不稳定的.

定理 6 若正常数平衡点 $\mathbf{E}^*(u^*, v^*, w^*)$满足以下条件:

$(\mathbf{H_2})$ $\quad \varepsilon c > b_1\delta_2,\ 1 + kv^* > b_i(1-u^*),\ e^i > a\delta_i\ \ (i = 1, 2)$

$(\mathbf{H_3})$ $\quad 1 + au^* + b_1v^* + b_2w^* > a(1-u^*)$

$(\mathbf{H_4})$ $\quad b_2\delta_2(e_1 - a\delta_1) > (e_2 - a\delta_2)(c + b_2\delta_1)$

$(\mathbf{H_5})$ $\quad b_1\delta_1[(1+kv^*) - b_1(1-u^*)]v^* > (\varepsilon c - b_1\delta_2)[(1+kv^*) - b_2(1-u^*)]w^*$

则正常数平衡点$\mathbf{E}^*(u^*,v^*,w^*)$是局部渐近稳定的.

证明 系统 (1.3) 在平衡点 $\mathbf{E}^*$ 处的 Jacobi 矩阵为

$$\mathbf{J}_{\mathbf{E}^*}=\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}. \tag{8}$$

其中

$c_{11}=\frac{u^*}{1+kv^*}\left[\frac{a(1-u^*)}{1+au^*+b_1v^*+b_2w^*}-1\right]$, $c_{12}=-\frac{u^*}{1+kv^*}\left[\frac{k(1-u^*)}{1+kv^*}+\frac{(1+kv^*)-b_1(1-u^*)}{1+au^*+b_1v^*+b_2w^*}\right]$,
$c_{13}=\frac{u^*}{1+kv^*}\left[\frac{b_2(1-u^*)-(1+kv^*)}{1+au^*+b_1v^*+b_2w^*}\right]$, $c_{21}=\frac{(e_1-a\delta_1)v^*}{1+au^*+b_1v^*+b_2w^*}$, $c_{22}=-\frac{b_1\delta_1v^*}{1+au^*+b_1v^*+b_2w^*}<0$,
$c_{23}=-\frac{(c+b_2\delta_1)v^*}{1+au^*+b_1v^*+b_2w^*}<0$, $c_{31}=\frac{(e_2-a\delta_2)w^*}{1+au^*+b_1v^*+b_2w^*}$, $c_{32}=\frac{(\varepsilon c-b_1\delta_2)}{1+au^*+b_1v^*+b_2w^*}$,
$c_{33}=-\frac{b_2\delta_2w^*}{1+au^*+b_1v^*+b_2w^*}<0$.

矩阵 (8) 的特征方程为

$$\lambda^3+P_1\lambda^2+P_2\lambda+P_3=0. \tag{9}$$

其中

$$\begin{aligned} P_1&=-(c_{11}+c_{22}+c_{33})\\ &=\frac{u^*}{1+kv^*}\left[1-\frac{a(1-u^*)}{1+au^*+b_1v^*+b_2w^*}\right]+\frac{b_1\delta_1v^*+b_2\delta_2w^*}{1+au^*+b_1v^*+b_2w^*},\end{aligned}$$

$$\begin{aligned} P_2&=c_{11}c_{22}+c_{11}c_{33}+c_{22}c_{33}-c_{12}c_{21}-c_{13}c_{31}-c_{23}c_{32}\\ &=\frac{u^*(b_1\delta_1v^*+b_2\delta_2w^*)}{(1+kv^*)(1+au^*+b_1v^*+b_2w^*)}+\frac{k(e_1-a\delta_1)(1-u^*)u^*v^*}{(1+kv^*)^2(1+au^*+b_1v^*+b_2w^*)}\\ &+\frac{u^*\left\{(1+kv^*)\left[(e_1-a\delta_1)v^*+(e_2-a\delta_2)w^*\right]-(1-u^*)(b_1e_1v^*+b_2e_2w^*)\right\}}{(1+kv^*)(1+au^*+b_1v^*+b_2w^*)}\\ &+\frac{\left[b_1b_2\delta_1\delta_2+(c\varepsilon-b_1\delta_2)(c+b_2\delta_1)\right]v^*w^*}{(1+au^*+b_1v^*+b_2w^*)^2},\end{aligned}$$

$$\begin{aligned} P_3&=c_{11}(c_{23}c_{32}-c_{22}c_{33})+c_{12}(c_{21}c_{33}-c_{23}c_{31})+c_{13}(c_{22}c_{31}-c_{32}c_{21})\\ &=-\frac{u^*}{1+kv^*}\left[\frac{a(1-u^*)}{1+au^*+b_1v^*+b_2w^*}-1\right]\frac{\left[(c+b_2\delta_1)(c\varepsilon-b_1\delta_2)+b_1b_2\delta_1\delta_2\right]v^*w^*}{(1+au^*+b_1v^*+b_2w^*)^2}\\ &-\frac{u^*}{1+kv^*}\left[\frac{k(1-u^*)}{1+kv^*}+\frac{(1+kv^*)-b_1(1-u^*)}{1+au^*+b_1v^*+b_2w^*}\right]\frac{\left[(e_2-a\delta_2)(c+b_2\delta_1)-b_2\delta_2(e_1-a\delta_2)\right]v^*w^*}{(1+au^*+b_1v^*+b_2w^*)^2}\\ &-\frac{u^*}{1+kv^*}\left[\frac{b_2(1-u^*)-(1+kv^*)}{1+au^*+b_1v^*+b_2w^*}\right]\frac{\left[b_1\delta_1(e_2-a\delta_2)+(e_1-a\delta_1)(\varepsilon c-b_1\delta_2)\right]v^*w^*}{(1+au^*+b_1v^*+b_2w^*)^2}.\end{aligned}$$

由条件 ($\mathbf{H_2}$) 可知,

$$\operatorname{sgn}(\mathbf{J_{E^*}}) = \begin{pmatrix} - & - & - \\ + & - & - \\ + & + & - \end{pmatrix}. \tag{10}$$

由条件 ($\mathbf{H_2}$) ($\mathbf{H_3}$) ($\mathbf{H_4}$) 可知, $P_1 > 0, P_3 > 0$.

$$\begin{aligned} P_1P_2 - P_3 = {} & c_{11}^2(-c_{22} - c_{33}) + c_{22}^2(-c_{11} - c_{33}) + c_{33}^2(-c_{11} - c_{22}) \\ & + c_{12}(c_{23}c_{31} + c_{11}c_{21}) + c_{21}(c_{13}c_{32} + c_{22}c_{12}) + c_{22}(c_{23}c_{32} - c_{11}c_{33}) \\ & + c_{33}(c_{23}c_{32} - c_{11}c_{22} + c_{13}c_{31}) + c_{11}c_{13}c_{31} \end{aligned} \tag{11}$$

由条件($\mathbf{H_5}$) 与 (10) 的符号可知,

$$\begin{aligned} c_{13}c_{32} + c_{22}c_{12} = {} & \frac{(\varepsilon c - b_1\delta_2)\left[b_2(1-u^*) - (1+kv^*)\right]u^*w^*}{(1+kv^*)(1+au^*+b_1v^*+b_2w^*)^2} \\ & + \frac{b_1\delta_1u^*v^*}{(1+kv^*)(1+au^*+b_1v^*+b_2w^*)}\left[\frac{k(1-u^*)}{1+kv^*} + \frac{(1+kv^*) - b_1(1-u^*)}{1+au^*+b_1v^*+b_2w^*}\right] \\ = {} & \frac{u^*\left\{b_1\delta_1\left[(1+kv^*) - b_1(1-u^*)\right]v^* - (\varepsilon c - b_1\delta_2)\left[(1+kv^*) - b_2(1-u^*)\right]w^*\right\}}{(1+kv^*)(1+au^*+b_1v^*+b_2w^*)^2} \\ & + \frac{kb_1\delta_1(1-u^*)u^*v^*}{(1+kv^*)^2(1+au^*+b_1v^*+b_2w^*)} > 0 \end{aligned}$$

从而 $P_1P_2 - P_3 > 0$.

因此, 当条件 ($\mathbf{H_2}$) ($\mathbf{H_3}$) ($\mathbf{H_4}$) ($\mathbf{H_5}$) 成立时, $P_1 > 0$, $P_3 > 0$, $P_1P_2 - P_3 > 0$, 由 Routh-Hurwitz 判据 [42]可知, 特征方程 (9) 的根均具有负实部, 进而系统 (1.3) 的正常数平衡点 $\mathbf{E}^*$ 是局部渐近稳定的. 反之, 特征方程 (9) 的根不全有负实部且实部不为零, 因此正常数平衡点 $\mathbf{E}^*$ 是不稳定的.

2.4. 正常数平衡点的全局稳定性

这一部分我们主要研究使得正常数平衡点 $\mathbf{E}^*(u^*, v^*, w^*)$ 在 $\mathbb{R}_+^3$ 内全局渐近稳定的充分条件.

定理 7 若 ($\mathbf{H_3}$)与以下条件

($\mathbf{H_6}$) $$a\delta_i(1+kv^*) - b_ie_i(1-u^*) > 0$$

($\mathbf{H_7}$) $$2b_1\delta_1(1+kv^*)^2 < k(1-u^*)\left[a\delta_1(1+kv^*) - b_1e_1(1-u^*)\right]$$

($\mathbf{H_8}$)

$$4b_1b_2e_1e_2\delta_1\delta_2 > \left[e_1(c\varepsilon - b_1\delta_2) - e_2(c + b_2\delta_1)\right]^2$$

$$e_2(c + b_2\delta_1) > e_1(c\varepsilon - b_1\delta_2)$$

$$b_1e_1\delta_1\left[a\delta_2(1+kv^*) - b_2e_2(1-u^*)\right] > \left[e_2(c+b_2\delta_1) - e_1(\varepsilon c - b_1\delta_2)\right]\left[b_1e_1(1-u^*) - a\delta_1(1+kv^*)\right]$$

成立, 则正常数平衡点 $\mathbf{E}^*(u^*,v^*,w^*)$ 在 $\mathbb{R}_+^3=\{(u,v,w):u(t)>0,v(t)>0,w(t)>0\}$ 内是全局渐近稳定的.

证明 在 $\mathbb{R}_+^3=\{(u,v,w):u(t)>0,v(t)>0,w(t)>0\}$ 内定义 Lyapunov 函数为

$$V=(u-u^*-u^*\ln\frac{u}{u^*})+\frac{1}{e_1}(v-v^*-v^*\ln\frac{v}{v^*})+\frac{1}{e_2}(w-w^*-w^*\ln\frac{w}{w^*}) \tag{12}$$

对等式 (12) 沿着系统 (1.3) 的解关于时间 t 求导可得

$$\frac{\mathrm{d}V}{\mathrm{d}t}=(\frac{u-u^*}{u})\frac{\mathrm{d}u}{\mathrm{d}t}+(\frac{v-v^*}{v})\frac{\mathrm{d}v}{\mathrm{d}t}+(\frac{w-w^*}{w})\frac{\mathrm{d}w}{\mathrm{d}t}$$

由于 E^* 是系统 (1.3)的正常数平衡点, 有 $\dfrac{v^*+w^*}{1+au^*+b_1v^*+b_2w^*}=\dfrac{1-u^*}{1+kv^*}$,

$$\frac{e_1u^*-cw^*}{1+au^*+b_1v^*+b_2w^*}=\delta_1,\quad \frac{e_2u^*+\varepsilon cv^*}{1+au^*+b_1v^*+b_2w^*}=\delta_2.$$

另外, $uv^*-uv^*=v^*(u-u^*)-u^*(v-v^*)$, $u^*v-uv^*=u^*(v-v^*)-v^*(u-u^*)$

$vw^*-v^*w=w^*(v-v^*)-v^*(w-w^*)$, $v^*w-vw^*=v^*(w-w^*)-w^*(v-v^*)$

$uw^*-u^*w=w^*(u-u^*)-u^*(w-w^*)$, $u^*w-uw^*=u^*(w-w^*)-w^*(u-u^*)$.

经过简单计算得到,

$$\begin{aligned}(\frac{u-u^*}{u})\frac{\mathrm{d}u}{\mathrm{d}t}=&-\left[\frac{1}{1+kv}-\frac{a(1-u^*)}{(1+kv^*)(1+au+b_1v+b_2w)}\right](u-u^*)^2\\&-\left[\frac{k(1-u^*)}{(1+kv)(1+kv^*)}+\frac{(1+kv^*)-b_1(1-u^*)}{(1+kv^*)(1+au+b_1v+b_2w)}\right](u-u^*)(v-v^*)\\&-\left[\frac{(1+kv^*)-b_2(1-u^*)}{(1+kv^*)(1+au+b_1v+b_2w)}\right](u-u^*)(w-w^*)\end{aligned}$$

$$\begin{aligned}\frac{1}{e_1}(\frac{v-v^*}{v})\frac{\mathrm{d}v}{\mathrm{d}t}=&\frac{e_1-a\delta_1}{e_1(1+au+b_1v+b_2w)}(u-u^*)(v-v^*)-\frac{b_1\delta_1}{e_1(1+au+b_1v+b_2w)}(v-v^*)^2\\&-\frac{c+b_2\delta_1}{e_1(1+au+b_1v+b_2w)}(v-v^*)(w-w^*)\end{aligned}$$

$$\begin{aligned}\frac{1}{e_2}(\frac{w-w^*}{w})\frac{\mathrm{d}w}{\mathrm{d}t}=&\frac{e_2-a\delta_2}{e_2(1+au+b_1v+b_2w)}(u-u^*)(w-w^*)-\frac{b_2\delta_2}{e_2(1+au+b_1v+b_2w)}(w-w^*)^2\\&+\frac{c\varepsilon-b_1\delta_2}{e_2(1+au+b_1v+b_2w)}(v-v^*)(w-w^*)\end{aligned}$$

从而

$$\frac{\mathrm{d}V}{\mathrm{d}t}=-\left[\frac{1}{1+kv}-\frac{a(1-u^*)}{(1+kv^*)(1+au+b_1v+b_2w)}\right](u-u^*)^2-\frac{b_1\delta_1}{e_1(1+au+b_1v+b_2w)}(v-v^*)^2$$

$$-\frac{b_2\delta_2}{e_2(1+au+b_1v+b_2w)}(w-w^*)^2+\left[\frac{e_1-a\delta_1}{e_1(1+au+b_1v+b_2w)}\right](u-u^*)(v-v^*)$$

$$+\left[-\frac{k(1-u^*)}{(1+kv)(1+kv^*)}-\frac{(1+kv^*)-b_1(1-u^*)}{(1+kv^*)(1+au+b_1v+b_2w)}\right](u-u^*)(v-v^*)$$

$$+\left[\frac{e_2-a\delta_2}{e_2(1+au+b_1v+b_2w)}-\frac{(1+kv^*)-b_2(1-u^*)}{(1+kv^*)(1+au+b_1v+b_2w)}\right](u-u^*)(w-w^*)$$

$$+\left[\frac{c\varepsilon-b_1\delta_2}{e_2(1+au+b_1v+b_2w)}-\frac{c+b_2\delta_1}{e_1(1+au+b_1v+b_2w)}\right](v-v^*)(w-w^*)$$

令 $l_{11}=-\left[\frac{1}{1+kv}-\frac{a(1-u^*)}{(1+kv^*)(1+au+b_1v+b_2w)}\right]$, $l_{12}=l_{21}=\frac{1}{2}\left[\frac{b_1e_1(1-u^*)-a\delta_1(1+kv^*)}{e_1(1+kv^*)(1+au+b_1v+b_2w)}-\frac{k(1-u^*)}{(1+kv)(1+kv^*)}\right]$,

$l_{22}=-\frac{b_1\delta_1}{e_1(1+au+b_1v+b_2w)}$, $l_{13}=l_{31}=\frac{1}{2}\left[\frac{b_2e_2(1-u^*)-a\delta_2(1+kv^*)}{e_2(1+kv^*)(1+au+b_1v+b_2w)}\right]$,

$l_{33}=-\frac{b_2\delta_2}{e_2(1+au+b_1v+b_2w)}$, $l_{23}=l_{32}=\frac{1}{2}\left[\frac{e_1(c\varepsilon-b_1\delta_2)-e_2(c+b_2\delta_1)}{e_1e_2(1+au+b_1v+b_2w)}\right]$.

$$L=\begin{pmatrix} l_{11} & l_{12} & l_{13}\\ l_{12} & l_{22} & l_{23}\\ l_{13} & l_{23} & l_{33}\end{pmatrix}.$$

由 $(\mathbf{H_6})$ $(\mathbf{H_7})$可知, $\left| \ l_{11} \ \right| < 0.$

$$\begin{vmatrix} l_{11} & l_{12}\\ l_{12} & l_{22}\end{vmatrix}=l_{11}l_{22}-l_{12}^2=\frac{2e_1(1+kv)(1+au+b_1v+b_2w)\left\{2b_1\delta_1(1+kv^*)^2-k(1-u^*)[a\delta_1(1+kv^*)-b_1e_1(1-u^*)]\right\}}{4e_1^2(1+kv)^2(1+kv^*)(1+au+b_1v+b_2w)^2}$$

$$-\frac{4ab_1e_1\delta_1(1-u^*)(1+kv^*)(1+kv)^2}{4e_1^2(1+kv)^2(1+kv^*)(1+au+b_1v+b_2w)^2}-\frac{(1+kv)^2[b_1e_1(1-u^*)-a\delta_1(1+kv^*)]^2}{4e_1^2(1+kv)^2(1+kv^*)(1+au+b_1v+b_2w)^2}$$

$$-\frac{e_1^2k^2(1-u^*)^2(1+au+b_1v+b_2w)^2}{4e_1^2(1+kv)^2(1+kv^*)(1+au+b_1v+b_2w)^2}<0$$

由 $(\mathbf{H_6})$ $(\mathbf{H_8})$可知,

$$\begin{vmatrix} l_{11} & l_{12} & l_{13}\\ l_{12} & l_{22} & l_{23}\\ l_{13} & l_{23} & l_{33}\end{vmatrix}=l_{11}l_{22}l_{33}+2l_{12}l_{13}l_{23}-l_{11}l_{23}^2-l_{22}l_{13}^2-l_{33}l_{12}^2$$

$$=\frac{\left\{[e_1(c\varepsilon-b_1\delta_2)-e_2(c+b_2\delta_1)]^2-4b_1b_2e_1e_2\delta_1\delta_2\right\}(1+kv)(1+kv^*)[(1+kv^*)(1+au+b_1v+b_2w)-a(1-u^*)(1+kv)]}{4e_1^2e_2^2(1+kv)^2(1+kv^*)^2(1+au+b_1v+b_2w)^3}$$

$$+\frac{[e_1(c\varepsilon-b_1\delta_2)-e_2(c_+b_2\delta_1)][b_1e_1(1-u^*)-a\delta_1(1+kv^*)](1+kv)^2[b_2e_2(1-u^*)-a\delta_2(1+kv^*)]}{4e_1^2e_2^2(1+kv)^2(1+kv^*)^2(1+au+b_1v+b_2w)^3}$$

$+\frac{b_1e_1\delta_1(1+kv)^2[b_2e_2(1-u^*)-a\delta_2(1+kv^*)]^2}{4e_1^2e_2^2(1+kv)^2(1+kv^*)^2(1+au+b_1v+b_2w)^3}+\frac{b_2e_2\delta_2(1+kv)[b_1e_1(1-u^*)-a\delta_1(1+kv^*)]}{4e_1^2e_2^2(1+kv)^2(1+kv^*)^2(1+au+b_1v+b_2w)^3}$

$-\frac{ke_1(1+kv)(1-u^*)[e_1(c\varepsilon-b_1\delta_2)-e_2(c_+b_2\delta_1)][b_2e_2(1-u^*)-a\delta_2(1+kv^*)](1+au+b_1v+b_2w)}{4e_1^2e_2^2(1+kv)^2(1+kv^*)^2(1+au+b_1v+b_2w)^3}$

$+\frac{b_2e_1e_2k\delta_2(1-u^*)(1+au+b_1v+b_2w)}{4e_1^2e_2^2(1+kv)^2(1+kv^*)^2(1+au+b_1v+b_2w)^3}<0.$

记 $\tilde{u}=u-u^*$, $\tilde{v}=v-v^*$, $\tilde{w}=w-w^*$, 从而 $\frac{\mathrm{d}V}{\mathrm{d}t}=(\tilde{u},\tilde{u},\tilde{w})^\top L\,(\tilde{u},\tilde{u},\tilde{w})\leq 0$. 当且仅当 $u=u^*$, $v=v^*$, $w=w^*$ 时, $\frac{\mathrm{d}V}{\mathrm{d}t}=0$, 由 LaSalle 变分引理 [43], 我们得到 $\mathbf{E}^*$ 在 $\mathbb{R}_+^3$ 内是全局渐近稳定的.

2.5. Hopf 分支

这一部分我们以恐惧因子 k 作为分支参数, 运用 Sotomayor定理[44]讨论各平衡点处发生 Hopf 分支的条件.

定理 8 当分支参数 k 越过临界值 $k=k_1$ 时, 系统 (1.3) 围绕无 IG 捕食者平衡点 $\mathbf{E}_2$ 经历霍普夫分支, 这里 k_1 满足: $Q_1(k_1)=a_{11}(k_1)+a_{22}(k_1)=0$, $\lambda_{c_1}=\sqrt{a_{11}(k_1)a_{22}(k_1)-a_{12}(k_1)a_{21}(k_1)}$ $\det(J(\mathbf{E}_2))|_{k=k_1}=s_3(k_1)>0$, $\frac{\mathrm{d}\lambda_{a_1}}{\mathrm{d}k}|_{k=k_1}=\frac{\sigma_1\varphi-2\sigma_2\psi}{\sigma_1^2-4\sigma_2^2}\neq 0$.

证明 设 $\lambda(k)=\lambda_{a_1}(k)+i\lambda_{c_1}(k)$ 为 $(\lambda-a_{33})\left[\lambda^2-(a_{11}+a_{22})\lambda+a_{11}a_{22}-a_{12}a_{21}\right]=0$ 的特征值, 将 $\lambda(k)$ 代入并分离实部和虚部得,

$$\begin{cases}\lambda_{a_1}^3-3\lambda_{a_1}\lambda_{c_1}^2+s_1(\lambda_{c_1}^2-\lambda_{a_1}^2)+s_2\lambda_{a_1}-s_3=0,\\ 3\lambda_{c_1}\lambda_{a_1}^2-\lambda_{c_1}^3-2s_1\lambda_{a_1}\lambda_{c_1}+s_2\lambda_{c_1}=0,\end{cases}\tag{13}$$

而 $s_1=a_{11}(k)+a_{22}(k)+a_{33}(k)$,
$s_2=a_{11}(k)a_{22}(k)-a_{12}(k)a_{21}(k)+a_{33}(k)\left[a_{11}(k)+a_{22}(k)\right]$,
$s_3=a_{33}(k)\left[a_{11}(k)a_{22}(k)-a_{12}(k)a_{21}(k)\right]$.

当特征值的实部为零时, 该特征值是共轭的, 无 IG 捕食者平衡点 $\mathbf{E}_2$ 因产生 Hopf 分支而失去了它的稳定性. 定义 Hopf 分支临界值为 k_1, 就有 $\lambda_{a_1}(k_1)=0$ 且 $\lambda_{c_1}(k_1)\neq 0$, 代入 (13) 式得到如下方程组:

$$\begin{cases}s_1\lambda_{c_1}^2-s_3=0\\ -\lambda_{c_1}^3+s_2\lambda_{c_1}=0\end{cases}$$

当 $Q_1=a_{11}(k_1)+a_{22}(k_1)=0$时, 有 $s_1=a_{33}(k_1)$, 则 $\lambda_{c_1}=\sqrt{a_{11}(k_1)a_{22}(k_1)-a_{12}(k_1)a_{21}(k_1)}$, 从而 $\det(J(\mathbf{E}_2))|_{k=k_1}=s_3(k_1)>0$. 对 (13) 式两边关于参数 k 求导得

$$\begin{cases}(3\lambda_{a_1}^2-3\lambda_{c_1}^2-2s_1\lambda_{a_1}+s_2)\frac{\mathrm{d}\lambda_{a_1}}{\mathrm{d}k}+(2s_1\lambda_{c_1}-6\lambda_{a_1}\lambda_{c_1})\frac{\mathrm{d}\lambda_{c_1}}{\mathrm{d}k}=(\lambda_{a_1}^2-\lambda_{c_1}^2)\frac{\mathrm{d}s_1}{\mathrm{d}k}-\lambda_{a_1}\frac{\mathrm{d}s_2}{\mathrm{d}k}+\frac{\mathrm{d}s_3}{\mathrm{d}k}\\ (3\lambda_{a_1}^2-3\lambda_{c_1}^2-2s_1\lambda_{a_1}+s_2)\frac{\mathrm{d}\lambda_{c_1}}{\mathrm{d}k}+(6\lambda_{a_1}\lambda_{c_1}-2s_1\lambda_{c_1})\frac{\mathrm{d}\lambda_{a_1}}{\mathrm{d}k}=2\lambda_{a_1}\lambda_{c_1}\frac{\mathrm{d}s_1}{\mathrm{d}k}-\lambda_{c_1}\frac{\mathrm{d}s_2}{\mathrm{d}k}\end{cases}$$

由于 $\lambda_{a_1}=0$, 并令 $\sigma_1=s_2 3\lambda_{c_1}^2$, $\sigma_2=2s_1\lambda_{c_1}$, $\varphi=\frac{\mathrm{d}s_3}{\mathrm{d}k}-\lambda_{c_1}^2\frac{\mathrm{d}s_1}{\mathrm{d}k}$, $\psi=-\lambda_{c_1}\frac{\mathrm{d}s_2}{\mathrm{d}k}$. 则上式方程组变

为 $\lambda_{c_1}(k_1) \neq 0$, 代入 (13) 式得到如下方程组:

$$\begin{cases} \sigma_1 \dfrac{\mathrm{d}\lambda_{a_1}}{\mathrm{d}k} + \sigma_2 \dfrac{\mathrm{d}\lambda_{c_1}}{\mathrm{d}k} = \varphi \\ \sigma_1 \dfrac{\mathrm{d}\lambda_{c_1}}{\mathrm{d}k} - \sigma_2 \dfrac{\mathrm{d}\lambda_{a_1}}{\mathrm{d}k} = \psi \end{cases}$$

解得 $\dfrac{\mathrm{d}\lambda_{a_1}}{\mathrm{d}k}|_{k=k_1} = \dfrac{\sigma_1\varphi - 2\sigma_2\psi}{\sigma_1^2 - 4\sigma_2^2} \neq 0$, 证毕.

推论 当分支参数 k 越过临界值 $k = k_2$ 时, 系统 (1.3) 围绕无 IG 食饵平衡点 $\mathbf{E}_3$ 经历霍普夫分支, 这里 k_2 满足: $Q_2(k_2) = b_{11}(k_2) + b_{33}(k_2) = 0$, $\lambda_{c_2} = \sqrt{b_{11}(k_2)b_{33}(k_2) - b_{13}(k_2)b_{31}(k_2)}$ $\det(J(\mathbf{E}_3))|_{k=k_2} = \overline{s_3}(k_2) > 0$, $\dfrac{\mathrm{d}\lambda_{a_2}}{\mathrm{d}k}|_{k=k_2} = \dfrac{\overline{\sigma_1\varphi} - 2\overline{\sigma_2}\overline{\psi}}{\overline{\sigma_1}^2 - 4\overline{\sigma_2}^2} \neq 0$.

定理9 若以下条件成立

$(\mathbf{H_9})$

(i) $\quad P_1(k^*) > 0, P_3(k^*) > 0$

(ii) $\quad Q(k^*) = P_1(k^*)P_2(k^*) - P_3(k^*) = 0$

(iii) $\quad \dfrac{\mathrm{d}Q(k)}{\mathrm{d}k}|_{k=k^*} \neq 0$

且分支参数 k经过临界值 k^* 时, 系统 (1.3) 围绕正常数平衡点 $\mathbf{E}^*$ 产生 Hopf 分支.

证明 由条件 $(\mathbf{H_9})$ 中的(i)(ii)得出, 当 $k = k^*$时, (9) 式有一个负根和两个纯虚根, 则 (9)式就一定可以写成 $(\lambda^2 + P_2)(\lambda + P_1) = 0$ 的形式,并得出三个根为 $\lambda_1 = -P_1$, $\lambda_2 = i\sqrt{P_2}$, $\lambda_3 = -i\sqrt{P_2}$. 对所有的分支参数值 k, 特征值 λ_{23} 的形式为 $\lambda_2 = \alpha(k) + i\beta(k)$, $\lambda_3 = \alpha(k) - i\beta(k)$, $\alpha(k)$为实部, 将 $\lambda(k) = \alpha(k) + i\beta(k)$ 代入 (9) 式中得

$$\Big(\alpha(k) + i\beta(k)\Big)^3 + P_1(k)\Big(\alpha(k) + i\beta(k)\Big)^2 + P_2\Big(\alpha(k) + i\beta(k)\Big) + P_3 = 0 \tag{14}$$

对 (13) 式两边关于分支参数 k 求导并分离实部和虚部得到以下方程组

$$\begin{cases} l_1 \dfrac{\mathrm{d}\alpha(k)}{\mathrm{d}k} - l_2 \dfrac{\mathrm{d}\beta(k)}{\mathrm{d}k} + l_3(k) = 0 \\ l_2 \dfrac{\mathrm{d}\alpha(k)}{\mathrm{d}k} + l_1 \dfrac{\mathrm{d}\beta(k)}{\mathrm{d}k} + l_4(k) = 0 \end{cases}$$

其中

$l_1(k) = P_2(k) + 2P_1(k)\alpha(k) + 3\Big(\alpha(k)^2 - \beta(k)^2\Big)$, $l_2(k) = 2\Big(P_1(k) + 3\alpha(k)\Big)\beta(k)$,

$l_3(k) = \Big(\alpha(k)^2 - \beta(k)^2\Big)P_1(k) + \alpha(k)P_2(k) + P_3(k)$, $l_4(k) = \Big(2\alpha(k)P_1(k) + P_2(k)\Big)\beta(k)$.

解得 $\dfrac{\mathrm{d}\alpha(k)}{\mathrm{d}k}|_{k=k^*} = Re\left[\dfrac{\mathrm{d}\lambda(k)}{\mathrm{d}k}\right]_{k=k^*} = -\left[\dfrac{l_1(k)l_3(k) + l_2(k)l_4(k)}{l_1(k)^2 + l_2(k)^2}\right]_{k=k^*}$

$$= \left[\frac{P_2(k)P_1(k) + P_1(k)P_2(k) - P_3(k)}{2\big(P_1(k)^2 + P_2(k)^2\big)}\right]_{k=k^*} = -\left[\frac{\frac{\mathrm{d}Q(k)}{\mathrm{d}k}}{2\big(P_1(k)^2 + P_2(k)^2\big)}\right]_{k=k^*} \neq 0.$$

3. 自扩散系统 (1.4)的动力学分析

3.1. 持久性和有界性

为研究正不变吸引域的存在性, 系统 (1.4) 解的有界性和持久性使用了以下引理[45].

引理 1

设 $f(s)$ 对 $s \geq 0$ 是正的 C^1 类函数, $d > 0, \eta \geq 0$是常数, 进而, 让 $T \in [0, \infty)$, $\Phi \in C^{2,1}(\Omega \times (T, \infty)) \cap C^{1,0}(\overline{\Omega} \times [T, \infty))$ 是正函数.

(i) 若 Φ 满足

$$\begin{cases} \Phi_t - d\Delta\Phi \leq \Phi^{1+\eta} f(\Phi)(\theta - \Phi), & (x,t) \in \Omega \times (T, \infty) \\ \Phi_v = 0, & (x,t) \in \partial\Omega \times (T, \infty) \end{cases}$$

常数 $\theta > 0$, 则 $\lim\limits_{t\to\infty} \sup \max\limits_{\overline{\Omega}} \Phi(., t) \leq \theta$.

(ii) 若 Φ 满足

$$\begin{cases} \Phi_t - d\Delta\Phi \geq \Phi^{1+\eta} f(\Phi)(\theta - \Phi), & (x,t) \in \Omega \times (T, \infty) \\ \Phi_v = 0, & (x,t) \in \partial\Omega \times (T, \infty) \end{cases}$$

常数 $\theta > 0$, 则 $\lim\limits_{t\to\infty} \inf \min\limits_{\overline{\Omega}} \Phi(., t) \geq \theta$.

定理10 系统 (1.4) 在 $\mathbb{R}_3^+$ 内的所有解是一致有界的并最终进入正不变吸引域 $\sum = [0,1] \times \left[0, \frac{e_1}{b_1\delta_1}\right] \times \left[0, \frac{\varepsilon c e_1 + b_1 e_2 \delta_1}{b_1 b_2 \delta_1 \delta_2}\right]$.

证明 我们必须证明对 $\forall t > 0$, $\big(u(x,0), v(x,0), w(x,0)\big) \in \sum$, $\big(u(x,t), v(x,t), w(x,t)\big) \in \sum$. 直接可以看到 $u(x,t) > 0$, $v(x,t) > 0$, $w(x,t) > 0$, 因此，初值 $u(x,0), v(x,0)$和 $w(x,0)$是非负的, 现从系统 (1.4) 的第一个方程中, 我们得到

$$u_t - \Delta u \leq u(1-u),$$

由引理 1 中的 (i)知,

$$\lim_{t\to\infty} \sup \max_{\overline{\Omega}} u(x,t) \leq 1.$$

即对 $\forall \epsilon > 0$, $\exists t_1 > 0$, 当 $t > t_1$, $(x,t) \in \overline{\Omega} \times [t_1, \infty]$时, 使得 $u(x,t) \leq 1 + \epsilon$.

因此, 在 $(x,t)\in\overline{\Omega}\times[t_1,\infty]$上, v 的方程满足

$$v_t-d_2\Delta v\le v\left[\frac{e_1(1+\epsilon)}{1+a(1+\epsilon)+b_1v}-\delta_1\right]\le\left[\frac{e_1(1+\epsilon)}{b_1}-\delta_1 v\right]=\delta_1\left[\frac{e_1(1+\epsilon)}{b_1\delta_1}-v\right]$$

由引理 1 中的 (i)知,

$$\lim_{t\to\infty}\sup\max_{\overline{\Omega}}v(x,t)\le\frac{e_1}{b_1\delta_1}.$$

即对 $\forall\epsilon_1$, $\exists t_2>t_1$, 当 $t>t_2$, $(x,t)\in\overline{\Omega}\times[t_2,\infty]$时, 使得 $v(x,t)\le\frac{e_1}{b_1\delta_1}+\epsilon_1$.

因此, 在 $(x,t)\in\overline{\Omega}\times[t_2,\infty]$上, w 的方程满足

$$w_t-d_3\Delta w\le w\left[\frac{e_2(1+\epsilon)+c\varepsilon(\frac{e_1}{b_1\delta_1}+\epsilon_1)}{b_2w}-\delta_2\right]\le\delta_2\left\{\frac{\left[e_2(1+\epsilon)+c\varepsilon\left(\frac{e_1}{b_1\delta_1}+\epsilon_1\right)\right]}{b_2\delta_2}-w\right\}$$

由于 ϵ,ϵ_1是任意的, 再应用引理 1的 (i)得,

$$\lim_{t\to\infty}\sup\max_{\overline{\Omega}}w(x,t)\le\frac{e_2+\frac{\varepsilon ce_1}{b_1\delta_1}}{b_2\delta_2}=\frac{\varepsilon ce_1+b_1e_2\delta_1}{b_1b_2\delta_1\delta_2}.$$

定义 对于具有非负初值且 $(u(x,0),v(x,0),w(x,0))\neq0$ 的系统 (1.4), 若 $\exists\sigma_0,\sigma_1,\sigma_2$ 均为正常数, 使得系统 (1.4) 的解 $u(x,t),v(x,t),w(x,t)$满足

$\lim\limits_{t\to\infty}\inf\min\limits_{\overline{\Omega}}(u,t)\ge\sigma_0$, $\lim\limits_{t\to\infty}\inf\min\limits_{\overline{\Omega}}(v,t)\ge\sigma_1$, $\lim\limits_{t\to\infty}\inf\min\limits_{\overline{\Omega}}(w,t)\ge\sigma_2$, 则称系统 (1.4) 是持久的.

定理11 若 $l_u=1-\frac{(b_1+b_2)(b_1\delta_1-ke_1)}{b_1^2b_2\delta_1}>0$, $l_v=\frac{b_1e_1\delta_1\delta_2l_u-(c+b_2\delta_1)[b_1\delta_1(\delta_2+a\delta_2+e_2)+\varepsilon ce_1]}{b_1^2\delta_1\delta_2(c+b_2\delta_1)}>0$, $l_w=\frac{e_2\delta_1l_u+\varepsilon c\delta_1l_v-\delta_1\delta_2(1+a)-e_1\delta_2}{b_2\delta_1\delta_2}>0$, 则系统 (1.4) 是持久的.

因为

$$u_t-d_1\Delta u=u\left(\frac{1-u}{1+kv}-\frac{v+w}{1+au+b_1v+b_2w}\right)\ge u(\frac{1-u}{1+kv}-\frac{1}{b_1}-\frac{1}{b_2})$$

$$\ge u\left[\frac{b_1\delta_1(1-u)}{b_1\delta_1+ke_1}-\frac{b_1+b_2}{b_1b_2}\right]\ge\frac{b_1\delta_1u}{b_1\delta_1+ke_1}\left[1-\frac{(b_1+b_2)(b_1\delta_1+ke_1)}{b_1^2b_2\delta_1}-u\right],$$

且令 $l_u=1-\frac{(b_1+b_2)(b_1\delta_1+ke_1)}{b_1^2b_2\delta_1}>0$, 则由引理 1的 (ii)知,

$$\lim_{t\to\infty}\inf\min_{\overline{\Omega}}(u,t)\ge1-\frac{(b_1+b_2)(b_1\delta_1+ke_1)}{b_1^2b_2\delta_1}$$

即对 $\forall0<\epsilon^{(0)}<l_u$, $\exists t_0$, 当 $t>t_0$, $(x,t)\in\overline{\Omega}\times[t_0,\infty]$时, 使得 $u(x,t)\ge l_u-\epsilon^{(0)}$.

在 $(x,t)\in\overline{\Omega}\times[t_0,\infty]$上, 系统 (1.4) 的第二个关于 v 的方程满足:

$$v_t-d_2\Delta v\ge v\left[\frac{b_1e_1\delta_1\delta_2(l_u-\epsilon^{(0)})}{b_1\delta_1\delta_2(1+a)+(b_1e_2\delta_1+\varepsilon ce_1)+b_1^2\delta_2\delta_2v)}-\frac{c+b_2\delta_1}{b_2}\right]\ge$$

$$\frac{b_1^2\delta_1\delta_2(c+b_2\delta_1)v}{b_2\left[b_1\delta_1\delta_2(1+a)+(b_1e_2\delta_1+\varepsilon ce_1)+b_1^2\delta_1\delta_2v\right]}\left\{\frac{b_1e_1\delta_1\delta_2l_u-(c+b_2\delta_1)\left[b_1\delta_1(\delta_2+a\delta_2+e_2)+\varepsilon ce_1\right]}{b_1^2\delta_1\delta_2(c+b_2\delta_1)}-v\right\}$$

令 $l_v = \frac{b_1e_1\delta_1\delta_2 l_u - (c+b_2\delta_1)[b_1\delta_1(\delta_2+a\delta_2+e_2)+\varepsilon ce_1]}{b_1^2\delta_1\delta_2(c+b_2\delta_1)} > 0$,由引理 1的 (ii)知,

$$\lim_{t\to\infty}\inf\min_{\overline{\Omega}}(v,t) \geq \frac{b_1e_1\delta_1\delta_2 l_u - (c+b_2\delta_1)\left[b_1\delta_1(\delta_2+a\delta_2+e_2)+\varepsilon ce_1\right]}{b_1^2\delta_1\delta_2(c+b_2\delta_1)}$$

即对 $\forall\epsilon^{(1)} > 0$, $\exists t_0^{(1)} > t_0$, 当 $t > t_0^{(1)}$, $(x,t) \in \overline{\Omega} \times \left[t_0^{(1)}, \infty\right]$ 时, 使得 $v(x,t) \geq l_v - \epsilon^{(1)}$.

类似地, 系统 (1.4) 的第三个关于 w方程满足:

$$w_t - d_3\Delta w \geq w\left[\frac{e_2\delta_1(l_u-\epsilon^{(0)})+\varepsilon c\delta_1(l_v-\epsilon^{(1)})}{\delta_1(1+a)+e_1+b_2\delta_1 w} - \delta_2\right]$$

$$\geq \frac{b_2\delta_1\delta_2 w}{\delta_1(1+a)+e_1+b_2\delta_1 w}\left[\frac{e_2\delta_1(l_u-\epsilon^{(0)})+\varepsilon c\delta_1(l_v-\epsilon^{(1)})-\delta_1\delta_2(1+a)-e_1\delta_2}{b_2\delta_1\delta_2} - w\right]$$

令 $l_w = \frac{e_2\delta_1 l_u+\varepsilon c\delta_1 l_v-\delta_1\delta_2(1+a)-e_1\delta_2}{b_2\delta_1\delta_2} > 0$, 由引理 1的 (ii)知,

$$\lim_{t\to\infty}\inf\min_{\overline{\Omega}}(w,t) \geq \frac{e_2\delta_1 l_u+\varepsilon c\delta_1 l_v-\delta_1\delta_2(1+a)-e_1\delta_2}{b_2\delta_1\delta_2}$$

证毕.

3.2. 局部稳定性

这一部分, 讨论了系统 (1.4) 正常数平衡态的稳定性, 很容易看到系统 (1.4) 的正常数平衡点与系统 (1.3) 的一致. 现表示 $\mathbf{E} = \left(u(x,t), v(x,t), w(x,t)\right)^\top$, $\mathbf{F}(\mathbf{E}) = \left(f_1(\mathbf{E}), f_2(\mathbf{E}), f_3(\mathbf{E})\right)^\top$, $\mathbf{D} = \mathrm{diag}(d_1,d_2,d_3)$, 系统 (1.4)在 $\mathbf{E}^*$ 处的线性化形式如下:

$$\begin{cases} \mathbf{E}_t = L\mathbf{E}, & x\in\Omega, \\ \mathbf{n}\cdot\nabla\mathbf{E} = 0, & x\in\partial\Omega. \end{cases}$$

而 $L = \mathbf{D}\Delta + \mathbf{J}_{\mathbf{E}^*}$.

设 $0 = \mu_0 < \mu_1 < \mu_2 < ... < \mu_n < ...$, 是 Ω 上具有齐次 Neumann 边界条件的 $-\Delta$ 算子的特征值, $E(\mu_i)$ 为 $H^1(\Omega)$上 $\mu_i(i = 0,1,2,\ldots)$ 对应的特征空间, X 为 $[H^1(\Omega)]$ 上的 $\left[C^1(\bar{\Omega})\right]^3$ 的闭包, $X_{ij} = \{\mathbf{c}\phi_{ij}|\mathbf{c}\in R^3\}$, $\{\phi_{ij} : j = 1,2,3,\ldots, dimE(\mu_i)\}$ 是 X_i 的一组标准正交基,则

$$X = \bigoplus_{i=1}^{\infty} X_i, \qquad X_i = \bigoplus_{j=1}^{\dim[E(\mu_i)]} X_{ij}.$$

对每一个 $i \geq 1$, X_i 在算子 L 下是不变的, 且对某个 $i \geq 1$来说, λ 是算子 L 的特征值当且仅当它是矩阵 $-\mu_i\mathbf{D} + \mathbf{J}_{\mathbf{E}^*}$ 的特征值时, 该特征值并对应 X_i 中一个特征向量. $-\mu_i\mathbf{D} + \mathbf{J}_{\mathbf{E}^*}$ 的特征方程为

$$\Psi_i(\lambda) = \lambda^3 + P_{1i}\lambda^2 + P_{2i}\lambda + P_{3i} = 0 \tag{15}$$

而

$$P_{1i} = (d_1 + d_2 + d_3)\mu_i + P_1$$

$$P_{2i} = (d_1d_2 + d_1d_3 + d_2d_3)\mu_i^2 - [c_{11}(d_2 + d_3) + c_{22}(d_1 + d_3) + c_{33}(d_1 + d_2)]\,\mu_i + P_2$$

$$\begin{aligned} P_{3i} &= d_1d_2d_3\mu_i^3 - (d_1d_2c_{33} + d_1d_3c_{22} + d_2d_3c_{11})\mu_i^2 \\ &\quad + [d_1(c_{22}c_{33} - c_{23}c_{32}) + d_2(c_{11}c_{33} - c_{13}c_{31}) + d_3(c_{11}c_{22} - c_{12}c_{21})]\,\mu_i + P_3 \\ &= q_1\mu_i^3 + q_2\mu_i^2 + q_3\mu_i + q_4. \end{aligned}$$

$$Q_i = P_{1i}P_{2i} - P_{3i} = m_1\mu_i^3 + m_2\mu_i^2 + m_3\mu_i + m_4$$

而

$$m_1 = (d_1 + d_2)(d_1 + d_3)(d_2 + d_3) - d_1d_2d_3 > 0$$

$$m_2 = -2(d_1d_2 + d_1d_3 + d_2d_3)(c_{11} + c_{22} + c_{33}) - d_1^2(c_{22} + c_{33}) - d_2^2(c_{11} + c_{33}) - d_3^2(c_{11} + c_{22})$$

$$\begin{aligned} m_3 &= d_1\left[2(c_{11}c_{22} + c_{11}c_{33} + c_{22}c_{33}) - c_{12}c_{21} - c_{13}c_{31} + c_{22}^2 + c_{33}^2\right] \\ &\quad + d_2\left[2(c_{11}c_{22} + c_{11}c_{33} + c_{22}c_{33}) - c_{12}c_{21} - c_{13}c_{31} + c_{11}^2 + c_{33}^2\right] \\ &\quad + d_3\left[2(c_{11}c_{22} + c_{11}c_{33} + c_{22}c_{33}) - c_{23}c_{32} - c_{13}c_{31} + c_{11}^2 + c_{22}^2\right] \end{aligned}$$

$$m_4 = P_1P_2 - P_3.$$

由 (10) 中的符号可知 m_2, m_3, m_4 均大于零, 有 $Q_i > 0$, 因此, $P_{1i}, P_{2i}, P_{3i} > 0$, 由 Routh-Hurwitz 准则可知方程 (15) 的三个根都具有负实部, 对每个 $i \geq 1$.

令 $\lambda = \mu_i\zeta$, 则

$$\Psi_i(\lambda) = \mu_i^3\zeta^3 + P_{1i}\mu_i^2\zeta^2 + P_{2i}\mu_i\zeta + P_{3i} := \widetilde{\Psi}_i(\zeta)$$

让 $\mu_i \longrightarrow \infty$, 当 $i \longrightarrow \infty$ 时,

$$\lim_{t\to\infty} \frac{\widetilde{\Psi}_i(\zeta)}{\mu_i^3} = \zeta^3 + (d_1 + d_2 + d_3)\zeta^2 + (d_1d_2 + d_1d_3 + d_2d_3)\zeta + d_1d_2d_3 := \widetilde{\Psi}(\zeta)$$

显然, $\widetilde{\Psi}(\zeta) = 0$有三个根 $-d_1$, $-d_2$, $-d_3$, 由连续性知 $\exists i_0$, 使得 $\widetilde{\Psi}_i(\zeta) = 0$ 的三个根 $\zeta_{1i}, \zeta_{2i}, \zeta_{3i}$ 满足 $\mathbf{Re}\{\zeta_{1i}\}, \mathbf{Re}\{\zeta_{2i}\}, \mathbf{Re}\{\zeta_{3i}\} \leq -\frac{\overline{d}}{2}$, $\forall i \geq i_0$, 这里 $\overline{d} = \min\{d_1, d_2, d_3\}$, 则 $\mathbf{Re}\{\lambda_{1i}\}, \mathbf{Re}\{\lambda_{2i}\}, \mathbf{Re}\{\lambda_{3i}\} \leq -\mu_i\frac{\overline{d}}{2} \leq -\frac{\overline{d}}{2}$, $\forall i \geq i_0$.令

$$-\widetilde{d} = \max\left\{\mathbf{Re}\{\lambda_{1i}\}, \mathbf{Re}\{\lambda_{2i}\}, \mathbf{Re}\{\lambda_{3i}\} \leq -\mu_i\frac{\overline{d}}{2} \leq -\frac{\overline{d}}{2}\right\}$$

则 $\widetilde{d}>0$, 且 $\mathbf{Re}\{\lambda_{1i}\}, \mathbf{Re}\{\lambda_{2i}\}$, $\mathbf{Re}\{\lambda_{3i}\} \leq -\mu_i\frac{\overline{d}}{2} \leq -\frac{\overline{d}}{2} \leq -d = \min\left\{\widetilde{d}, \frac{\overline{d}}{2}\right\} < 0,\ \forall i \geq 1$ 由 L 算子的特征值组成的谱位于半平面 $\mathbf{Re}\{\lambda \leq -d\}$内. 因此, 由 Dan Henry [46]的定理 5.1.1, 我们有以下 $\mathbf{E}^*$ 的稳定性结果.

定理12 若条件 $(\mathbf{H_2})$- $(\mathbf{H_6})$ 成立, 则自扩散系统 (1.4) 的正常数平衡点 $\mathbf{E}^*$ 是局部渐近稳定的.

注: 上述定理表明自扩散系统可能不会改变系统的稳定性, 也就是说，自扩散不能产生 Turing 不稳定性.

4. 结论

本文为了解释 IG 食饵和 IG 捕食者的搜索效率对所涉及三个物种, 即受恐惧干扰的共享资源, IG食饵, IG捕食者的依赖性, 考虑了在齐次 Newman 边界条件下具有恐惧效应和 Bedington-DeAngelis 功能反应的时空共位群内捕食模型. 对于这个 IGP模型，我们导出了所有可行平衡点的存在性和(局部/全局)稳定性的条件, 包括平凡平衡点、边界平衡点和正常数平衡点. 此外, 还得到了以恐惧因子为参数的 Hopf 分支的条件. 通过研究发现, 时间系统 (1.3) 的可行平衡点也是时空系统 (1.4) 的可行平衡点. 另外, 我们从定理 6 和 9 可以看到, 无论扩散是否存在, 正常数平衡点的局部稳定性的条件并没有受到扩散的影响.

定理 10 时空系统 (1.4) 一致持久性的存在也预示着受到恐惧影响的共享资源, IG 食饵, IG 捕食者三物种可以共存.

本文进行了严格的理论推理论证, 从理论上来说具有可行性和时效性, 但不足之处是缺少具体的数值分析.

基金项目

国家自然科学基金(Nos.11761063)。

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