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Fuzzifying拓扑中的θ-半分离定理

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摘 要

本文首先引入不分明化拓扑空间中$T_0^{S\theta},T_1^{S\theta},T_2^{S\theta},R_S^{\theta},N_1^{S\theta},R_0^{S\theta},R_1^{S\theta}$分离公理的定义，再利用Fuzzifying拓扑空间理论和连续值逻辑语义方法进行研究，得到不分明化θ-半分离相关定理。

关键词

半分离公理，不分明化半$\theta-R_0$分离性，不分明化拓扑空间

θ-Semiseparation Axioms in Fuzzifying Topology

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Abstract

We introduce the definitions of $T_0^{S\theta},T_1^{S\theta},T_2^{S\theta},R_S^{\theta},N_1^{S\theta},R_0^{S\theta},R_1^{S\theta}$ separation axioms in fuzzifying topology space, the fuzzy topological space theory and logical semantics of continuous values are used to prove main results, and fuzzifying θ-semiseparation axioms are obtained.

Keywords

Semiseparation Axioms, Fuzzifying Semi $\theta-R_0$ Separation Axioms, Fuzzifying Topology

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1. 引言

1968 年C.L. Chang提出不分明拓扑空间的概念，此后不分明拓扑学得到了迅速的发展，而且对问题的分析讨论也在逐步深化，各个不同方向的研究都得出了一些比较深刻的结果。应明生教授[1] [2] [3]提出了不分明化拓扑的概念，并从不同的角度发展了不分明集框架下的拓扑学。1982 年，Dorsett C提出一般拓扑空间中的半 T_0,T_1,T_2,R_0,R_1 分离定理。1984 年，胡庆平提出一般拓扑空间中 S_3,S_4 分离定理。此后，张广济和F.H. Khedr提出Fuzzifying拓扑空间中的半 T_0,T_1,T_2,T_3,T_4 分离定理和半 R_0,R_1 分离定理。同时Alkazragy A和Caldas M在一般拓扑中提出 $\theta-semiT_0$， $\theta-semiT_1$， $\theta-semiT_2$， $\theta-semiR$， $\theta-semiN$， $\theta-semiR_0$， $\theta-semiR_1$ 分离定理，并展开相关研究。于是在前人基础上，如何将一般拓扑空间中 θ-半分离定理推广到不分明化拓扑空间中得到不分明化 θ-半分离定理，这对于丰富不分明化拓扑空间理论是重要的。

本文在前人工作的基础上在 Fuzzifying 拓扑空间中引入 θ-半分离定理，得到 Fuzzifying θ-半分离定理的一些好的性质和结论。

2. 预备知识

定理 1 [4]设 $(X,\mathscr{T})$ 是不分明化拓扑空间， $\forall A\in\mathscr{P}(X)$， $\vDash A\in N_x^{\theta}\to A\in N_x$。

定义 1 [5] A 的半闭包 $\underline{A}$ 定义为： $x\in\underline{A}:=\forall B\big((B\supseteq A)\wedge(B\in\mathscr{F}_S)\to x\in B\big)$。

定义 2 [5]设 Ω 是 Fuzzifying 拓扑空间类，一元模糊谓词 $S_i\in\mathscr{F}(\Omega),i=0,1,2$ 被称为是 S_i 分离的，以下为一些等价定理：

$$\vDash S_0(X,\mathscr{T})\leftrightarrow(\forall x)(\forall y)\Big((x\neq y)\to\neg\big(x\in\underline{\{y\}}\big)\vee\neg\big(y\in\underline{\{x\}}\big)\Big)$$

$$\vDash S_1(X,\mathscr{T})\leftrightarrow(\forall x)\big(\{x\}\in\mathscr{F}_S\big)$$

$$\vDash S_2(X,\mathscr{T})\leftrightarrow(\forall x)(\forall y)\Big((x\neq y)\to\exists B\big(B\in \mathrm{B}_x^S\big)\wedge\big(y\notin\underline{B}\big)\Big)$$

定义 3 [6] A 的 θ-半闭包定义为对所有满足 $x\in X$ 且 $\underline{U}^S\cap A\neq\varnothing$ 的集合，表示为 $\underline{A}^{S\theta}$， $\underline{U}^S$ 为 U 的半闭包。

定义 4 [7]当存在 X 的一个 θ-开集 U 满足 $U\subseteq A\subseteq Cl(U)$，则子集 A 称为 θ-半开集。其中 $Cl(U)$ 为 U 的闭包。

定义 5 [8]设 $(X,\mathscr{T})$ 是一个拓扑空间， $x\in X$， $M\subset X$，则 M 称为 x 的 θ-半邻域当存在一个包含 x 的 θ-半开集 A 满足 $x\in A\subset M$。

定义 6 [9]设 $(X,\mathscr{T})$ 是一个 Fuzzifying 拓扑空间，则一元模糊谓词 $\mathscr{T}_S\in\mathscr{F}(\mathscr{P}(X))$ 称为 Fuzzifying 半开集，若 $A\in\mathscr{T}_S:=(\exists B)\big((B\in\mathscr{T})\wedge(B\subseteq A)\wedge(\forall x)(x\in A\to x\in\bar{B})\big)$。

定义 7 [9]设 $x\in X$， $N_x^S\in\mathscr{F}(\mathscr{P}(X))$ 表示 x 的半邻域系，定义为：

$$A\in N_x^S:=\exists B\big((B\in\mathscr{T}_S)\wedge(x\in B\subseteq A)\big)$$

定理 2 [10] (1) $A\subseteq\underline{A}$；(2) $A\equiv\underline{A}\leftrightarrow A\in\mathscr{F}_S$；(3) $\vDash(A\subseteq B)\to\big(A\in N_x^S\to B\in N_x^S\big)$。

3. 主要结果及其证明

首先，为了方便书写，下面给出一些简记记号：

$$SK_{x,y}^{\theta} := (\exists A)\left(\left(A \in N_x^{\theta S}\right) \wedge (y \notin A) \vee \left(\left(A \in N_y^{\theta S}\right) \wedge (x \notin A)\right)\right)$$

$$SH_{x,y}^{\theta} = (\exists A)(\exists B)\left(\left(A \in N_x^{\theta S}\right) \wedge (y \notin A) \wedge \left(B \in N_y^{\theta S}\right) \wedge (x \notin B)\right)$$

$$SM_{x,y}^{\theta} := (\exists A)(\exists B)\left(\left(A \in N_x^{\theta S}\right) \wedge \left(B \in N_y^{\theta S}\right) \wedge (A \cap B = \varnothing)\right)$$

定义 1 设$(X,\mathscr{T})$是一个 Fuzzifying 拓扑空间，则称一元不分明谓词$\mathscr{T}_S^{\theta} \in \mathscr{F}\left(\mathscr{P}(X)\right)$为 Fuzzifying θ-半开集，若$A \in \mathscr{T}_S^{\theta} := (\exists B)\left(\left(B \in \mathscr{T}_{\theta}\right) \wedge (B \subseteq A) \wedge (\forall x)\left(x \in A \to x \in \overline{B}\right)\right)$。

定义 2 设$x \in X$，$N_x^{\theta S} \in \mathscr{F}\left(\mathscr{P}(X)\right)$表示$x$的 Fuzzifying θ-半邻域系，定义为：

$$A \in N_x^{\theta S} := \exists B\left(\left(B \in N_x^S\right) \otimes \left(\underline{B} \subseteq A\right)\right)$$

定理 1 对$\forall A \in \mathscr{P}(X)$，$\vDash \left(A \in \mathscr{T}_S^{\theta}\right) \leftrightarrow (\forall x)\left(x \in A \to A \in N_x^{\theta S}\right)$。

定义 3 设Ω是 Fuzzifying 拓扑空间类，分别称一元模糊谓词$T_i^{S\theta} \in \mathscr{F}(\Omega), i = 0,1,2$为是 Fuzzifying $T_i^{S\theta}$分离的，定义为：

$$T_0^{S\theta}(X,\mathscr{T}) := (\forall x)(\forall y)\left((x \neq y) \to SK_{x,y}^{\theta}\right)$$

$$T_1^{S\theta}(X,\mathscr{T}) := (\forall x)(\forall y)\left((x \neq y) \to SH_{x,y}^{\theta}\right)$$

$$T_2^{S\theta}(X,\mathscr{T}) := (\forall x)(\forall y)\left((x \neq y) \to SM_{x,y}^{\theta}\right)$$

定义 4 $\forall A \in X$，Fuzzifying 半θ-闭包定义为：

$$x \in Cl_S^{\theta}(A) := (\forall B)\left(\left(B \in N_x^S\right) \to \neg\left(A \cap \underline{B} \equiv \varnothing\right)\right)$$

定理 2 $\vDash T_0^{S\theta}(X,\mathscr{T}) \leftrightarrow (\forall x)(\forall y)\left((x \neq y) \to \neg\left(x \in Cl_S^{\theta}\{y\}\right) \vee \neg\left(y \in Cl_S^{\theta}\{x\}\right)\right)$

证明

$$\begin{aligned}
\left[T_0^{S\theta}(X,\mathscr{T})\right] &= \inf_{x \neq y} \max\left(\sup_{y \notin A} N_x^{\theta S}(A), \sup_{x \notin A} N_y^{\theta S}(A)\right) \\
&= \inf_{x \neq y} \max\left(N_x^{\theta S}\left(X \sim \{y\}\right), N_y^{\theta S}\left(Y \sim \{x\}\right)\right) \\
&= \inf_{x \neq y} \max\left(\left(1 - Cl_S^{\theta}\{y\}\right)(x), \left(1 - Cl_S^{\theta}\{x\}\right)(y)\right) \\
&= \left[(\forall x)(\forall y)\left((x \neq y) \to \neg\left(x \in Cl_S^{\theta}\{y\}\right) \vee \neg\left(y \in Cl_S^{\theta}\{x\}\right)\right)\right]
\end{aligned}$$

定理 3 $\vDash x \in \underline{A} \to x \in Cl_S^{\theta}(A)$

证明

$$\begin{aligned}
\underline{A}(x) &= \inf_{B \in \mathscr{P}(X)} \min\left(1, 1 - N_x^S(B) + \sup_{y \in A} B(y)\right) \\
&\leq \inf_{B \in \mathscr{P}(X)} \min\left(1, 1 - N_x^S(B) + \sup_{y \in A} \underline{B}(y)\right) = Cl_S^{\theta}(A)(x)
\end{aligned}$$

定理 4 $\vDash T_0^{S\theta}(X,\mathscr{T}) \to S_0(X,\mathscr{T})$

证明 $\left[T_0^{S\theta}(X,\mathscr{T})\right] = \inf_{x \neq y} \max\left(\left(1 - Cl_S^{\theta}\{y\}\right)(x), \left(1 - Cl_S^{\theta}\{x\}\right)(y)\right)$

$\left[S_0\left(X,\mathscr{T}\right)\right]=\inf_{x\neq y}\max\left(\left(1-\underline{\{y\}}\right)(x),\left(1-\underline{\{x\}}\right)(y)\right)$，则 $\vDash T_0^{S\theta}\left(X,\mathscr{T}\right)\rightarrow S_0\left(X,\mathscr{T}\right)$

定理 5 $\vDash T_1^{S\theta}\left(X,\mathscr{T}\right)\leftrightarrow(\forall x)\left(\{x\}\in\mathcal{F}_S^\theta\right)$

证明 对任意 $x_1\neq x_2$，

$$\begin{aligned}\left[(\forall x)\left(\{x\}\in\mathcal{F}_S^\theta\right)\right]&=\inf_{x\in X}\mathscr{T}_S^\theta\left(X\sim\{x\}\right)=\inf_{x\in X}\inf_{y\in X\sim\{x\}}N_y^{\theta S}\left(X\sim\{x\}\right)\\&\leq\inf_{x\in X\sim\{x_2\}}N_y^{\theta S}\left(X\sim\{x_2\}\right)\leq N_{x_1}^{\theta S}\left(X\sim\{x_2\}\right)\\&=\sup_{x_2\notin A}N_{x_1}^{\theta S}(A)\end{aligned}$$

同理 $\left[(\forall x)\left(\{x\}\in\mathcal{F}_S^\theta\right)\right]\leq N_{x_2}^{\theta S}\left(X\sim\{x_1\}\right)=\sup_{x_1\notin B}N_{x_2}^{\theta S}(B)$

则

$$\begin{aligned}\left[(\forall x)\left(\{x\}\in\mathcal{F}_S^\theta\right)\right]&\leq\inf_{x_1\neq x_2}\min\left(\sup_{x_2\notin A}N_{x_1}^{\theta S}(A),\sup_{x_1\notin B}N_{x_2}^{\theta S}(B)\right)\\&=\inf_{x\neq y}\min\left(\sup_{y\notin A}N_x^{\theta S}(A),\sup_{x\notin B}N_y^{\theta S}(B)\right)=\left[T_1^{S\theta}\left(X,\mathscr{T}\right)\right]\end{aligned}$$

反过来

$$\begin{aligned}\left[T_1^{S\theta}\left(X,\mathscr{T}\right)\right]&=\inf_{x_1\neq x_2}\min\left(\sup_{x_2\notin A}N_{x_1}^{\theta S}(A),\sup_{x_1\notin B}N_{x_2}^{\theta S}(B)\right)\\&=\inf_{x_1\neq x_2}\min\left(N_{x_1}^{\theta S}\left(X\sim\{x_2\}\right),N_{x_2}^{\theta S}\left(X\sim\{x_1\}\right)\right)\\&\leq\inf_{x_1\neq x_2}\left(N_{x_1}^{\theta S}\left(X\sim\{x_2\}\right)\right)=\inf_{x_2\in x_1}\inf_{x_1\in X\sim\{x_2\}}\left(N_{x_1}^{\theta S}\left(X\sim\{x_2\}\right)\right)\\&=\inf_{x_2\in X}\mathscr{T}_S^\theta\left(X\sim\{x_2\}\right)\end{aligned}$$

所以，$\vDash T_1^{S\theta}\left(X,\mathscr{T}\right)\leftrightarrow(\forall x)\left(\{x\}\in\mathcal{F}_S^\theta\right)$。

定义 5 设 Σ 是一类不分明化拓扑空间，一元模糊谓词 $\mathcal{F}_S^\theta\left(\mathscr{T}_S^\theta\right)\in\mathscr{F}\left(\mathscr{P}(X)\right)$ 称为 Fuzzifying θ-半闭集，定义为：$A\in\mathcal{F}_S^\theta:=A\equiv Cl_S^\theta(A)$，即 $\mathcal{F}_S^\theta(A)=\int_{\mathscr{P}(X)}\inf_{x\in X\sim A}\left(1-Cl_S^\theta(A)(x)\right)/A$。

定理 6 $\vDash A\in\mathcal{F}_S^\theta\rightarrow A\in\mathcal{F}$。

证明 由定义 5 和定理 3 易证。

定理 7 $\vDash T_1^{S\theta}\left(X,\mathscr{T}\right)\rightarrow S_1\left(X,\mathscr{T}\right)$

证明 $\left[T_1^{S\theta}\left(X,\mathscr{T}\right)\right]=\inf_x\mathcal{F}_S^\theta\left(\{x\}\right)\leq\inf_x\mathcal{F}_S\left(\{x\}\right)=\left[S_1\left(X,\mathscr{T}\right)\right]$。

定理 8 (1) $\vDash T_1^{S\theta}\left(X,\mathscr{T}\right)\rightarrow T_0^{S\theta}\left(X,\mathscr{T}\right)$；(2) $\vDash T_2^{S\theta}\left(X,\mathscr{T}\right)\rightarrow T_1^{S\theta}\left(X,\mathscr{T}\right)$

证明 (1) (2)由定义显然得证。

定义 6 设 Ω 是不分明化拓扑空间类，一元模糊谓词 $R_S^\theta\in\mathscr{F}(\Omega)$ 称为 Fuzzifying 半 $\theta-R$ 分离的，定义为：

$$R_S^\theta\left(X,\mathscr{T}\right):=(\forall x)(\forall D)\left(\left(D\in\mathcal{F}_S^\theta\right)\wedge(x\notin D)\rightarrow(\exists A)\left(\left(A\in N_x^{\theta S}\right)\wedge\left(Cl_S^\theta(A)\cap D=\varnothing\right)\right)\right)$$

定理 9 $\vDash R_S^\theta\left(X,\mathscr{T}\right)\leftrightarrow(\forall x)(\forall A)\left(\left(A\in\mathscr{T}_S^\theta\right)\wedge(x\in A)\rightarrow\exists B\left(\left(B\in N_x^{S\theta}\right)\wedge\left(Cl_S^\theta(B)\subseteq A\right)\right)\right)$

证明

$$\left[R_S^\theta(X,\mathscr{T})\right]=\inf_{x\notin D}\min\left(1,1-\mathcal{F}_S^\theta(D)+\sup_{B\in\mathscr{P}(X)}\min\left(N_x^{\theta S}(B),\inf_{y\in D}N_y^{\theta S}(B^c)\right)\right)$$

$$=\inf_{x\in A}\min\left(1,1-\mathscr{T}_S^\theta(A)+\sup_{B\in\mathscr{P}(X)}\min\left(N_x^{\theta S}(B),\inf_{y\in A^c}N_y^{\theta S}(B^c)\right)\right)$$

$$=\left[(\forall x)(\forall A)\left(\left(A\in\mathscr{T}_S^\theta\right)\wedge(x\in A)\to\exists B\left(\left(B\in N_x^{S\theta}\right)\wedge\left(Cl_S^\theta(B)\subseteq A\right)\right)\right)\right]$$

定理 10 $\vDash R_S^\theta(X,\mathscr{T})\dot{\wedge}T_1^{S\theta}(X,\mathscr{T})\to T_2^{S\theta}(X,\mathscr{T})$

证明

$$\left[R_S^\theta(X,\mathscr{T})+T_1^{S\theta}(X,\mathscr{T})\right]$$

$$=\inf_{x\notin A^c}\min\left(1,1-\mathscr{T}_S^\theta(A^c)+\sup_{B\in\mathscr{P}(X)}\min\left(N_x^{\theta S}(B),\inf_{y\in A^c}N_y^{\theta S}(B^c)\right)\right)+\inf_{z\in X}\mathscr{T}_S^\theta\left(\{z\}^c\right)$$

$$\le\inf_{x\in X,y\neq x}\inf_{y\in X}\min\left(1,1-\mathscr{T}_S^\theta\left(\{y\}^c\right)+\sup_{B\in\mathscr{P}(X)}\min\left(N_x^{\theta S}(B),N_y^{\theta S}(B^c)\right)\right)+\mathscr{T}_S^\theta\left(\{y\}^c\right)$$

$$\le\inf_{y\neq x}\left(1,1+\sup_{B\in\mathscr{P}(X)}\min\left(N_x^{\theta S}(B),N_y^{\theta S}(B^c)\right)\right)\le\inf_{y\neq x}\sup_{B\cap C\neq\varnothing}\min\left(N_x^{\theta S}(B),N_y^{\theta S}(C)\right)+1$$

$$=\left[T_2^{S\theta}(X,\mathscr{T})\right]+1$$

所以，$\left[T_2^{S\theta}(X,\mathscr{T})\right]\ge\left[R_S^\theta(X,\mathscr{T})\right]+\left[T_1^{S\theta}(X,\mathscr{T})\right]-1$。

定义 7 设Ω是不分明化拓扑空间类，那么称一元模糊谓词 $R_S^\theta\in\mathscr{F}(\Omega)$为 Fuzzifying 半$\theta-N$分离的，定义为：

$$N_S^\theta(X,\mathscr{T}):=(\forall A)(\forall B)\left(\left(A\in\mathcal{F}_S^\theta\right)\wedge\left(B\in\mathcal{F}_S^\theta\right)\wedge(A\cap B=\varnothing)\to(\exists G)\left(\left(G\in\mathscr{T}_S^\theta\right)\wedge(A\subseteq G)\wedge\left(Cl_S^\theta(G)\cap B=\varnothing\right)\right)\right)$$

定理 11 设$(X,\mathscr{T})$是 Fuzzifying 空间，则

$$N_S^\theta(X,\mathscr{T})\leftrightarrow(\forall A)(\forall B)\left(\left(A\in\mathcal{F}_S^\theta\right)\wedge\left(B\in\mathscr{T}_S^\theta\right)\wedge(A\subseteq B)\to(\exists G)\left(\left(G\in\mathscr{T}_S^\theta\right)\wedge(A\subseteq G)\wedge\left(Cl_S^\theta(G)\subseteq B\right)\right)\right)$$

证明

$$\left[N_S^\theta(X,\mathscr{T})\right]$$

$$=\inf_{A\cap B^c=\varnothing}\min\left(1,1-\mathcal{F}_S^\theta(A)\wedge\mathcal{F}_S^\theta(B^c)+\sup_{A\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{x\in B^c}N_x^{\theta S}(G^c)\right)\right)$$

$$=\inf_{A\subseteq B}\min\left(1,1-\mathcal{F}_S^\theta(A)\wedge\mathscr{T}_S^\theta(B)+\sup_{A\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{x\notin B}N_x^{\theta S}(G^c)\right)\right)$$

$$=\inf_{A\subseteq B}\min\left(1,1-\mathcal{F}_S^\theta(A)\wedge\mathscr{T}_S^\theta(B)+\sup_{A\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{x\notin B}\left(1-Cl_S^\theta(G)(x)\right)\right)\right)$$

$$=\left[(\forall A)(\forall B)\left(\left(A\in\mathcal{F}_S^\theta\right)\wedge\left(B\in\mathscr{T}_S^\theta\right)\wedge(A\subseteq B)\to\exists G\left(\left(G\in\mathscr{T}_S^\theta\right)\wedge(A\subseteq G)\wedge\left(Cl_S^\theta(G)\subseteq B\right)\right)\right)\right]$$

定理 12 $\vDash N_S^\theta(X,\mathscr{T})\dot{\wedge}T_1^{S\theta}(X,\mathscr{T})\to R_S^\theta(X,\mathscr{T})$

证明

$$\left[N_S^\theta(X,\mathscr{T})\right]+\left[T_1^{S\theta}(X,\mathscr{T})\right]$$
$$=\inf_{A\subseteq D^c}\min\left(1,1-\mathscr{T}_S^\theta(A^c)\wedge\mathscr{T}_S^\theta(D^c)+\sup_{A\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)\right)+\inf_{z\in X}\mathscr{T}_S^\theta\left(\{z\}^c\right)$$
$$\le\inf_{x\notin B}\min\left(1,1-\min\left(\mathscr{T}_S^\theta\left(\{x\}^c\right),\mathscr{T}_S^\theta(D^c)\right)+\sup_{x\in G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)\right)+\inf_{z\in X}\mathscr{T}_S^\theta\left(\{z\}^c\right)$$
$$=\inf_{x\notin B}\min\left(1,1-\max\left(1-\mathscr{T}_S^\theta\left(\{x\}^c\right)\right)+\sup_{x\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right),\right.$$
$$\left.1-\mathscr{T}_S^\theta(D^c)+\sup_{x\in G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)\right)+\inf_{z\in X}\mathscr{T}_S^\theta\left(\{z\}^c\right)$$
$$\le\inf_{x\notin B}\max\left(\min\left(1,1-\left(\mathscr{T}_S^\theta\left(\{x\}^c\right)\right)\right)+\sup_{x\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)+\inf_{z\in X}\mathscr{T}_S^\theta\left(\{z\}^c\right),\right.$$
$$\left.\min\left(1,1-\mathscr{T}_S^\theta(D^c)\right)+\sup_{x\in G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)+\inf_{z\in X}\mathscr{T}_S^\theta\left(\{z\}^c\right)\right)$$
$$\le\inf_{x\notin B}\max\left(1+\sup_{x\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right),\min\left(1,1-\mathscr{T}_S^\theta(D^c)\right)+\sup_{x\in G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)+\inf_{z\in X}\mathscr{T}_S^\theta\left(\{z\}^c\right)\right)$$
$$\le\inf_{x\notin B}\max\left(1+\sup_{x\subseteq G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right),\min\left(1,1-\mathscr{T}_S^\theta(D^c)\right)+\sup_{x\in G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)+1\right)$$
$$\le\inf_{x\notin B}\left(\min\left(1,1-\mathscr{T}_S^\theta(D^c)\right)+\sup_{x\in G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)+1\right)$$
$$\le\inf_{x\notin B}\left(\min\left(1,1-\mathscr{F}_S^\theta(D)\right)+\sup_{x\in G}\min\left(\mathscr{T}_S^\theta(G),\inf_{y\in D}N_y^{\theta S}(G^c)\right)+1\right)=\left[R_S^\theta(X,\mathscr{T})\right]+1$$

定义 8 设Ω是不分明化拓扑空间类，那么称一元模糊谓词 $R_0^{S\theta}\in\mathscr{F}(\Omega)$ 为 Fuzzifying $\theta-R_0$ 分离的，定义为：$R_0^{S\theta}(X,\mathscr{T}):=(\forall x)(\forall y)\left((x\ne y)\to\left(SK_{x,y}^\theta\to SH_{x,y}^\theta\right)\right)$。

定义 9 设Ω是不分明化拓扑空间类，一元模糊谓词 $R_1^{S\theta}\in\mathscr{F}(\Omega)$ 称为 Fuzzifying 半 $\theta-R_1$ 分离的，定义为：$R_1^{S\theta}(X,\mathscr{T}):=(\forall x)(\forall y)\left((x\ne y)\to\left(SK_{x,y}^\theta\to SM_{x,y}^\theta\right)\right)$。

定理 13 $\vDash R_1^{S\theta}(X,\mathscr{T})\to R_0^{S\theta}(X,\mathscr{T})$

证明

$$\left[R_0^{S\theta}(X,\mathscr{T})\right]$$
$$=\inf_{x\ne y}\min\left(1,1-\max\left(\left(1-Cl_S^\theta\{y\}\right)(x),\left(1-Cl_S^\theta\{x\}\right)(y)\right)+\inf_x\mathscr{F}_S^\theta\left(\{x\}\right)\right)$$
$$\ge\inf_{x\ne y}\min\left(1,1-\max\left(\left(1-Cl_S^\theta\{y\}\right)(x),\left(1-Cl_S^\theta\{x\}\right)(y)\right)+\inf_{x\ne y}\min\left(\sup_{y\notin A}N_x^{\theta S}(A),\sup_{x\notin B}N_y^{\theta S}(B)\right)\right)$$
$$\ge\inf_{x\ne y}\left(1,1-\max\left(\left(1-Cl_S^\theta\{y\}\right)(x),\left(1-Cl_S^\theta\{x\}\right)(y)\right)+\inf_{x\ne y}\sup_{A\cap B\ne\varnothing}\min\left(N_x^{\theta S}(A),N_x^{\theta S}(B)\right)\right)=\left[R_1^{S\theta}(X,\mathscr{T})\right]$$

定理 14 $\vDash T_1^{S\theta}(X,\mathscr{T})\to R_0^{S\theta}(X,\mathscr{T})$

证明

$$\left[R_0^{S\theta}(X,\mathscr{T})\right]=\inf_{x\ne y}\min\left(1,1-\max\left(\left(1-Cl_S^\theta\{y\}\right)(x),\left(1-Cl_S^\theta\{x\}\right)(y)\right)+\inf\mathscr{F}_S^\theta\left(\{x\}\right)\right)$$
$$\ge\inf_{x\ne y}\min\left(1,1-\max\left(\left(1-Cl_S^\theta\{y\}\right)(x),\left(1-Cl_S^\theta\{x\}\right)(y)\right)+\inf_{x\ne y}\min\left(\sup_{y\notin A}N_x^{\theta S}(A),\sup_{x\notin B}N_y^{\theta S}(B)\right)\right)$$
$$\ge\inf_{x\ne y}\min\left(\sup_{y\notin A}N_x^{\theta S}(A),\sup_{x\notin B}N_y^{\theta S}(B)\right)=\left[T_1^{S\theta}(X,\mathscr{T})\right]$$

定理 15 $\vDash R_0^{S\theta}(X,\mathscr{T}) \dot{\wedge} T_0^{S\theta}(X,\mathscr{T}) \to T_1^{S\theta}(X,\mathscr{T})$

证明

$$\begin{aligned}&\left[R_0^{S\theta}(X,\mathscr{T})\right]+\left[T_0^{S\theta}(X,\mathscr{T})\right]\\&=\inf_{x\neq y}\min\left(1,1-\max\left(\left(1-Cl_S^{\theta}\{y\}\right)(x),\left(1-Cl_S^{\theta}\{x\}\right)(y)\right)+\inf\mathcal{F}_S^{\theta}(\{x\})\right)\\&\quad+\inf_{x\neq y}\max\left(\left(1-Cl_S^{\theta}\{y\}\right)(x),\left(1-Cl_S^{\theta}\{x\}\right)(y)\right)\leq\inf_{x\neq y}\min\left(1,1+\inf\mathcal{F}_S^{\theta}(\{x\})\right)\\&\leq 1+\left[T_1^{S\theta}(X,\mathscr{T})\right]\end{aligned}$$

所以，$\vDash R_0^{S\theta}(X,\mathscr{T}) \dot{\wedge} T_0^{S\theta}(X,\mathscr{T}) \to T_1^{S\theta}(X,\mathscr{T})$

定理 16 $\vDash T_0^{S\theta}(X,\mathscr{T}) \to \left(R_0^{S\theta}(X,\mathscr{T}) \to T_1^{S\theta}(X,\mathscr{T})\right)$

证明

$$\begin{aligned}&\left[T_0^{S\theta}(X,\mathscr{T})\to\left(R_0^{S\theta}(X,\mathscr{T})\to T_1^{S\theta}(X,\mathscr{T})\right)\right]\\&=\min\left(1,1-\left[T_0^{S\theta}(X,\mathscr{T})\right]+\min\left(1,1-\left[R_0^{S\theta}(X,\mathscr{T})\right]+\left[T_1^{S\theta}(X,\mathscr{T})\right]\right)\right)\\&=\min\left(1,1-\left(\left[T_0^{S\theta}(X,\mathscr{T})\right]+\left[R_0^{S\theta}(X,\mathscr{T})\right]-1\right)+\left[T_1^{S\theta}(X,\mathscr{T})\right]\right)=1\end{aligned}$$

定理 17 $\vDash R_0^{S\theta}(X,\mathscr{T}) \to \left(T_0^{S\theta}(X,\mathscr{T}) \to T_1^{S\theta}(X,\mathscr{T})\right)$

证明 证明类似于定理 16。

定理 18 $\vDash T_2^{S\theta}(X,\mathscr{T}) \to R_1^{S\theta}(X,\mathscr{T})$

证明

$$\begin{aligned}\left[R_1^{S\theta}(X,\mathscr{T})\right]&=\inf_{x\neq y}\left(1,1-\max\left(\left(1-Cl_S^{\theta}\{y\}\right)(x),\left(1-Cl_S^{\theta}\{x\}\right)(y)\right)+\inf_{x\neq y}\sup_{A\cap B\neq\varnothing}\min\left(N_x^{\theta S}(A),N_x^{\theta S}(B)\right)\right)\\&\geq\inf_{x\neq y}\sup_{A\cap B=\varnothing}\min\left(N_{x_1}^{\theta S}(A),N_{x_2}^{\theta S}(B)\right)=\left[T_2^{S\theta}(X,\mathscr{T})\right]\end{aligned}$$

定理 19 $\vDash R_1^{S\theta}(X,\mathscr{T}) \dot{\wedge} T_0^{S\theta}(X,\mathscr{T}) \to T_2^{S\theta}(X,\mathscr{T})$

证明 证明类似于定理 15。

定理 20 $\vDash T_0^{S\theta}(X,\mathscr{T}) \to \left(R_1^{S\theta}(X,\mathscr{T}) \to T_2^{S\theta}(X,\mathscr{T})\right)$

证明 证明类似于定理 16。

定理 21 $\vDash R_1^{S\theta}(X,\mathscr{T}) \to \left(T_0^{S\theta}(X,\mathscr{T}) \to T_2^{S\theta}(X,\mathscr{T})\right)$

证明 证明类似于定理 16。

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