

Blowup of Solutions for a System of Doubly Nonlinear Degenerate Parabolic Equations with p-Laplacian

Longfei Qi, Jing Su, Qingying Hu

College of Science, Henan University of Technology, Zhengzhou Henan
Email: slxhqy@163.com

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Abstract

This paper is concerned with a system of doubly nonlinear degenerate parabolic equations with p-Laplacian. We prove that, under suitable conditions on the nonlinearity and certain initial datum, the lower bound for the blowup time is given if blowup does occur by using a modification of Levine's concavity method.

Keywords

Blowup of Solution, Doubly Nonlinear Parabolic Equations, Levine's Concavity Method

多重非线性退化的p-Laplacian抛物方程组解的爆破

齐龙飞, 苏 琛, 呼青英

河南工业大学理学院, 河南 郑州
Email: slxhqy@163.com

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摘要

本文研究了一类多重非线性退化的 p-Laplacian 抛物方程组解的爆破, 利用修正的 Levine 凸性方法, 在非线性和初始条件的适当条件下, 给出了解爆破时间的下界。

关键词

爆破, 多重非线性抛物方程组, Levine 凸性方法

1. 引言

本文研究如下非线性抛物方程组解的爆破性

$$\frac{\partial}{\partial t} (|u|^{m-2} u) - \Delta_{p_1} u = f_1(u, v), \quad (1.1)$$

$$\frac{\partial}{\partial t} (|v|^{m_2-2} v) - \Delta_{p_2} v = f_2(u, v), \quad (1.2)$$

$$u(x, t) = v(x, t) = 0, \quad x \in \partial\Omega, t \geq 0, \quad (1.3)$$

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad x \in \Omega, \quad (1.4)$$

其中 Ω 是 R^n ($n \geq 1$) 上的有界区域且有光滑边界 $\partial\Omega$, $\Delta_{p_i} u = \operatorname{div}(|\nabla u|^{p_i-2} \nabla u)$, $2 \leq p_1, p_2 < +\infty, m_1, m_2 > 2$, $f_i(u, v), i = 1, 2$ 为以后给定的函数, $u_0(x), v_0(x)$ 为正的初始函数。

近几十年来, 非线性抛物方程解的爆破问题引起人们的极大兴趣, 考虑爆破性自然要研究解是否会爆破以及解的爆破时间 t^* , 在这方面已经有大量的文献, 如文献[1]-[5]。问题(1.1)~(1.4)可以描述诸多化学反应、热传导过程和种群动力学过程(见文献[6]-[17]及其参考文献), 它有多项非线性项, 处理难度较大。对型如(1.1)的单个多重非线性抛物方程

$$\frac{\partial}{\partial t} (|u|^{m-2} u) - \Delta u = f(u), \quad (1.5)$$

已有许多结果, 如文献[6]-[10]给出了方程(1.5)的初边值问题和初值问题局部和整体解的存在性, 文献[11]-[14]则研究了其整体吸引子的存在性和正则性, 类似方程(1.1), (1.2)的方程组问题的整体吸引子的存在性和正则性也有一些研究[15]-[17]。但关于该类问题解的爆破性研究则相对较少, Iami 和 Mochizuki [18]给出了 Neumann 初边值条件下方程(1.5)的爆破条件, Levine [19]-[21]则用凸性方法证明了单个方程(1.1)及其等价的如下问题解的爆破性

$$\frac{\partial u}{\partial t} - \Delta \varphi(u) = f(u).$$

Korpusov 和 Svishnikov [22] [23]及 Polat [24]则对如下方程的初边值问题

$$(u - \Delta u + |u|^{k-2} u)_t - \Delta u = f(u)$$

在负初始能量时得到解的爆破条件。Wang 和 Ge [25]及其参考文献则利用比较原理讨论了非线性边界时方程(1.1), (1.2)中非线性项为 $u^\alpha v^\beta$ 时解的爆破问题。

本文利用修正的 Levine 凸性方法讨论问题(1.1)~(1.4)解的爆破条件, 推广了文献[22]-[25]的结果。在第二节将给出一些假设和基本引理, 第三节给出主要结果和证明。

2. 假设和引理

本文所用符号均同文献[6], 记 $W_0^{1,p}(\Omega)$ 和 $L^m(\Omega)$ 为通常的 Soblev 空间, 其范数分别记为 $\|\cdot\|_{W_0^{1,p}} = \|\nabla\cdot\|_p$ 和 $\|\cdot\|_m$, 特别是当 $m=2$ 时, 记 $\|\cdot\|_m = \|\cdot\|$ 。

本文始终假设 $2 \leq p_1, p_2 < +\infty, m_1, m_2 > 2$ 。关于非线性项 $f_i(u, v), i=1, 2$ 的假设如下:

(A1) $f_i(u, v) \in C^1(\mathbb{R}^2), i=1, 2$, 存在函数 $F(u, v) \in C^2(\mathbb{R}^2)$ 使得

$$f_1(u, v) = \frac{\partial F(u, v)}{\partial u}, \quad f_2(u, v) = \frac{\partial F(u, v)}{\partial v},$$

且存在常数 $\beta_0 > 2, \beta_1 > 0$ 使得

$$\beta_1(|u|^{p+1} + |v|^{p+1}) \leq F(u, v) \leq \frac{1}{\beta_0}(uf_1(u, v) + vf_2(u, v)),$$

其中, $p > 0$ 当 $n=1, 2$ 时, $0 < p < \frac{n+2}{n-2}$ 当 $n > 2$ 时。

注: 满足条件(A1)的函数是存在的。事实上, 一个典型的例子是取

$$F(u, v) = a|u+v|^{p+1} + 2b|uv|^{\frac{p+1}{2}}$$

且 $f_1(u, v) = \frac{\partial F(u, v)}{\partial u}, f_2(u, v) = \frac{\partial F(u, v)}{\partial v}$, 即

$$f_1(u, v) = (p+1)\left(a|u+v|^{p-1}(u+v) + b|u|^{\frac{p-3}{2}}|v|^{\frac{p+1}{2}}\right),$$

$$f_2(u, v) = (p+1)\left(a|u+v|^{p-1}(u+v) + b|v|^{\frac{p-3}{2}}|u|^{\frac{p+1}{2}}\right),$$

这时, $(p+1)F(u, v) = uf_1(u, v) + vf_2(u, v)$, 其中 $a > 0, b > 0, p \geq 1, \beta_0 = p+1$ 。该例的详细情况可见文献[26]。

利用 Galerkin 方法, 结合单调性理论和紧性方法[2], 类似文献[27]可得问题(1.1)~(1.4)解的局部存在性。

定理 2.1. 假设条件(A1)成立且 $\min(p_1, p_2) > n(p+1)/(n + \max(m_1, m_2))$, $\max(m_1, m_2) \leq p+1$, $u_0 \in W_0^{1,p_1}, v_0 \in W_0^{1,p_2}$, 则问题(1.1)~(1.4)存在弱解 (u, v) , 即, 存在 $T > 0$ 使得

$$u \in L^\infty(0, T; W_0^{1,p_1}), \quad v \in L^\infty(0, T; W_0^{1,p_2}), \quad \left(|u|^{\frac{m_1}{2}}\right)_t, \left(|v|^{\frac{m_2}{2}}\right)_t \in L^2((0, T) \times \Omega),$$

且对任意 $\varphi(t) \in D(0, T)$ 和任意分别的 $\phi(x) \in W_0^{1,p_1}$ (或 W_0^{1,p_2}) 成立:

$$\int_0^T \left[\int_\Omega \left[\frac{2(m_1-1)}{m_1} |u|^{m_1/2-2} u \left(|u|^{m_1/2}\right)_t \phi + |\nabla u|^{p_1-2} \nabla u \nabla \phi - f_1(u, v) \phi \right] dx \varphi(t) \right] dt = 0,$$

$$\int_0^T \left[\int_\Omega \left[\frac{2(m_2-1)}{m_2} |v|^{m_2/2-2} v \left(|v|^{m_2/2}\right)_t \phi + |\nabla v|^{p_2-2} \nabla v \nabla \phi - f_2(u, v) \phi \right] dx \varphi(t) \right] dt = 0,$$

以及 $u(x, 0) = u_0 \in W_0^{1,p_1}, v(x, 0) = v_0 \in W_0^{1,p_2}$ 。

下面给出本文的基本引理。

引理 2.2. [5] [27] 设 $H(t)$ 是 \mathbb{R} 上非负二次连续可导函数且满足不等式

$$\Phi''(t)\Phi(t) - \alpha[\Phi'(t)]^2 + \beta\Phi(t) \geq 0$$

其中 $\alpha > 1, \beta > 0$ 为常数。若 $\Phi(0) > 0$, $\Phi'(0) > 0, [\Phi'(0)]^2 > \frac{2\beta}{2\alpha-1}\Phi(0)$, 则必存在时刻 $T < T_1 = A^{-1}\Phi^{1-\alpha}(0)$, 使当 $t \rightarrow T^-$ 时有 $\Phi(t) \rightarrow +\infty$, 其中

$$A^2 = (\alpha - 1)^2 \Phi^{-2\alpha}(0) \left[(\Phi'(0))^2 - \frac{2\beta}{2\alpha - 1} \Phi(0) \right] > 0.$$

3. 主要结果及证明

首先引入泛函

$$\Phi(t) = \int_0^t \left(\frac{m_1 - 1}{m_1} \|u\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v\|_{m_2}^{m_2} \right) ds + \frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2}, \quad (3.1)$$

$$J(t) = \int_0^t \int_{\Omega} \left[(m_1 - 1) |u|^{m_1 - 2} u_t^2 + (m_2 - 1) |v|^{m_2 - 2} v_t^2 \right] dx ds + \frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2}, \quad (3.2)$$

$$E(t) = \frac{m_1 - 1}{m_1} \|u\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v\|_{m_2}^{m_2} + \frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} - \int_{\Omega} F(u, v) dx, \quad (3.3)$$

$$E(0) = \frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2} + \frac{1}{p_1} \|\nabla u_0\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v_0\|_{p_2}^{p_2} - \int_{\Omega} F(u_0, v_0) dx. \quad (3.4)$$

现给出主要引理。

引理 3.1. 对任意 $t \in [0, T)$, 下面不等式成立

$$[\Phi'(t)]^2 \leq (m_1 + m_2) \Phi(t) J(t). \quad (3.5)$$

证明 注意到

$$\begin{aligned} \Phi'(t) &= \frac{m_1 - 1}{m_1} \|u\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v\|_{m_2}^{m_2} \\ &= \int_0^t \frac{d}{ds} \left[\frac{m_1 - 1}{m_1} \|u\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v\|_{m_2}^{m_2} \right] ds + \frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2}, \end{aligned} \quad (3.6)$$

注意到由 Holder 不等式得下列不等式

$$\left| \int_0^t \frac{m_1 - 1}{m_1} \frac{d}{ds} \|u\|_{m_1}^{m_1} ds \right| \leq (m_1 - 1) \int_0^t \int_{\Omega} |u|^{\frac{m_1}{2}} \left(|u|^{\frac{m_1 - 2}{2}} u_t \right) dx ds \leq (m_1 - 1) \left(\int_0^t \|u\|_{m_1}^{m_1} ds \right)^{\frac{1}{2}} \left(\int_0^t \int_{\Omega} |u|^{m_1 - 2} |u_t|^2 dx ds \right)^{\frac{1}{2}}, \quad (3.7)$$

$$\left| \int_0^t \frac{m_2 - 1}{m_2} \frac{d}{ds} \|v\|_{m_2}^{m_2} ds \right| \leq (m_2 - 1) \left(\int_0^t \|v\|_{m_2}^{m_2} ds \right)^{\frac{1}{2}} \left(\int_0^t \int_{\Omega} |v|^{m_2 - 2} |v_t|^2 dx ds \right)^{\frac{1}{2}}. \quad (3.8)$$

考虑到(3.7)~(3.8), 则由(3.6)得

$$\begin{aligned} \Phi'(t) &\leq (m_1 - 1) \left(\int_0^t \|u\|_{m_1}^{m_1} ds \right)^{\frac{1}{2}} \left(\int_0^t \int_{\Omega} |u|^{m_1 - 2} |u_t|^2 dx ds \right)^{\frac{1}{2}} \\ &\quad + (m_2 - 1) \left(\int_0^t \|v\|_{m_2}^{m_2} ds \right)^{\frac{1}{2}} \left(\int_0^t \int_{\Omega} |v|^{m_2 - 2} |v_t|^2 dx ds \right)^{\frac{1}{2}} \\ &\quad + \frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2}, \end{aligned}$$

再利用不等式

$$\left(\sum_{i=1}^k a_i b_i\right)^2 \leq \left(\sum_{i=1}^k a_i^2\right)\left(\sum_{i=1}^k b_i^2\right),$$

得

$$\begin{aligned} (\Phi'(t))^2 &\leq \left[\int_0^t \left((m_1 - 1) \|u\|_{m_1}^{m_1} + (m_2 - 1) \|v\|_{m_2}^{m_2} \right) ds + \frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2} \right] \\ &\quad \cdot \left[\int_0^t \int_{\Omega} \left((m_1 - 1) |u|^{m_1 - 2} |u_t|^2 + (m_2 - 1) |v|^{m_2 - 2} |v_t|^2 \right) dx ds + \frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2} \right] \\ &\leq (m_1 + m_2) \left[\int_0^t \left(\frac{m_1 - 1}{m_1 + m_2} \|u\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_1 + m_2} \|v\|_{m_2}^{m_2} \right) ds + \frac{m_1 - 1}{m_1(m_1 + m_2)} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2(m_1 + m_2)} \|v_0\|_{m_2}^{m_2} \right] J \\ &\leq (m_1 + m_2) \Phi J. \end{aligned}$$

于是, 引理得证。

下面, 给出主要定理。

定理 3.2. 设定理 2.1 的条件成立, 且

$$\left[4(m_1 + m_2) - 5 \right] \left[\frac{m_1 - 1}{m_1} \|u_0\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \|v_0\|_{m_2}^{m_2} \right] \geq 4 \left[\frac{1}{p_1} \|\nabla u_0\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v_0\|_{p_2}^{p_2} - \int_{\Omega} F(u_0, v_0) dx \right], \quad (3.9)$$

则问题(1.1)~(1.4)的弱解 (u, v) 必在某有限时刻 $T < T_1 = A^{-1} \Phi^{1-\alpha}(0)$ 爆破, 即

$$\limsup_{t \rightarrow T} \left(\frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} \right) = +\infty.$$

证明 注意到

$$\begin{aligned} \frac{m_1 - 1}{m_1} \frac{d}{dt} |u|^{m_1} &= (m_1 - 1) u |u|^{m_1 - 2} u_t = u \frac{d}{dt} (u |u|^{m_1 - 2}), \\ \frac{m_2 - 1}{m_2} \frac{d}{dt} |v|^{m_2} &= (m_2 - 1) v |v|^{m_2 - 2} v_t = v \frac{d}{dt} (v |v|^{m_2 - 2}), \end{aligned}$$

由(3.1), 得

$$\Phi''(t) = \frac{m_1 - 1}{m_1} \frac{d}{dt} \|u\|_{m_1}^{m_1} + \frac{m_2 - 1}{m_2} \frac{d}{dt} \|v\|_{m_2}^{m_2} = \int_{\Omega} \left(u \frac{d}{dt} (u |u|^{m_1 - 2}) + v \frac{d}{dt} (v |v|^{m_2 - 2}) \right) dx.$$

把(1.1), (1.2)代入得

$$\Phi''(t) + \|\nabla u\|_{p_1}^{p_1} + \|\nabla v\|_{p_2}^{p_2} = \int_{\Omega} (f_1(u, v)u + f_2(u, v)v) dx, \quad (3.10)$$

利用

$$\frac{d}{dt} (u |u|^{m_1 - 2}) = (m_1 - 1) |u|^{m_1 - 2} u_t, \quad \frac{d}{dt} (v |v|^{m_2 - 2}) = (m_2 - 1) |v|^{m_2 - 2} v_t,$$

由(3.2)及方程(1.1), (1.2)得

$$\begin{aligned} J'(t) &= (m_1 - 1) \int_{\Omega} |u|^{m_1 - 2} |u_t|^2 dx + (m_2 - 1) \int_{\Omega} |v|^{m_2 - 2} |v_t|^2 dx \\ &= \int_{\Omega} u_t (|u|^{m_1 - 2} u)_t dx + \int_{\Omega} v_t (|v|^{m_2 - 2} v)_t dx \\ &= - \int_{\Omega} |\nabla u|^{p_1 - 2} \nabla u \nabla u_t dx - \int_{\Omega} |\nabla v|^{p_2 - 2} \nabla v \nabla v_t dx \\ &\quad + \int_{\Omega} (f_1(u, v)u_t + f_2(u, v)v_t) dx, \end{aligned}$$

即

$$J'(t) + \frac{d}{dt} \left(\frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} \right) = \frac{d}{dt} \int_{\Omega} F(u, v) dx, \quad (3.11)$$

(3.11)关于 t 积分得

$$J(t) + \frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} - E(0) = \int_{\Omega} F(u, v) dx, \quad (3.12)$$

再利用条件(A1)得

$$\beta_0 \left(J(t) + \frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} - E(0) \right) \leq \int_{\Omega} (f_1(u, v)u + f_2(u, v)v) dx, \quad (3.13)$$

(3.10)结合(3.13), 并用到 $\beta_0 > 2$, 得

$$\begin{aligned} \Phi''(t) + \frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} &\geq 2 \left(J(t) + \frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} - E(0) \right) \\ &\geq 2J(t) + \|\nabla u\|^2 + \|\nabla v\|^2 - 2E(0), \end{aligned}$$

即

$$\Phi''(t) - 2J(t) + 2E(0) \geq 0, \quad (3.14)$$

注意到 $\Phi(t) \geq 0$ 得

$$\Phi''(t)\Phi(t) - 2J(t)\Phi(t) + 2E(0)\Phi(t) \geq 0, \quad (3.15)$$

利用引理 3.1, 得

$$\Phi''(t)\Phi(t) - \alpha [\Phi'(t)]^2 + \beta\Phi(t) \geq 0,$$

其中 $\alpha = 2(m_1 + m_2), \beta = 2E(0)$ 。

如果 $E(0) > 0$, 由(3.1), (3.6)和(3.9)知 $\Phi(0) > 0, \Phi'(0) > 0, [\Phi'(0)]^2 > \frac{2\beta}{2\alpha - 1}\Phi(0)$, 于是, 由引理 2.2 得结论。如果 $E(0) \leq 0$, 取 $\beta = 0$, 则(3.15)变为

$$\Phi(t)\Phi(t) - \alpha [\Phi'(t)]^2 \geq 0, \quad (3.16)$$

于是, 由(3.16)和标准的凸性引理得结论。综合两种情况即得 $\limsup_{t \rightarrow \tau} \left(\frac{1}{p_1} \|\nabla u\|_{p_1}^{p_1} + \frac{1}{p_2} \|\nabla v\|_{p_2}^{p_2} \right) = +\infty$ 。

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